Y2S2 XMQs and MS

(Total: 65 marks)

1. P3_Sample Q4 . 10 marks - Y2S2 Conditional probability

2. P3_Specimen Q4 . 11 marks - Y2S2 Conditional probability

3. P31_2019 Q1 . 8 marks - Y1S5 Probability

4. P31_2020 Q1 . 8 marks - Y2S2 Conditional probability

5. P31_2021 Q1 . 7 marks - Y1S6 Statistical distributions

6. P31_2021 Q4 . 11 marks - Y2S2 Conditional probability

7. P31_2022 Q5 . 10 marks - Y2S2 Conditional probability

4. Given that

$$P(A) = 0.35 \qquad P(B) = 0.45 \qquad \text{and} \qquad P(A \cap B) = 0.13$$

find

(a) $P(A' \mid B')$

(2)

(b) Explain why the events A and B are not independent.

(1)

The event C has $P(C) = 0.20$

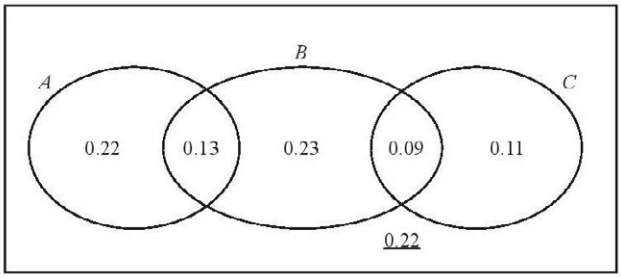
The events A and C are mutually exclusive and the events B and C are statistically independent.

(c) Draw a Venn diagram to illustrate the events A, B and C, giving the probabilities for each region.

(5)

(d) Find $P([B \cup C]')$

(2)

Question	Scheme	Marks	AOs
4(a)	$\mathrm{P}(A' \mid B') = \dfrac{\mathrm{P}(A' \cap B')}{\mathrm{P}(B')}$ or $\dfrac{0.33}{0.55}$	M1	3.1a
	$= \dfrac{3}{5}$ or 0.6	A1	1.1b
		(2)	
(b)	e.g. $\mathrm{P}(A) \times \mathrm{P}(B) = \frac{7}{20} \times \frac{9}{20} = \frac{63}{400} \neq \mathrm{P}(A \cap B) = 0.13 = \frac{52}{400}$ or $\mathrm{P}(A' \mid B') = 0.6 \neq \mathrm{P}(A') = 0.65$	B1	2.4
		(1)	
(c)		B1	2.5
		M1	3.1a
		A1	1.1b
		M1	1.1b
		A1	1.1b
		(5)	
(d)	$\mathrm{P}(B \cup C)' = 0.22 + 0.22$ or $1-[0.56]$ or $1-[0.13+0.23+0.09+0.11]$ o.e.	M1	1.1b
	= 0.44	A1	1.1b
		(2)	
			(10 marks)

Notes:

(a)

M1: for a correct ratio of probabilities formula and at least one correct value.

A1: a correct answer

(b)

for a fully correct explanation: correct probabilities and correct comparisons.

(c)

B1: for box with B intersecting A and C but C not intersecting A.(Or accept three intersecting circles, but with zeros entered for $A \cap C$ and $A \cap B \cap C$)No box is B0

M1: for method for finding P($B \cap C$)

