

Y2P4 XMQs and MS

(Total: 50 marks)

1. P2_Sample Q7 . 5 marks - Y2P4 Binomial expansion
2. P1_Specimen Q2 . 8 marks - Y2P4 Binomial expansion
3. P1_2018 Q11. 10 marks - Y2P4 Binomial expansion
4. P1_2019 Q4 . 6 marks - Y2P4 Binomial expansion
5. P1_2020 Q1 . 5 marks - Y2P4 Binomial expansion
6. P1_2021 Q9 . 11 marks - Y2P4 Binomial expansion
7. P2_2022 Q7 . 5 marks - Y2P4 Binomial expansion

Question	Scheme	Marks	AOs
7(a)	$\sqrt{(4-x)} = 2\left(1 - \frac{1}{4}x\right)^{\frac{1}{2}}$	M1	2.1
	$\left(1 - \frac{1}{4}x\right)^{\frac{1}{2}} = 1 + \frac{1}{2}\left(-\frac{1}{4}x\right) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\left(-\frac{1}{4}x\right)^2 + \dots$	M1	1.1b
	$\sqrt{(4-x)} = 2\left(1 - \frac{1}{8}x - \frac{1}{128}x^2 + \dots\right)$	A1	1.1b
	$\sqrt{(4-x)} = 2 - \frac{1}{4}x - \frac{1}{64}x^2 + \dots \text{ and } k = -\frac{1}{64}$	A1	1.1b
		(4)	
(b)	The expansion is valid for $ x < 4$, so $x = 1$ can be used	B1	2.4
		(1)	
(5 marks)			
Notes:			
(a)			
M1: Takes out a factor of 4 and writes $\sqrt{(4-x)} = 2(1 \pm \dots)^{\frac{1}{2}}$			
M1: For an attempt at the binomial expansion with $n = \frac{1}{2}$			
Eg. $(1+ax)^{\frac{1}{2}} = 1 + \frac{1}{2}(ax) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}(ax)^2 + \dots$			
A1: Correct expression inside the bracket $1 - \frac{1}{8}x - \frac{1}{128}x^2 + \dots$ which may be left unsimplified			
A1: $\sqrt{(4-x)} = 2 - \frac{1}{4}x - \frac{1}{64}x^2 + \dots$ and $k = -\frac{1}{64}$			
(b)			
B1: The expansion is valid for $ x < 4$, so $x = 1$ can be used			

Question	Scheme	Marks	AOs
2 (a)	$(4 + 5x)^{\frac{1}{2}} = (4)^{\frac{1}{2}} \left(1 + \frac{5x}{4}\right)^{\frac{1}{2}} = 2 \left(1 + \frac{5x}{4}\right)^{\frac{1}{2}}$	B1	1.1b
	$= \{2\} \left[1 + \left(\frac{1}{2}\right)\left(\frac{5x}{4}\right) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!} \left(\frac{5x}{4}\right)^2 + \dots \right]$	M1	1.1b
	$= 2 + \frac{5}{4}x - \frac{25}{64}x^2 + \dots$	A1	2.1
		(4)	
(b)(i)	$\left\{ x = \frac{1}{10} \Rightarrow \right\} (4 + 5(0.1))^{\frac{1}{2}}$	M1	1.1b
	$= \sqrt{4.5} = \frac{3}{2}\sqrt{2} \text{ or } \frac{3}{\sqrt{2}}$		
	$\frac{3}{2}\sqrt{2} \text{ or } 1.5\sqrt{2} \text{ or } \frac{3}{\sqrt{2}} = 2 + \frac{5}{4}\left(\frac{1}{10}\right) - \frac{25}{64}\left(\frac{1}{10}\right)^2 + \dots \{= 2.121\dots\}$ $\Rightarrow \frac{3}{2}\sqrt{2} = \frac{543}{256} \text{ or } \frac{3}{\sqrt{2}} = \frac{543}{256} \Rightarrow \sqrt{2} = \dots$	M1	3.1a
	So, $\sqrt{2} = \frac{181}{128} \text{ or } \sqrt{2} = \frac{256}{181}$	A1	1.1b
(b)(ii)	$x = \frac{1}{10}$ satisfies $ x < \frac{4}{5}$ (o.e.), so the approximation is valid.	B1	2.3
		(4)	
(8 marks)			

