Y1P9 XMQs and MS

(Total: 29 marks)

1. P1(AS)_2018 Q7 . 6 marks - Y1P9 Trigonometric ratios
2. P1(AS)_2019 Q6 . 6 marks - Y1P9 Trigonometric ratios
3. P1(AS)_2020 Q5 . 6 marks - Y1P9 Trigonometric ratios
4. P1(AS)_2021 Q7 . 5 marks - Y1P9 Trigonometric ratios
5. P1(AS)_2022 Q4 . 6 marks - Y1P9 Trigonometric ratios

7. In a triangle ABC, side AB has length 10 cm, side AC has length 5 cm, and angle $BAC = \theta$ where θ is measured in degrees. The area of triangle ABC is 15 cm^2

(a) Find the two possible values of $\cos\theta$

(4)

Given that BC is the longest side of the triangle,

(b) find the exact length of BC.

(2)

Question	Scheme	Marks	AOs
7 (a)	Uses $15 = \frac{1}{2} \times 5 \times 10 \times \sin\theta$	M1	1.1b
	$\sin\theta = \frac{3}{5}$ oe	A1	1.1b
	Uses $\cos^2\theta = 1 - \sin^2\theta$	M1	2.1
	$\cos\theta = \pm\frac{4}{5}$	A1	1.1b
		(4)	
(b)	Uses $BC^2 = 10^2 + 5^2 - 2\times 10\times 5\times "-\frac{4}{5}"$	M1	3.1a
	$BC = \sqrt{205}$	A1	1.1b
		(2)	
			(6 marks)

Notes

(a)

M1: Uses the formula $\text{Area} = \frac{1}{2}ab\sin C$ in an attempt to find the value of $\sin\theta$ or θ

A1: $\sin\theta = \frac{3}{5}$ oe This may be implied by $\theta =$ awrt 36.9° or awrt 0.644 (radians)

M1: Uses their value of $\sin\theta$ to find two values of $\cos\theta$ This may be scored via the formula $\cos^2\theta = 1 - \sin^2\theta$ or by a triangle method. Also allow the use of a graphical calculator or good candidates may just write down the **two values**. The values must be symmetrical $\pm k$

A1: $\cos\theta = \pm\frac{4}{5}$ or ± 0.8 Condone these values appearing from $\pm 0.79....$

(b)

M1: Uses a suitable method of finding the longest side. For example chooses the negative value (or the obtuse angle) and proceeds to find *BC* using the cosine rule. Alternatively works out *BC* using both values and chooses the larger value. If stated the cosine rule should be correct (with a minus sign). Note if the sign is +ve and the acute angle is chosen the correct value will be seen. It is however M0 A0

A1: $BC = \sqrt{205}$

6.

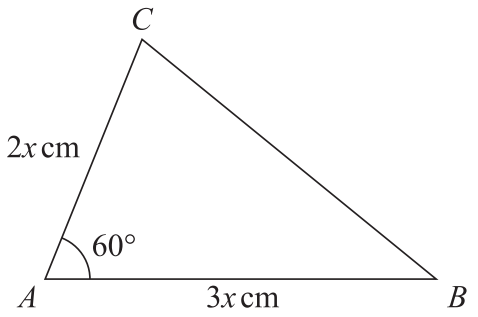


Figure 1

Figure 1 shows a sketch of a triangle ABC with $AB = 3x$ cm, $AC = 2x$ cm and angle $CAB = 60°$

Given that the area of triangle ABC is $18\sqrt{3}$ cm^2

(a) show that $x = 2\sqrt{3}$

(3)

(b) Hence find the exact length of BC, giving your answer as a simplified surd.

(3)

Question	Scheme	Marks	AOs
6 (a)	Uses $18\sqrt{3}=\frac{1}{2}\times 2x\times 3x\times \sin 60°$	M1	1.1a
	Sight of $\sin 60°=\frac{\sqrt{3}}{2}$ and proceeds to $x^2=k$ oe	M1	1.1b
	$x=\sqrt{12}=2\sqrt{3}$ *	A1*	2.1
		(3)	
(b)	Uses $BC^2=\left(6\sqrt{3}\right)^2+\left(4\sqrt{3}\right)^2-2\times 6\sqrt{3}\times 4\sqrt{3}\times \cos 60°$	M1	1.1b
	$BC^2=84$	A1	1.1b
	$BC=2\sqrt{21}$ (cm)	A1	1.1b
		(3)	
			(6 marks)

Notes

(a)

M1: Attempts to use the formula $A=\frac{1}{2}ab\sin C$.