A1: for 0.09

M1: for 0.13 and their 0.09 in correct places and method for their 0.23

A1: fully correct

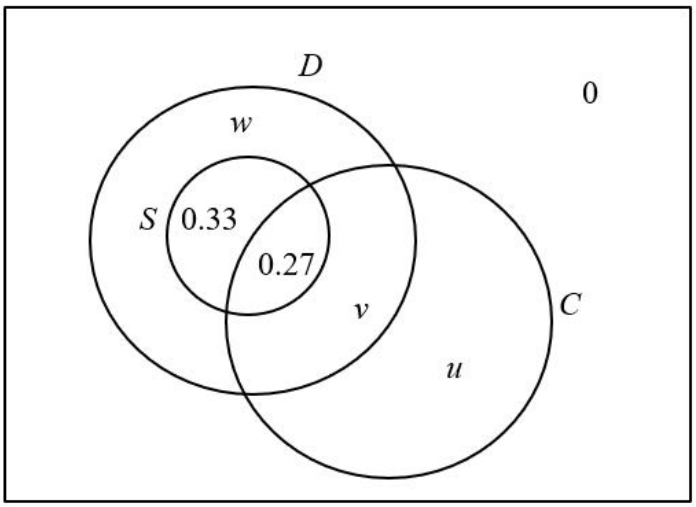
(d)

M1: for a correct expression – ft their probabilities from their Venn diagram.

A1: cao

4. The Venn diagram shows the probabilities of students' lunch boxes containing a drink, sandwiches and a chocolate bar.

D is the event that a lunch box contains a drink,
S is the event that a lunch box contains sandwiches,
C is the event that a lunch box contains a chocolate bar,
u, v and w are probabilities.



(a) Write down $P(S \cap D')$ **(1)**

One day, 80 students each bring in a lunch box.
Given that all 80 lunch boxes contain sandwiches and a drink,

(b) estimate how many of these 80 lunch boxes will contain a chocolate bar. **(3)**

Given that the events S and C are independent and that $P(D \mid C) = \frac{14}{15}$

(c) calculate the value of u, the value of v and the value of w. **(7)**

Question	Scheme	Marks	AOs
4(a)	$P(S \cap D') = 0$	B1	1.1b
		(1)	
(b)	$P(C \mid S \cap D) = \frac{0.27}{0.6} = \frac{9}{20} = 0.45$	M1	3.1b
	$\therefore 80 \times "0.45"$	M1	1.1b
	$=36$	A1	1.1b
		(3)	
(c)	$[P(C) \times P(S) = P(C \cap S)]$		
	$P(S) = 0.6$, $P(C) = 0.27 + v + u$, $P(S \cap C) = 0.27$	M1	3.1a
	$0.6 \times (0.27 + u + v) = 0.27$ or $u + v = 0.18$ o.e	A1	1.1b
	$\left[P(D \mid C) = \frac{P(D \cap C)}{P(C)} \right]$ $P(D \cap C) = 0.27 + v$	M1	3.1a
	$\frac{14}{15} = \frac{0.27 + v}{0.27 + v + u}$ or $14u - v = 0.27$ o.e	A1	1.1b
	$15u = 0.45$	M1dd	1.1b
	$u = 0.03$ $v = 0.15$	A1	1.1b
	$w = 0.22$	A1ft	1.1b
		(7)	
			(11 marks)

Notes:

(a) B1: correct answer only

(b) M1: for a correct ratio of probabilities formula with at least one correct value and multiplying by 80

A1: a correct answer

(c) M1: for translating the problem and realising the equation $P(C) \times P(S) = P(C \cap S)$ needs to be used with at least 2 parts correct.

A1: a correct equation

M1: for a correct probability formula with $P(D \cap C) = 0.27 + v$

A1: a second correct equation

M1dd: dependent on the previous 2 method marks being awarded. Solving the two simultaneous equations by eliminating one variable. May be implied by either *u* or *v* correct

A1: *u* correct

A1: *v* correct

A1ft: w = 0.22, ft *their* u, v provided that $u + v + w < 0.4$

Answer ALL questions. Write your answers in the spaces provided.

1. Three bags, *A*, *B* and *C*, each contain 1 red marble and some green marbles.

Bag *A* contains 1 red marble and 9 green marbles only
Bag *B* contains 1 red marble and 4 green marbles only
Bag *C* contains 1 red marble and 2 green marbles only

Sasha selects at random one marble from bag *A*.
If he selects a red marble, he stops selecting.
If the marble is green, he continues by selecting at random one marble from bag *B*.
If he selects a red marble, he stops selecting.
If the marble is green, he continues by selecting at random one marble from bag *C*.