Question 2 Notes:

(a)	
B1:	Manipulates $(4 + 5x)^{\frac{1}{2}}$ by taking out a factor of $(4)^{\frac{1}{2}}$ or 2
M1:	Expands $(... + \lambda x)^{\frac{1}{2}}$ to give at least 2 terms which can be simplified or un-simplified, E.g. $1 + \left(\frac{1}{2}\right)(\lambda x)$ or $\left(\frac{1}{2}\right)(\lambda x) + \frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(\lambda x)^2$ or $1 + ... + \frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(\lambda x)^2$ where λ is a numerical value and where $\lambda \neq 1$.
A1ft:	A correct simplified or un-simplified $1 + \left(\frac{1}{2}\right)(\lambda x) + \frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(\lambda x)^2$ expansion with consistent (λx)
A1:	Fully correct solution leading to $2 + \frac{5}{4}x + kx^2$, where $k = -\frac{25}{64}$
(b)(i)	
M1:	Attempts to substitute $x = \frac{1}{10}$ or 0.1 into $(4 + 5x)^{\frac{1}{2}}$
M1:	A complete method of finding an approximate value for $\sqrt{2}$. E.g. <ul style="list-style-type: none"> • substituting $x = \frac{1}{10}$ or 0.1 into their part (a) binomial expansion and equating the result to an expression of the form $\alpha\sqrt{2}$ or $\frac{\beta}{\sqrt{2}}$; $\alpha, \beta \neq 0$ • followed by re-arranging to give $\sqrt{2} = ...$
A1:	$\frac{181}{128}$ or any equivalent fraction , e.g. $\frac{362}{256}$ or $\frac{543}{384}$ Also allow $\frac{256}{181}$ or any equivalent fraction
(b)(ii)	
B1:	Explains that the approximation is valid because $x = \frac{1}{10}$ satisfies $ x < \frac{4}{5}$

Question	Scheme	Marks	AOs
11 (a)	$\sqrt{\frac{1+4x}{1-x}} = (1+4x)^{0.5} \times (1-x)^{-0.5}$	B1	3.1a
	$(1+4x)^{0.5} = 1 + 0.5 \times (4x) + \frac{0.5 \times -0.5}{2} \times (4x)^2$	M1	1.1b
	$(1-x)^{-0.5} = 1 + (-0.5)(-x) + \frac{(-0.5) \times (-1.5)}{2} (-x)^2$	M1	1.1b
	$(1+4x)^{0.5} = 1 + 2x - 2x^2$ and $(1-x)^{-0.5} = 1 + 0.5x + 0.375x^2$ oe	A1	1.1b
	$(1+4x)^{0.5} \times (1-x)^{-0.5} = (1+2x-2x^2 \dots) \times \left(1 + \frac{1}{2}x + \frac{3}{8}x^2 \dots\right)$ $= 1 + \frac{1}{2}x + \frac{3}{8}x^2 + 2x + x^2 - 2x^2 + \dots$ $= A + Bx + Cx^2$	dM1	2.1
	$= 1 + \frac{5}{2}x - \frac{5}{8}x^2 \dots$ *	A1*	1.1b
		(6)	
(b)	Expression is valid $ x < \frac{1}{4}$ Should not use $x = \frac{1}{2}$ as $\frac{1}{2} > \frac{1}{4}$	B1	2.3
		(1)	
(c)	Substitutes $x = \frac{1}{11}$ into $\sqrt{\frac{1+4x}{1-x}} \approx 1 + \frac{5}{2}x - \frac{5}{8}x^2$	M1	1.1b
	$\sqrt{\frac{3}{2}} = \frac{1183}{968}$	A1	1.1b
	(so $\sqrt{6}$ is) $\frac{1183}{484}$ or $\frac{2904}{1183}$	A1	2.1
		(3)	