If the candidate writes $18\sqrt{3}=\frac{1}{2}\times 5x\times \sin 60°$ **without** sight of a previous correct line then this would be M0

M1: Sight of $\sin 60°=\frac{\sqrt{3}}{2}$ or awrt 0.866 and proceeds to $x^2=k$ oe such as $px^2=q$

This may be awarded from the correct formula or $A=ab\sin C$

A1*: Look for $x^2=12\Rightarrow x=2\sqrt{3}$, $x^2=4\times 3\Rightarrow x=2\sqrt{3}$ or $x=\sqrt{12}=2\sqrt{3}$

This is a given answer and all aspects must be correct including one of the above intermediate lines. It cannot be scored by using decimal equivalents to $\sqrt{3}$

Alternative using the given answer of $x=2\sqrt{3}$

M1: Attempts to use the formula $A=\frac{1}{2}\times 4\sqrt{3}\times 6\sqrt{3}\sin 60°$ oe

M1: Sight of $\sin 60°=\frac{\sqrt{3}}{2}$ and proceeds to $A=18\sqrt{3}$

A1*: Concludes that $x=2\sqrt{3}$

(b)

M1: Attempts the cosine rule with the sides in the correct position.

This can be scored from $BC^2=\left(3x\right)^2+\left(2x\right)^2-2\times 3x\times 2x\times \cos 60°$ as long as there is some attempt to substitute x in later. Condone slips on the squaring

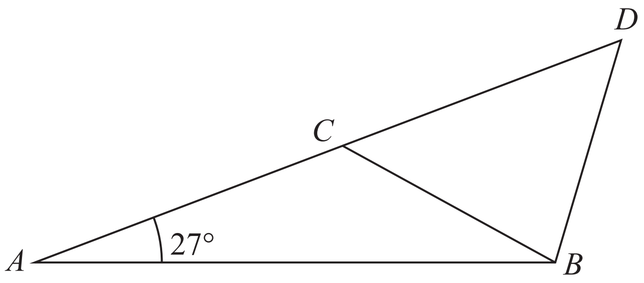
A1: $BC^2=84$ Accept $BC^2=7\times 12$, $BC=\sqrt{84}$ or $BC=2\sqrt{21}$

If they replace the surds with decimals they can score the A1 for $BC^2=$ awrt 84.0

A1: $BC=2\sqrt{21}$

Condone other variables, say $x=2\sqrt{21}$, but it cannot be scored via decimals.

5.



Not to scale

Figure 1

Figure 1 shows the design for a structure used to support a roof.

The structure consists of four steel beams, AB, BD, BC and AD.

Given $AB = 12\,\text{m}$, $BC = BD = 7\text{m}$ and angle $BAC = 27°$

(a) find, to one decimal place, the size of angle ACB.

(3)

The steel beams can only be bought in whole metre lengths.

(b) Find the minimum length of steel that needs to be bought to make the complete structure.

(3)

Question	Scheme	Marks	AOs
5 (a)	States $\dfrac{\sin\theta}{12}=\dfrac{\sin 27}{7}$	M1	1.1b
	Finds $\theta=$ awrt $51°$ or awrt $129°$	A1	1.1b
	$=$ awrt $128.9°$	A1	1.1b
		(3)	
(b)	Attempts to find part or all of AD Eg $AD^2=7^2+12^2-2\times12\times7\cos101.9=(AD=15.09)$ Eg $(AC)^2=7^2+12^2-2\times12\times7\cos(180-"128.9"-27)$ Eg $12\cos27$ or $7\cos"51"$	M1	1.1b
	Full method for the total length = $12+7+7+"15.09"=$	dM1	3.1a
	= 42 m	A1	3.2a
		(3)	
			(6 marks)

Notes

(a)

M1: States $\dfrac{\sin\theta}{12}=\dfrac{\sin 27}{7}$ oe with the sides and angles in the correct positions

Alternatively they may use the cosine rule on $\angle ACB$ and then solve the subsequent quadratic to find AC and then use the cosine rule again

A1: awrt $51°$ or awrt $129°$

A1: Awrt $128.9°$ only (must be seen in part a))

(b)

M1: Attempts a "correct" method of finding either AD or a part of AD eg (AC or CD or forming a perpendicular to split the triangle into two right angled triangles to find AX or XD) which may be seen in (a).
You should condone incorrect labelling of the side.
Look for attempted application of the cosine rule

$$(AD)^2=7^2+12^2-2\times12\times7\cos("128.9"-27)$$
$$\text{or } (AC)^2=7^2+12^2-2\times12\times7\cos(180-"128.9"-27)$$

Or an attempted application of the sine rule $\dfrac{(AD)}{\sin("128.9"-27)}=\dfrac{7}{\sin 27}$