(a) Draw a tree diagram to represent this information.

(2)

(b) Find the probability that Sasha selects 3 green marbles.

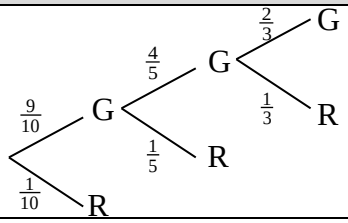
(2)

(c) Find the probability that Sasha selects at least 1 marble of each colour.

(2)

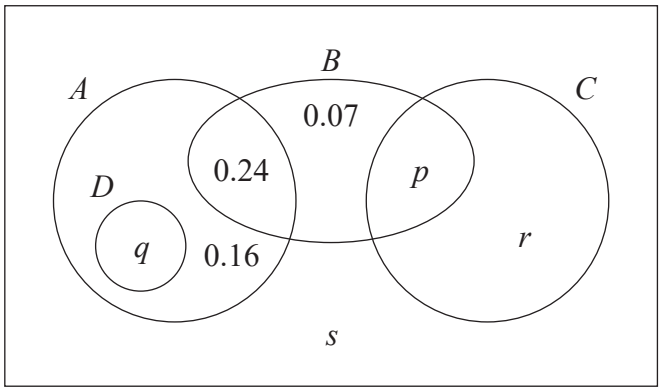
(d) Given that Sasha selects a red marble, find the probability that he selects it from bag *B*.

(2)

Question	Scheme	Marks	AOs
1(a)		B1 dB1	1.1b 1.1b
		(2)	
(b)	$\frac{9}{10}\times\frac{4}{5}\times\frac{2}{3}$	M1	1.1b
	$=\frac{12}{25}$ (= 0.48)	A1	1.1b
		(2)	
(c)	$\frac{9}{10}\times\frac{1}{5}+\frac{9}{10}\times\frac{4}{5}\times\frac{1}{3}$ or $1-\left(\frac{1}{10}+\frac{9}{10}\times\frac{4}{5}\times\frac{2}{3}\right)$	M1	3.1b
	$=\frac{21}{50}$ (= 0.42)	A1	1.1b
		(2)	
(d)	[P(Red from *B*|Red selected)] $=\frac{\frac{9}{10}\times\frac{1}{5}}{\frac{1}{10}+\frac{9}{10}\times\frac{1}{5}+\frac{9}{10}\times\frac{4}{5}\times\frac{1}{3}}\left[=\frac{\frac{9}{50}}{\frac{13}{25}}\right]$	M1	3.1b
	$=\frac{9}{26}$	A1	1.1b
		(2)	
			(8 marks)

Notes	
	Allow decimals or percentages throughout this question.
(a)	**B1:** for correct shape (3 pairs) and at least **one** label on **at least two** pairs G(reen) and R(ed) allow G and G' **or** R and R' as labels, etc. condone 'extra' pairs if they are labelled with a probability of 0 **dB1:** (dep on previous B1) all correct i.e. for all 6 correct probabilities on the correct branches with at least one label on **each** pair
(b)	**M1:** Multiplication of 3 correct probabilities (allow ft from their tree diagram) **A1:** $\frac{12}{25}$ oe
(c)	**M1:** Either addition of only two correct products (product of two probs + product of three probs) which may ft from their tree diagram or for $1-('\frac{1}{10}'+'(b)')$ **A1:** $\frac{21}{50}$ oe
(d)	**M1:** Correct ratio of probabilities **or** correct ft ratio of probabilities e.g. $\frac{'\frac{9}{10}'\times'\frac{1}{5}'}{1-'(b)'}$ or $\frac{'\frac{9}{10}'\times'\frac{1}{5}'}{'\frac{1}{10}'+'(c)'}$ with num < den **A1:** $\frac{9}{26}$ (allow awrt 0.346)