(10 marks)

(a)	<p>B1: Scored for key step in setting up the process so that it can be attempted using binomial expansions</p> <p>This could be achieved by $\sqrt{\frac{1+4x}{1-x}} = (1+4x)^{0.5} \times (1-x)^{-0.5}$ See end for other alternatives</p> <p>It may be implied by later work.</p> <p>M1: Award for an attempt at the binomial expansion $(1+4x)^{0.5} = 1 + 0.5 \times (4x) + \frac{(0.5) \times (-0.5)}{2} \times (4x)^2$</p> <p>There must be three (or more terms). Allow a missing bracket on the $(4x)^2$ and a sign slip so the correct application may be implied by $1 + 2x \pm 0.5x^2$</p> <p>M1: Award for an attempt at the binomial expansion $(1-x)^{-0.5} = 1 + (-0.5)(-x) + \frac{(-0.5) \times (-1.5)}{2} (-x)^2$</p> <p>There must be three (or more terms). Allow a missing bracket on the $(-x)^2$ and a sign slips so the method may be awarded on $1 \pm 0.5x \pm 0.375x^2$</p> <p>A1: Both correct and simplified. This may be awarded for a correct final answer if a candidate does all their simplification at the end</p> <p>dM1: In the main scheme it is for multiplying their two expansions to reach a quadratic. It is for the key step in adding 'six' terms to produce the quadratic expression. Higher power terms may be seen. Condone slips on</p>
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the multiplication on one term only. It is dependent upon having scored the first B and one of the other two M's

In the alternative it is for multiplying $\left(1 + \frac{5}{2}x - \frac{5}{8}x^2\right)(1-x)^{0.5}$ and comparing it to $(1+4x)^{0.5}$

It is for the key step in adding 'six' terms to produce the quadratic expression.

A1*: Completes proof with no errors or omissions. In the alternative there must be some reference to the fact that both sides are equal.

(b)

B1: States that the expansion may not / is not valid when $|x| > \frac{1}{4}$

This may be implied by a statement such as $\frac{1}{2} > \frac{1}{4}$ or stating that the expansion is only valid when $|x| < \frac{1}{4}$

Condone, for this mark a candidate who substitutes $x = \frac{1}{2}$ into the $4x$ **and** states it is not valid as $2 > 1$ oe

Don't award for candidates who state that $\frac{1}{2}$ is too big without any reference to the validity of the expansion.

As a rule you should see some reference to $\frac{1}{4}$ or $4x$

(c)(i)

M1: Substitutes $x = \frac{1}{11}$ into BOTH sides $\sqrt{\frac{1+4x}{1-x}} \approx 1 + \frac{5}{2}x - \frac{5}{8}x^2$ and attempts to find at least one side.

As the left hand side is $\frac{\sqrt{6}}{2}$ they may multiply by 2 first which is acceptable

A1: Finds both sides leading to a correct equation/statement $\sqrt{\frac{15}{10}} = \frac{1183}{968}$ oe $\sqrt{6} = 2 \times \frac{1183}{968}$

A1: $\sqrt{6} = \frac{1183}{484}$ or $\sqrt{6} = \frac{2904}{1183}$ $\sqrt{6} = 2 \times \frac{1183}{968} = \frac{1183}{484}$ would imply all 3 marks

Watch for other equally valid alternatives for 11(a) including

B1: $(1+4x)^{0.5} \approx \left(1 + \frac{5}{2}x - \frac{5}{8}x^2\right)(1-x)^{0.5}$ then the M's are for $(1+4x)^{0.5}$ and $(1-x)^{0.5}$