Or $\dfrac{(AC)}{\sin(180-"128.9"-27)}=\dfrac{7}{\sin 27}$

Or an attempt using trigonometry on a right-angled triangle to find part of AD

$12\cos 27$ or $7\cos"51"$

This method can be implied by sight of awrt 15.1 or awrt 6.3 or awrt 8.8 or awrt 10.7 or awrt 4.4

dM1: A complete method of finding the TOTAL length.
There must have been an attempt to use the correct combination of angles and sides.
Expect to see $7+7+12+"AD"$ found using a correct method.
This is scored by either $7+7+12+"AD"$ if $\angle ACB = 128.9°$ in a) or
$7+7+12+\text{awrt}15.1$ by candidates who may have assumed $\angle ACB = 51.1°$ in a)

A1: Rounds correct 41.09 m (or correct expression) up to 42 m to find steel **bought**

Candidates who assumed $\angle ACB = 51.1°$ (acute) in (a):
Full marks can still be achieved as candidates may have restarted in (b) or not used the acute angle in their calculation which is often unclear. We are condoning any reference to $AC = 15.1$ so ignore any labelling of the lengths they are finding.

Diagram of the correct triangle with lengths and angles:

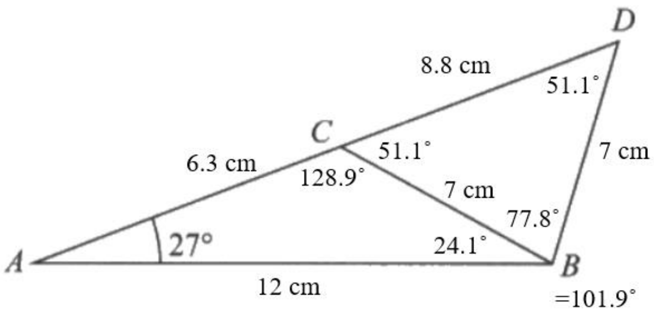
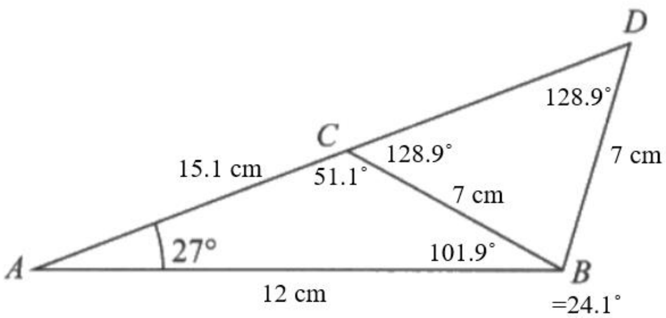


Diagram using the incorrect acute angle:



7. A parallelogram *PQRS* has area 50 cm^2

Given

- *PQ* has length 14 cm
- *QR* has length 7 cm
- angle *SPQ* is obtuse

find

(a) the size of angle *SPQ*, in degrees, to 2 decimal places,

(3)

(b) the length of the diagonal *SQ*, in cm, to one decimal place.

(2)

Question	Scheme	Marks	AOs
7 (a)	Sets $50 = 7\times14\sin(SPQ)$ oe	B1	1.2
	Finds $180° - \arcsin\left("\frac{50}{98}"\right)$	M1	1.1b
	$=149.32°$	A1	1.1b
		(3)	
(b)	Method of finding SQ $SQ^2 = 14^2 + 7^2 - 2\times14\times7\cos"149.32"$	M1	1.1b
	$= 20.3$ cm	A1	1.1b
		(2)	
			(5 marks)
Alt(a)	States or uses $14h = 50$ or $7h_1 = 50$	B1	1.2
	Full method to find obtuse $\angle SPQ$. In this case it is $90° + \arccos\left(\frac{h}{7}\right)$ or $90° + \arccos\left(\frac{h_1}{14}\right)$	M1	1.1b
	awrt 149.32°	A1	1.1b

Notes

(a)

B1: Sets $50 = 7\times14\sin(SPQ)$ oe

M1: Attempts the correct method of finding obtuse $\angle SPQ$. See scheme.

A1: awrt 149.32°

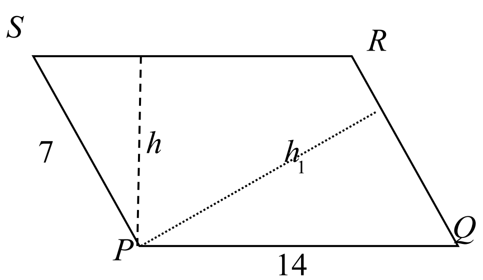
(b)

M1: A correct method of finding SQ using their $\angle SPQ$.

$SQ^2 = 14^2 + 7^2 - 2\times14\times7\cos"149.32"$ scores this mark.

A1: awrt 20.3 cm (condone lack of units)

Alt(a)



B1: States or uses $14h = 50$ or $7h_1 = 50$

M1: Full method to find obtuse $\angle SPQ$.