1. The Venn diagram shows the probabilities associated with four events, A, B, C and D



(a) Write down any pair of mutually exclusive events from A, B, C and D

(1)

Given that $P(B) = 0.4$

(b) find the value of p

(1)

Given also that A and B are independent

(c) find the value of q

(2)

Given further that $P(B' | C) = 0.64$

(d) find

(i) the value of r

(ii) the value of s

(4)

Qu 1	Scheme	Marks	AO
(a)	*A*, *C* <u>or</u> *D*, *B* <u>or</u> *D*,*C*	B1 **(1)**	1.2
(b)	[$p = 0.4 - 0.07 - 0.24 =$] **<u>0.09</u>**	B1 **(1)**	1.1b
(c)	*A* and *B* independent implies		1.1b
	$P(A)\times 0.4 = 0.24$ <u>or</u> $(q + 0.16 + 0.24)\times 0.4 = 0.24$	M1	
	so P(*A*) = 0.6 and *q* = **<u>0.20</u>**	A1cso **(2)**	1.1b
(d)(i)	$P(B' \mid C) = 0.64$ gives $\frac{r}{r+p} = 0.64$ <u>or</u> $\frac{r}{r+"0.09"} = 0.64$	M1	3.1a
	$r = 0.64r + 0.64$ "*p*" so $0.36r = 0.0576$ so *r* = **<u>0.16</u>**	A1	1.1b
(ii)	Using sum of probabilities = 1 e.g. "0.6" + 0.07 + "0.25" + *s* =1	M1	1.1b
	so *s* = **<u>0.08</u>**	A1 **(4)**	1.1b
		(8 marks)	

Notes

(a) B1 for one correct pair. If more than one pair they must all be correct.
Condone in a correct probability statement such as $P(A \cap C) = 0$
or correct use of set notation e.g. $A \cap C = \varnothing$
BUT e.g. "P(*A*) and P(*C*) are mutually exclusive" alone is B0

(b) B1 for $p = 0.09$ (Maybe stated in Venn Diagram [VD])
[If values in VD and text conflict, take text or a value <u>used</u> in a later part]

(c) M1 for a correct equation in one variable for P(*A*) or *q* using independence
<u>or</u> for seeing **both** $P(A \cap B) = P(A)\times P(B)$ <u>and</u> $0.24 = 0.6\times 0.4$
A1cso for *q* = 0.20 or exact equivalent (dep on correct use of independence)
Beware Use of P(*A*) = 1 – P(*B*) = 0.6 leading to *q* = 0.2 scores M0A0

(d)(i) 1st M1 for use of $P(B' \mid C) = 0.64$ leading to a correct equation in *r* and possibly *p*.
Can ft their *p* provided $0 < p < 1$
1st A1 for *r* = 0.16 or exact equivalent
(ii) 2nd M1 for use of total probability = 1 to form a linear equation in *s*. Allow *p*, *q*, *r* etc
Can follow through their values provided each of *p*, *q*, *r* are in [0, 1)
2nd A1 for *s* = 0.08 or exact equivalent

1. (a) State one disadvantage of using quota sampling compared with simple random sampling.

(1)

In a university 8% of students are members of the university dance club.

A random sample of 36 students is taken from the university.

The random variable X represents the number of these students who are members of the dance club.

(b) Using a suitable model for X, find

(i) $P(X = 4)$

(ii) $P(X \geqslant 7)$

(3)

Only 40% of the university dance club members can dance the tango.

(c) Find the probability that a student is a member of the university dance club and can dance the tango.

(1)

A random sample of 50 students is taken from the university.

(d) Find the probability that fewer than 3 of these students are members of the university dance club and can dance the tango.