M1: $(1-x)^{0.5} = 1 + (0.5)(-x) + \frac{(0.5) \times (-0.5)}{2}(-x)^2$

Or

B1: $\sqrt{\frac{1+4x}{1-x}} = \sqrt{1 + \frac{5x}{1-x}} = \left(1 + 5x(1-x)^{-1}\right)^{\frac{1}{2}}$ then the first M1 for one application of binomial and the second would be for both $(1-x)^{-1}$ and $(1-x)^{-2}$

Or

B1: $\sqrt{\frac{1+4x}{1-x}} \times \frac{\sqrt{1-x}}{\sqrt{1-x}} = \sqrt{(1+3x-4x^2)} \times (1-x)^{-1} = \left(1 + (3x-4x^2)\right)^{\frac{1}{2}} \times (1-x)^{-1}$

4 (a)	$\frac{1}{\sqrt{4-x}} = (4-x)^{-\frac{1}{2}} = 4^{-\frac{1}{2}} \times (1 \pm \dots)$	M1	2.1
	Uses a "correct" binomial expansion for their $(1+ax)^n = 1+nax + \frac{n(n-1)}{2} a^2 x^2 +$	M1	1.1b
	$\left(1 - \frac{x}{4}\right)^{-\frac{1}{2}} = 1 + \left(-\frac{1}{2}\right)\left(-\frac{x}{4}\right) + \frac{\left(-\frac{1}{2}\right) \times \left(-\frac{3}{2}\right)}{2} \left(-\frac{x}{4}\right)^2$	A1	1.1b
	$\frac{1}{\sqrt{4-x}} = \frac{1}{2} + \frac{1}{16}x + \frac{3}{256}x^2$	A1	1.1b
		(4)	
(b) (i)	States $x = -14$ and gives a valid reason. Eg explains that the expansion is not valid for $ x > 4$	B1	2.4
		(1)	
(b)(ii)	States $x = -\frac{1}{2}$ and gives a valid reason. Eg. explains that it is closest to zero	B1	2.4
		(1)	
			(6 marks)

(a)

M1: For the strategy of expanding $\frac{1}{\sqrt{4-x}}$ using the binomial expansion.

You must see $4^{-\frac{1}{2}}$ oe and an expansion which may or may not be combined.

M1: Uses a correct binomial expansion for their $(1 \pm ax)^n = 1 \pm nax \pm \frac{n(n-1)}{2} a^2 x^2 +$

Condone sign slips and the "a" not being squared in term 3. Condone $a = \pm 1$

Look for an attempt at the correct binomial coefficient for their n , being combined with the correct power of ax

A1: $\left(1 - \frac{x}{4}\right)^{-\frac{1}{2}} = 1 + \left(-\frac{1}{2}\right)\left(-\frac{x}{4}\right) + \frac{\left(-\frac{1}{2}\right) \times \left(-\frac{3}{2}\right)}{2} \left(-\frac{x}{4}\right)^2$ unsimplified

FYI the simplified form is $1 + \frac{x}{8} + \frac{3x^2}{128}$ Accept the terms with commas between.

A1: $\frac{1}{\sqrt{4-x}} = \frac{1}{2} + \frac{1}{16}x + \frac{3}{256}x^2$ Ignore subsequent terms. Allow with commas between.

Note: Alternatively $(4-x)^{-\frac{1}{2}} = 4^{-\frac{1}{2}} + \left(-\frac{1}{2}\right)4^{-\frac{3}{2}}(-x) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}4^{-\frac{5}{2}}(-x)^2 + \dots$

M1: For $4^{-\frac{1}{2}} + \dots$ **M1:** As above but allow slips on the sign of x and the value of n **A1:** Correct unsimplified (as above) **A1:** As main scheme

(b) Any evaluations of the expansions are irrelevant.

Look for a suitable value and a suitable reason for both parts.

(b)(i)

B1: Requires $x = -14$ with a suitable reason.

Eg. $x = -14$ as the expansion is only valid for $|x| < 4$ or equivalent.