In this case it is $90° + \arccos\left(\frac{h}{7}\right)$ or $90° + \arccos\left(\frac{h_1}{14}\right)$

A1: awrt 149.32°

Question	Scheme	Marks	AOs

4.

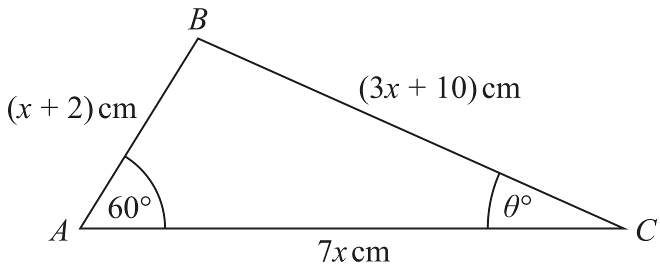


Figure 1

Figure 1 shows a sketch of triangle ABC with $AB = (x + 2)$ cm, $BC = (3x + 10)$ cm, $AC = 7x$ cm, angle $BAC = 60°$ and angle $ACB = \theta°$

(a) (i) Show that $17x^2 - 35x - 48 = 0$

(3)

(ii) Hence find the value of x.

(1)

(b) Hence find the value of θ giving your answer to one decimal place.

(2)

Question	Scheme	Marks	AOs
4(a)(i)	$(3x+10)^2=(x+2)^2+(7x)^2-2(x+2)(7x)\cos 60°$ oe	M1	3.1a
	Uses cos 60° = ½, expands the brackets and proceeds to a 3 term quadratic equation	dM1	1.1b
	$17x^2-35x-48=0$ *	A1*	2.1
		(3)	
(ii)	$x = 3$	B1	3.2a
		(1)	
(b)	$\frac{5}{\sin ACB}=\frac{19}{\sin 60°}\Rightarrow \sin ACB=...\left(\frac{5\sqrt{3}}{38}\right)$ or e.g. $5^2=21^2+19^2-2\times 19\times 21\cos ACB\Rightarrow \cos ACB=...\left(\frac{37}{38}\right)$	M1	1.1b
	θ = awrt 13.2	A1	1.1b
		(2)	
		(6 marks)	

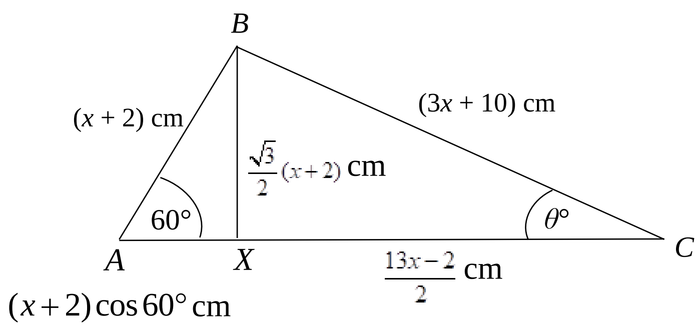
Notes

(a)(i) Mark (a) and (b) together

M1: Recognises the need to apply the cosine rule and attempts to use it with the sides in the correct positions and the formula applied correctly. Condone invisible brackets and slips on $3x+10$ as $3x-10$.
Alternatively, uses trigonometry to find *AX* and then equates two expressions for the length *BX*. You may see variations of this if they use Pythagoras or trigonometry to find *BX* and then apply Pythagoras to the triangle *BXC*. See the diagram below to help you.
The angles and lengths must be in the correct positions. Cos 60 may be $\frac{1}{2}$ from the start

dM1: Uses cos 60° = ½, expands the brackets and proceeds to a 3TQ. You may see the use of cos 60° = ½ in earlier work, but they must proceed to a 3TQ as well to score this mark. It is dependent on the first method mark.

A1*: Obtains the correct quadratic equation with the = 0 with no errors seen in the main body of their solution. Condone the recovery of invisible brackets as long as the intention is clear. You do not need to explicitly see cos 60 to score full marks.



(a)(ii)

B1: Selects the appropriate value i.e. $x = 3$ only. The other root must **either** be rejected if found **or** $x = 3$ must be the only root used in part (b). Can be implied by awrt 13.2 in (b)

(b)

M1: Using their value for x this mark is for either:

- applying the sine rule correctly (or considers 2 right angled triangles) and proceeding to obtain a value for sin *ACB* or
- applying the cosine rule correctly and proceeding to obtain a value for cos *ACB*.

Condone slips calculating the lengths *AB*, *BC* and *AC*. At least one of them should be found correctly for their value for x

(Also allow if the sine rule or cosine rule is applied correctly to find a value for sin *ABC* $\left(=\frac{21\sqrt{3}}{38}\right)$ or cos *ACB* $\left(=-\frac{11}{38}\right)$)

A1: awrt 13.2 (answers with little working eg just lengths on the diagram can score M1A1)