(2)

Qu 1	Scheme	Marks	AO
(a)	**Disadvantage:** e.g. Not random; cannot use (reliably) for inferences	B1 **(1)**	1.1b
(b)	[Sight or correct use of] $X \sim B(36, 0.08)$	M1	3.3
(i)	$P(X = 4) = 0.167387\ldots$ awrt **0.167**	A1	1.1b
(ii)	$[P(X \geqslant 7) = 1 - P(X \leqslant 6) =]$ 0.022233… awrt **0.0222**	A1 **(3)**	1.1b
(c)	P(In dance club and dance tango) = 0.4×0.08 = **0.032** or $\frac{4}{125}$ or **3.2%**	B1 **(1)**	1.1b
(d)	[Let T = those who can dance the Tango. Sight or use of] $T \sim B(50, \text{“}0.032\text{”})$	M1	3.3
	$[P(T < 3) = P(T \leqslant 2) =]$ 0.7850815… awrt **0.785**	A1 **(2)**	1.1b
		(7 marks)	

Notes

(a) B1 for a suitable disadvantage:

Allow (B1)	Do NOT allow (B0)
Not random or less random (o.e.)	Not representative
Cannot use (reliably) for inferences	Less accurate
(More likely to be) biased	Any comment based on time or cost
	Any mention of skew
	Any mention of non-response

(b) M1 for sight of B(36, 0.08) Allow in words: binomial with $n = 36$ and $p = 0.08$
may be implied by one correct answer to 2sf or sight of $P(X \leqslant 6) = 0.97776...$ i.e. awrt 0.98
Allow for $36C4 \times 0.08^4 \times 0.92^{32}$ as this is "correct use"

(i) 1st A1 for awrt 0.167 NB An answer of just awrt 0.167 scores M1($\Rightarrow$)1st A1

(ii) 2nd A1 for awrt 0.0222

(c) B1 for 0.032 o.e. (Can allow for sight of 0.4×0.08)

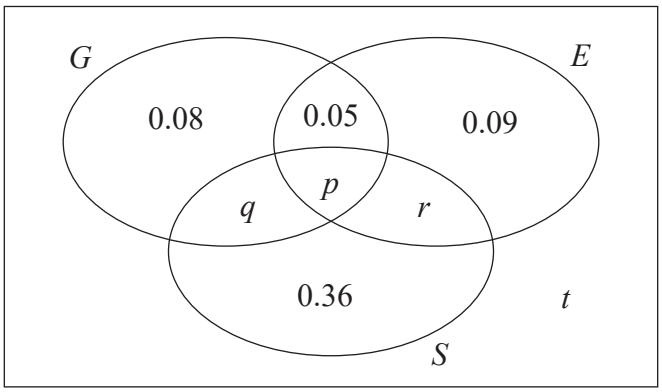
(d) M1 for sight of B(50, “0.032”) ft their answer to (c) provided it is a probability $\neq 0.08$
may be implied by correct answer
or sight of $[P(T \leqslant 3)] = 0.924348\ldots$ i.e. awrt 0.924 or $P(T \leqslant 2)$ as part of $1 - P(T \leqslant 2)$ calc.
A1 for awrt 0.785

MR Allow MR of 50 (e.g. 30) provided clearly attempting $P(T \leqslant 2)$ and score M1A0

4. A large college produces three magazines.
One magazine is about green issues, one is about equality and one is about sports.
A student at the college is selected at random and the events G, E and S are defined as follows

G is the event that the student reads the magazine about green issues
E is the event that the student reads the magazine about equality
S is the event that the student reads the magazine about sports

The Venn diagram, where p, q, r and t are probabilities, gives the probability for each subset.



(a) Find the proportion of students in the college who read exactly one of these magazines.