Eg. ' $x = -14$ as $|-14| > 4$ ' or 'I cannot use $x = -14$ as $\left|\frac{-14}{4}\right| > 1$ '

Eg. ' $x = -14$ as is outside the range $|x| < 4$ '

Do not allow ' -14 is too big' or ' $x = -14, |x| < 4$ ' either way around without some reference to the validity of the expansion.

(b)(ii)

B1: Requires $x = -\frac{1}{2}$ with a suitable reason.

Eg. $x = -\frac{1}{2}$ as it is 'the smallest/smaller value' or ' $x = -\frac{1}{2}$ as the value closest to zero' (that will give the more accurate approximation). The bracketed statement is not required.

Question		Marks	AOs
1 (a)	$(1+8x)^{\frac{1}{2}} = 1 + \frac{1}{2} \times 8x + \frac{\frac{1}{2} \times -\frac{1}{2}}{2!} \times (8x)^2 + \frac{\frac{1}{2} \times -\frac{1}{2} \times -\frac{3}{2}}{3!} \times (8x)^3$	M1 A1	1.1b 1.1b
	$= 1 + 4x - 8x^2 + 32x^3 + \dots$	A1	1.1b
		(3)	
(b)	Substitutes $x = \frac{1}{32}$ into $(1+8x)^{\frac{1}{2}}$ to give $\frac{\sqrt{5}}{2}$	M1	1.1b
	Explains that $x = \frac{1}{32}$ is substituted into $1 + 4x - 8x^2 + 32x^3$ and you multiply the result by 2	A1ft	2.4
		(2)	
(5 marks)			
Notes:			

(a)

M1: Attempts the binomial expansion with $n = \frac{1}{2}$ and obtains the correct structure for term 3 **or** term 4.

Award for the correct coefficient with the correct power of x . Do not accept nC_r notation for coefficients.

For example look for term 3 in the form $\frac{\frac{1}{2} \times -\frac{1}{2}}{2!} \times (*x)^2$ or $\frac{\frac{1}{2} \times -\frac{1}{2} \times -\frac{3}{2}}{3!} \times (*x)^3$

A1: Correct (unsimplified) expression. May be implied by correct simplified expression

A1: $1 + 4x - 8x^2 + 32x^3$

Award if there are extra terms (even if incorrect).

Award if the terms are listed $1, 4x, -8x^2, 32x^3$

(b)

M1: Score for substituting $x = \frac{1}{32}$ into $(1+8x)^{\frac{1}{2}}$ to obtain $\frac{\sqrt{5}}{2}$ or equivalent such as $\sqrt{\frac{5}{4}}$

Alternatively award for substituting $x = \frac{1}{32}$ into **both sides** and making a connection between the two sides by use of an = or \approx .

E.g. $\left(1 + \frac{8}{32}\right)^{\frac{1}{2}} = 1 + 4 \times \frac{1}{32} - 8 \times \left(\frac{1}{32}\right)^2 + 32 \times \left(\frac{1}{32}\right)^3$ following through on their expansion

Also implied by $\frac{\sqrt{5}}{2} = \frac{1145}{1024}$ for a correct expansion

It is not enough to state substitute $x = \frac{1}{32}$ into "the expansion" or just the rhs " $1 + 4x - 8x^2 + 32x^3$ "

A1ft: Requires a full (and correct) **explanation** as to how the expansion can be used to estimate $\sqrt{5}$

E.g. Calculates $1 + 4 \times \frac{1}{32} - 8 \times \left(\frac{1}{32}\right)^2 + 32 \times \left(\frac{1}{32}\right)^3$ and multiplies by 2.

This can be scored from an incorrect binomial expansion or a binomial expansion with more terms.

The explanation could be mathematical. So $\frac{\sqrt{5}}{2} = \frac{1145}{1024} \rightarrow \sqrt{5} = \frac{1145}{512}$ is acceptable.

SC: For 1 mark, M1,A0 score for a statement such as "substitute $x = \frac{1}{32}$ into both sides of part (a) and make $\sqrt{5}$ the subject"

Question	Scheme	Marks	AOs
9(a)(i)	$50x^2 + 38x + 9 \equiv A(5x+2)(1-2x) + B(1-2x) + C(5x+2)^2$ $\Rightarrow B = \dots$ or $C = \dots$	M1	1.1b
	$B = 1$ and $C = 2$	A1	1.1b
(a)(ii)	E.g. $x = 0$ $x = 0 \Rightarrow 9 = 2A + B + 4C$ $\Rightarrow 9 = 2A + 1 + 8 \Rightarrow A = \dots$	M1	2.1
	$A = 0^*$	A1*	1.1b
		(4)	
(b)(i)	$\frac{1}{(5x+2)^2} = (5x+2)^{-2} = 2^{-2} \left(1 + \frac{5}{2}x\right)^{-2}$ or $(5x+2)^{-2} = 2^{-2} + \dots$	M1	3.1a
	$\left(1 + \frac{5}{2}x\right)^{-2} = 1 - 2\left(\frac{5}{2}x\right) + \frac{-2(-2-1)}{2!}\left(\frac{5}{2}x\right)^2 + \dots$	M1	1.1b
	$2^{-2} \left(1 + \frac{5}{2}x\right)^{-2} = \frac{1}{4} - \frac{5}{4}x + \frac{75}{16}x^2 + \dots$	A1	1.1b
	$\frac{1}{(1-2x)} = (1-2x)^{-1} = 1 + 2x + \frac{-1(-1-1)}{2!}(2x)^2 + \dots$	M1	1.1b
	$\frac{1}{(5x+2)^2} + \frac{2}{1-2x} = \frac{1}{4} - \frac{5}{4}x + \frac{75}{16}x^2 + \dots + 2 + 4x + 8x^2 + \dots$	dM1	2.1
	$= \frac{9}{4} + \frac{11}{4}x + \frac{203}{16}x^2 + \dots$	A1	1.1b
(b)(ii)	$ x < \frac{2}{5}$	B1	2.2a
		(7)	
(11 marks)			
Notes			

(a)(i)

M1: Uses a correct identity and makes progress using an appropriate strategy (e.g. sub $x = \frac{1}{2}$) to find a value for B or C . May be implied by one correct value (cover up rule).

A1: Both values correct

(a)(ii)

M1: Uses an appropriate method to establish an equation connecting A with B and/or C and uses their values of B and/or C to find a suitable equation in A .

Amongst many different methods are:

Compare terms in $x^2 \Rightarrow 50 = -10A + 25C$ which would be implied by $50 = -10A + 25 \times 2$

Compare constant terms or substitute $x = 0 \Rightarrow 9 = 2A + B + 4C$ implied by $9 = 2A + 1 + 4 \times 2$

A1*: Fully correct proof with no errors.

Note: The second part is a proof so it is important that a suitable proof/show that is seen.

Candidates who write down 3 equations followed by three answers (with no working) will score M1 A1 M0 A0

(b)(i)

M1: Applies the key steps of writing $\frac{1}{(5x+2)^2}$ as $(5x+2)^{-2}$ and takes out a factor of 2^{-2} to form an expression of the form $(5x+2)^{-2} = 2^{-2}(1+*x)^{-2}$ where * is not 1 or 5

Alternatively uses direct expansion to obtain $2^{-2} + \dots$

M1: Correct attempt at the binomial expansion of $(1+*x)^{-2}$ up to the term in x^2

Look for $1 + (-2)*x + \frac{(-2)(-3)}{2}*x^2$ where * is not 5 or 1.

Condone sign slips and lack of $*^2$ on term 3.