(1)

No students read all three magazines and $P(G) = 0.25$

(b) Find

(i) the value of p

(ii) the value of q

(3)

Given that $P(S \mid E) = \frac{5}{12}$

(c) find

(i) the value of r

(ii) the value of t

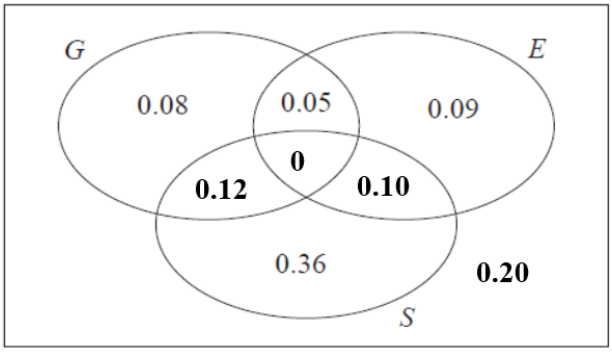
(4)

(d) Determine whether or not the events $(S \cap E')$ and G are independent.
Show your working clearly.

(3)

Qu 4	Scheme	Marks	AO
(a)	$0.08 + 0.09 + 0.36 =$ **<u>0.53</u>**	B1	1.1b
		(1)	
(b)(i)	$\left[\mathrm{P}(G \cap E \cap S) = 0 \ \Rightarrow\right]$ **<u>*p* = 0</u>**	B1	1.1b
(ii)	$[\mathrm{P}(G) = 0.25 \Rightarrow]\ 0.08 + 0.05 + q + "p" = 0.25$	M1	1.1b
	<u>*q* = 0.12</u>	A1	1.1b
		(3)	
(c)(i)	$\left[\mathrm{P}(S \mid E) = \frac{5}{12} \ \Rightarrow\right] \frac{r + "p"}{r + "p" + 0.09 + 0.05} = \frac{5}{12}$	M1	3.1a
		A1ft	1.1b
	$[12r = 5r + 5 \times 0.14 \ \Rightarrow]$ **<u>*r* = 0.10</u>**	A1	1.1b
(ii)	$[0.08 + 0.05 + "0.12" + "0" + 0.09 + "0.10" + 0.36 + t = 1 \ \Rightarrow]$ **<u>*t* = 0.20</u>**	B1ft	1.1b
		(4)	
(d)	$\mathrm{P}(S \cap E')$ = 0.36 + "*q*" [= 0.48]	B1ft	1.1b
	$\mathrm{P}\left(\left[(S \cap E')\right] \cap G\right) = "q"$ [=0.12] **and** P(*G*) = 0.25 **and**	M1	2.1
	$\mathrm{P}(S \cap E') \times \mathrm{P}(G) = "0.48" \times \frac{1}{4}$ <u>or</u> 0.12		
	$\mathrm{P}(S \cap E') \times \mathrm{P}(G) = 0.12 = \mathrm{P}\left(\left[(S \cap E')\right] \cap G\right)$ so **are independent**	A1	2.2a
		(3)	
		(11 marks)	

	Notes
(a)	B1 for 0.53 (or exact equivalent) [Allow 53%]
(b)(i)	B1 for *p* = 0 (may be placed in Venn diagram)
(ii)	M1 for a linear equation for *q* (ft letter "*p*" or their value if $0 \leqslant p \leqslant 0.12$) $\Rightarrow$ by *p* + *q* = 0.12
	A1 for *q* = 0.12 (may be placed in Venn diagram)
(c)(i)	M1 for a ratio of probabilities (*r* on num and den) (on LHS) with num < den **and** num <u>or</u> den correct ft. Allow ft of letter "*p*" <u>or</u> their *p* where $0 \leqslant p < 0.86$ but "+ 0" is not required.
	1st A1ft for a correct ratio of probabilities (on LHS) allowing ft of their *p* where $0 \leqslant p < 0.86$
	2nd A1 for *r* = 0.1(0) or exact equivalent (may be in Venn diagram) **Ans only** 3/3
(ii)	B1ft for *t* = 0.2(0) (o.e.) <u>or</u> correct ft i.e. 0.42 – (*p* + *q* + *r*) where *p*, *q*, *r* and *t* are all probs
(d)	B1ft for $\mathrm{P}(S \cap E')$ = 0.48 (with label) (ft letter "*q*" or their value if $0 \leqslant q \leqslant 0.12$)
	M1 for attempting all required probs (labelled) <u>and</u> using them in a correct test (allow ft of *q*)
	A1 for **all probs correct** and a correct deduction (no ft deduction here)
SC	**No "P"** If correct argument seen apart from P for probability for all 3 marks, award (B0M1A1)
	If unsure about an attempt using conditional probabilities, please send to review.