Alt Look for correct structure for 2^{nd} and 3^{rd} terms by direct expansion. See below

A1: For a fully correct expansion of $(2+5x)^{-2}$ which may be unsimplified. This may have been combined with their 'B'

A direct expansion would look like $(2+5x)^{-2} = 2^{-2} + (-2)2^{-3} \times 5x + \frac{(-2)(-3)}{2}2^{-4} \times (5x)^2$

M1: Correct attempt at the binomial expansion of $(1-2x)^{-1}$

Look for $1 + (-1)*x + \frac{(-1)(-2)}{2}*x^2$ where * is not 1

dM1: Fully correct strategy that is dependent on the previous **TWO** method marks.

There must be some attempt to use their values of B and C

A1: Correct expression or correct values for p, q and r.

(b)(ii)

B1: Correct range. Allow also other forms, for example $-\frac{2}{5} < x < \frac{2}{5}$ or $x \in \left(-\frac{2}{5}, \frac{2}{5}\right)$

Do not allow multiple answers here. The correct answer must be chosen if two answers are offered

Question	Scheme Page 19	Marks	AOs
7(a)	$\sqrt{4-9x} = 2(1 \pm \dots)^{\frac{1}{2}}$	B1	1.1b
	$\left(1 - \frac{9x}{4}\right)^{\frac{1}{2}} = \dots + \frac{\frac{1}{2} \times \left(-\frac{1}{2}\right) \left(-\frac{9x}{4}\right)^2}{2!}$ or $\dots + \frac{\frac{1}{2} \times \left(-\frac{1}{2}\right) \times \left(-\frac{3}{2}\right) \left(-\frac{9x}{4}\right)^3}{3!}$	M1	1.1b
	$1 + \frac{1}{2} \times \left(-\frac{9x}{4}\right) + \frac{\frac{1}{2} \times \left(-\frac{1}{2}\right) \left(-\frac{9x}{4}\right)^2}{2!} + \frac{\frac{1}{2} \times \left(-\frac{1}{2}\right) \times \left(-\frac{3}{2}\right) \left(-\frac{9x}{4}\right)^3}{3!}$	A1	1.1b
	$\sqrt{4-9x} = 2 - \frac{9x}{4} - \frac{81x^2}{64} - \frac{729x^3}{512}$	A1	1.1b
		(4)	
(b)	States that the approximation will be an overestimate since all terms (after the first one) in the expansion are negative (since $x > 0$)	B1	3.2b
		(1)	
			(5 marks)
Notes:			

(a)

B1: Takes out a factor of 4 and writes $\sqrt{4-9x} = 2(1 \pm \dots)^{\frac{1}{2}}$ or $\sqrt{4}(1 \pm \dots)^{\frac{1}{2}}$ or $4^{\frac{1}{2}}(1 \pm \dots)^{\frac{1}{2}}$

M1: For an attempt at the binomial expansion of $(1+ax)^{\frac{1}{2}}$ $a \neq 1$ to form term 3 or term 4 with the correct structure. Look for the correct binomial coefficient multiplied by the corresponding power of x e.g.

$$\frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)}{2!}(\dots x)^2 \text{ or } \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}(\dots x)^3 \text{ where } \dots \neq 1$$

Condone missing or incorrect brackets around the x terms but the binomial coefficients must be correct. Allow 2! and/or 3! or 2 and/or 6. Ignore attempts to find more terms.

Do not allow notation such as $\begin{pmatrix} \frac{1}{2} \\ 2 \end{pmatrix}$, $\begin{pmatrix} \frac{1}{2} \\ 2 \end{pmatrix}$ unless these are interpreted correctly.

A1: Correct expression for the expansion of $\left(1 - \frac{9x}{4}\right)^{\frac{1}{2}}$ e.g.

$$1 + \frac{1}{2} \times \left(-\frac{9x}{4}\right) + \frac{\frac{1}{2} \times \left(\frac{1}{2}-1\right) \left(-\frac{9x}{4}\right)^2}{2!} + \frac{\frac{1}{2} \times \left(\frac{1}{2}-1\right) \times \left(\frac{1}{2}-2\right) \left(-\frac{9x}{4}\right)^3}{3!}$$

which may be left unsimplified as shown but the bracketing must be correct unless any missing brackets are implied by subsequent work. If the 2 outside this expansion is only partially applied to this expansion then score A0 but if it is applied to all terms this A1 can be implied.

OR at least 2 correct simplified terms **for the final expansion** from, $-\frac{9x}{4}$, $-\frac{81x^2}{64}$, $-\frac{729x^3}{512}$

A1: $\sqrt{4-9x} = 2 - \frac{9x}{4} - \frac{81x^2}{64} - \frac{729x^3}{512}$ oe and condone e.g. $2 + \frac{-9x}{4} - \frac{81x^2}{64} - \frac{729x^3}{512}$

Allow equivalent mixed numbers and/or decimals for the coefficients e.g.:

$$\left(\frac{9}{4}, 2\frac{1}{4}, 2.25\right), \left(\frac{81}{64}, 1\frac{17}{64}, 1.265625\right), \left(\frac{729}{512}, 1\frac{217}{512}, 1.423828125\right)$$

Ignore any extra terms if found. Allow terms to be “listed” and apply isw once a correct expansion is seen. Allow recovery if applicable e.g. if an “x” is lost then “reappears”.

Direct expansion in (a) can be marked in a similar way:

$$\sqrt{4-9x} = (4-9x)^{\frac{1}{2}} = 4^{\frac{1}{2}} + \left(\frac{1}{2}\right)4^{-\frac{1}{2}} \times (-9x)^1 + \left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)4^{-\frac{3}{2}} \times \frac{(-9x)^2}{2!} + \left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)4^{-\frac{5}{2}} \times \frac{(-9x)^3}{3!}$$

B1: For 2 or $\sqrt{4}$ or $4^{\frac{1}{2}}$ as the constant term in the expansion.

M1: Correct form for term 3 or term 4.

E.g. $\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right) \times \frac{(\dots x)^2}{2!}$ or $\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right) \times \frac{(\dots x)^3}{3!}$ where $\dots \neq 1$

Condone missing brackets around the x terms but the binomial coefficients must be correct. Allow 2! and/or 3! or 2 and/or 6. Ignore attempts to find more terms.

Do not allow notation such as $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$ unless these are interpreted correctly.

A1: Correct expansion (unsimplified as above)

OR at least 2 correct simplified terms from, $-\frac{9x}{4}$, $-\frac{81x^2}{64}$, $-\frac{729x^3}{512}$

A1: $\sqrt{4-9x} = 2 - \frac{9x}{4} - \frac{81x^2}{64} - \frac{729x^3}{512}$ oe and condone e.g. $2 + \frac{-9x}{4} - \frac{81x^2}{64} - \frac{729x^3}{512}$

Allow equivalent mixed numbers and/or decimals for the coefficients e.g.:

$$\left(\frac{9}{4}, 2\frac{1}{4}, 2.25\right), \left(\frac{81}{64}, 1\frac{17}{64}, 1.265625\right), \left(\frac{729}{512}, 1\frac{217}{512}, 1.423828125\right)$$

Ignore any extra terms if found. Allow terms to be “listed” and apply isw once a correct expansion is seen. Allow recovery if applicable e.g. if an “x” is lost then “reappears”.

(b)

B1: States that the approximation will be an **overestimate** due to the fact that all terms (after the first one) in the expansion are negative or equivalent statements e.g.

- Overestimate because the terms are negative
- Overestimate as the terms are being taken away (from 2)

Condone “overestimate as every term is negative”

If you think a response is worthy of credit but are unsure then use Review.

This mark depends on having obtained an expansion in (a) of the form

$k - px - qx^2 - rx^3$ $k, p, q, r > 0$ but note that if e.g. one of the algebraic terms is zero or was “lost” or there are extra negative terms this mark is still available.

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