5. A company has 1825 employees.
The employees are classified as professional, skilled or elementary.

The following table shows

- the number of employees in each classification
- the two areas, *A* or *B*, where the employees live

	A	***B***
Professional	740	380
Skilled	275	90
Elementary	260	80

An employee is chosen at random.

Find the probability that this employee

(a) is skilled,

(1)

(b) lives in area *B* and is not a professional.

(1)

Some classifications of employees are more likely to work from home.

- 65% of professional employees in both area *A* and area *B* work from home
- 40% of skilled employees in both area *A* and area *B* work from home
- 5% of elementary employees in both area *A* and area *B* work from home
- Event *F* is that the employee is a professional
- Event *H* is that the employee works from home
- Event *R* is that the employee is from area *A*

(c) Using this information, complete the Venn diagram on the opposite page.

(4)

(d) Find $P(R' \cap F)$

(1)

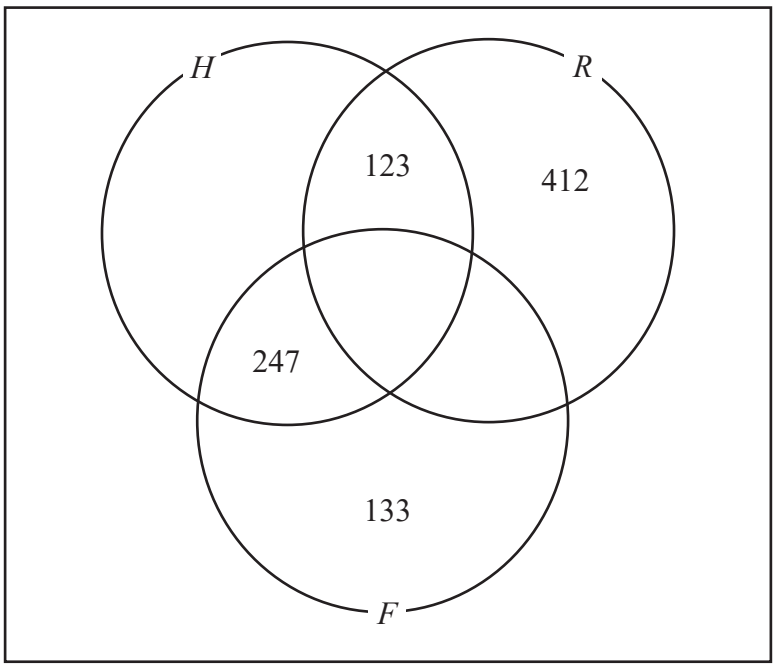
(e) Find $P([H \cup R]')$

(1)

(f) Find $P(F \mid H)$

(2)

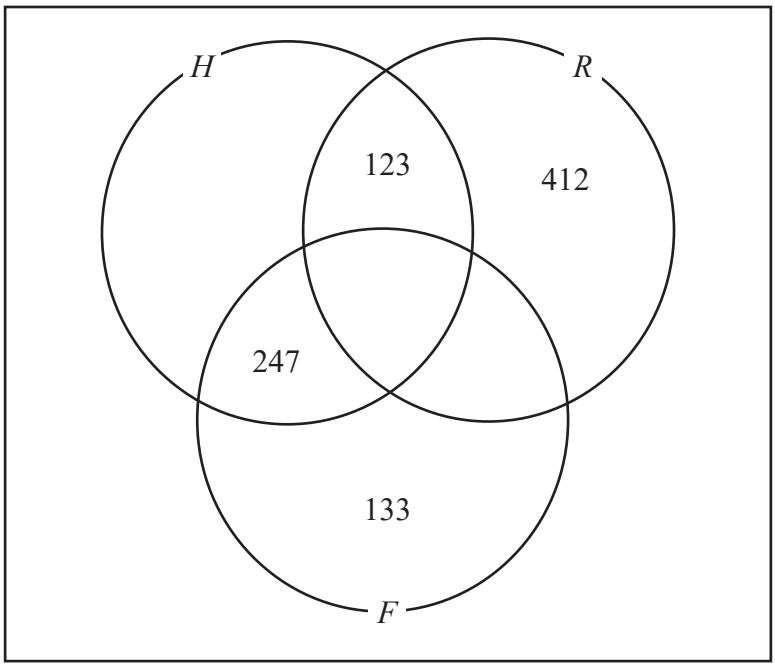
Question 5 continued



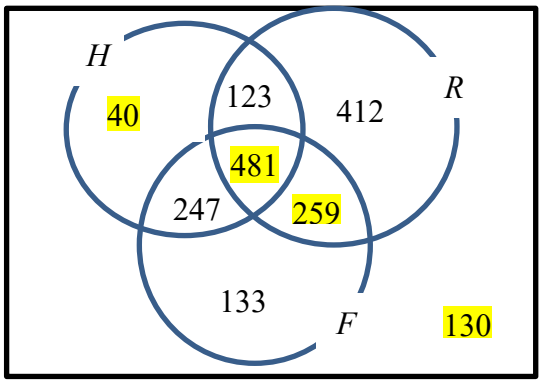
Turn over for a spare diagram if you need to redraw your Venn diagram.

Question 5 continued

Only use this diagram if you need to redraw your Venn diagram.



(Total for Question 5 is 10 marks)

Question	Scheme	Marks	AOs
5(a)	$\frac{365}{1825}$ or $\frac{1}{5}$ or 0.2 oe	B1	1.1b
		(1)	
(b)	$\frac{170}{1825}$ or $\frac{34}{365}$ or awrt 0.093	B1	1.1b
		(1)	
(c)	$90\times0.4+80\times0.05[=40]$ or $90\times0.6+80\times0.95[=130]$ or $740\times0.65[=481]$ or $740\times0.35[=259]$ 	M1 B1 B1 A1	3.1b 1.1b 1.1b 1.1b
		(4)	
(d)	$P(R'\cap F)=\frac{380}{1825}\left[=\frac{76}{365}=0.208...\right]$ oe awrt 0.208	B1	1.1b
		(1)	
(e)	$\left[\frac{133\ +\ "130"}{1825}=\right]\frac{"263"}{1825}$ awrt 0.144	B1ft	1.1b
		(1)	
(f)	$\frac{247+"481"}{247+"481"+123+"40"}$	M1	3.4
	$=\frac{728}{891}$ awrt 0.817	A1	1.1b
		(2)	

Notes: **(10 marks)**

		Look out for answers given in the question. If you see answers in the question and in the answer space those in the answer space take precedence.
(a)	**B1**	Allow equivalent
(b)	**B1**	Allow equivalent
(c)	**M1**	Correct method to find one of the values 40 or 130 or 481or 259 Implied by 40, 481, 259 or130 seen in correct place on diagram
	B1	One of the highlighted correct
	B1	A second value highlighted correct or their $("259"+"\ 481")=740$ or their $("40"+"\ 481")=521$ or their $("40"+"130")=170$
	A1	Fully correct
(d)	**B1**	380/1825oe or awrt 0.208
(e)	**B1ft**	Correct answer or Ft their 130 (> 0) do not allow if blank Allow ft correct to 3 sf.
(f)	**M1**	For a single fraction with the numerator < denominator and *n* is an integer we will award for *n*/ 891 **or** *n*/(sum of their 4 values in *H*, each > 0) or awrt 0.817
	A1	728/891 oe or awrt 0.817