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Y1P4 XMQs and MS

(Total: 32 marks)

1. P1_2022 Q11. 7 marks - Y1P4 Graphs and transformations
2. P1(AS)_2019 Q7 . 8 marks - Y1P4 Graphs and transformations
3. P1(AS)_2019 Q11. 10 marks - Y1P4 Graphs and transformations
4. P1(AS)_2022 Q7 . 7 marks - Y1P4 Graphs and transformations

Question	Scheme	Marks	AOs
11 (a)	Substitutes $x = \frac{1}{2}$ into $y = 2x^3 + 10$ and $y = 42x - 15x^2 - 7$ and finds the y values for both	M1	1.1b
	Achieves $\frac{41}{4}$ o.e. for both and makes a valid conclusion. *	A1*	2.4
		(2)	
(b)	Sets $42x - 15x^2 - 7 = 2x^3 + 10 \Rightarrow 2x^3 + 15x^2 - 42x + 17 = 0$	M1	1.1b
	Deduces that $(2x - 1)$ is a factor and attempts to divide	dM1	2.1
	$2x^3 + 15x^2 - 42x + 17 = (2x - 1)(x^2 + 8x - 17)$	A1	1.1b
	Solves their $x^2 + 8x - 17 = 0$ using suitable method	M1	1.1b
	Deduces $x = -4 + \sqrt{33}$ (see note)	A1	2.2a
	(5)		
			(7 marks)
Notes:			

(a)

M1: Substitutes $x = \frac{1}{2}$ into both $y = 2x^3 + 10$ and $y = 42x - 15x^2 - 7$ and finds y values

Sight of just the y values at each is sufficient for this mark only.

Alternative: Sets $42x - 15x^2 - 7 = 2x^3 + 10 \Rightarrow$ cubic and substitutes $x = \frac{1}{2}$ into the expression,

attempts $f\left(\frac{1}{2}\right)$ or else attempts to divide the cubic $= 0$ by $(2x - 1)$ or $\left(x - \frac{1}{2}\right)$. Condone $f\left(\frac{1}{2}\right) = 0$

without calculations for this mark only.

A1*: Correct calculations must be seen with a minimal conclusion that curves intersect (at $x = \frac{1}{2}$).

E.g. $2\left(\frac{1}{2}\right)^3 + 10 = 10.25$ and $42\left(\frac{1}{2}\right) - 15\left(\frac{1}{2}\right)^2 - 7 = 10.25$ so curves intersect.

Acceptable alternatives are:

$f(x) = 42x - 15x^2 - 7 - 2x^3 - 10, f\left(\frac{1}{2}\right) = 42\left(\frac{1}{2}\right) - 15\left(\frac{1}{2}\right)^2 - 7 - 2\left(\frac{1}{2}\right)^3 - 10 = 0 \Rightarrow$ so curves intersect

$f(x) = 2x^3 + 15x^2 - 42x + 17 \Rightarrow \left(x - \frac{1}{2}\right)(2x^2 + 16x - 34)$ so $x = \frac{1}{2}$ is a root so curves intersect

$f(x) = 2x^3 + 15x^2 - 42x + 17 \Rightarrow (2x - 1)(x^2 + 8x - 17)$ so $(2x - 1)$ is a factor hence curves intersect

Only accept verified, QED etc if there is a preamble mentioning intersection about how it will be shown.

Special case: Scores M1 A0 with or without a conclusion

This is presumably done using a calculator and requires all three roots exact or correct to 3sf

$$f(x) = 2x^3 + 15x^2 - 42x + 17 = 0$$

$$\Rightarrow x = 0.5, 1.74, -9.74$$

(b) This part requires candidates to show all stages of their working.

Answers without working will not score any marks

A method must be seen which could be from part (a) which must then be continued in (b)

M1: Sets $42x - 15x^2 - 7 = 2x^3 + 10$ and proceeds to 4 term cubic equation.

Condone slips, e.g. signs. Terms do not have to be on one side of the equation.

dM1: For the key step in attempting to "divide" the cubic by $(2x - 1)$

If attempted via inspection look for correct first and last terms

E.g. $2x^3 + 15x^2 - 42x + 17 = (2x - 1)(x^2 + \dots \pm 17)$ if cubic expression is correct

If attempted via division look for correct first and second terms

$$2x - 1 \overline{) \begin{array}{r} x^2 + 8x \\ 2x^3 + 15x^2 - 42x + 17 \end{array}} \quad \text{if cubic expression is correct}$$

It is acceptable for an attempt to divide by $\left(x - \frac{1}{2}\right)$. It is easily marked using the same

guidelines, e.g. $2x^3 + 15x^2 - 42x + 17 = \left(x - \frac{1}{2}\right)(2x^2 + 16x \dots)$

$$A1: 2x^3 + 15x^2 - 42x + 17 = (2x - 1)(x^2 + 8x - 17) \quad \text{o.e.} \quad \left(x - \frac{1}{2}\right)(2x^2 + 16x - 34)$$

This may be implied by sight of $(x^2 + 8x - 17)$ or $(2x^2 + 16x - 34)$ in a "division" sum.

M1: Solves their quadratic $x^2 + 8x - 17 = 0$ using a suitable method including calculator. You may need to check this. It is not completely dependent upon the previous M's but an attempt at a full method must have been seen. So look for

- the two equations being set equal to each other and some attempt made to combine
- some attempt to "divide" the result by $(2x - 1)$ o.e. allowing for flaws in the method

A1: Gives $x = -4 + \sqrt{33}$ o.e. only. The $x = -4 - \sqrt{33}$ must not be included in the final answer.

Allow exact unsimplified equivalents such as $x = \frac{-8 + \sqrt{132}}{2}$. ISW for instance if they then put this in decimal form.

7. The curve C has equation

$$y = \frac{k^2}{x} + 1 \quad x \in \mathbb{R}, x \neq 0$$

where k is a constant.

(a) Sketch C stating the equation of the horizontal asymptote.

(3)

The line l has equation $y = -2x + 5$

(b) Show that the x coordinate of any point of intersection of l with C is given by a solution of the equation

$$2x^2 - 4x + k^2 = 0$$

(2)

(c) Hence find the exact values of k for which l is a tangent to C .

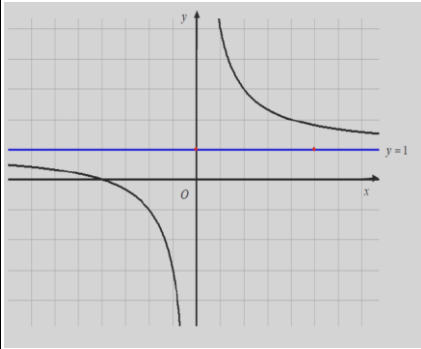
(3)

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Question	Scheme	Marks	AOs	
7 (a)		$\frac{1}{x}$ shape in 1st quadrant	M1	1.1b
		Correct	A1	1.1b
		Asymptote $y = 1$	B1	1.2
		(3)		
(b)	Combines equations $\Rightarrow \frac{k^2}{x} + 1 = -2x + 5$	M1	1.1b	
	$(\times x) \Rightarrow k^2 + 1x = -2x^2 + 5x \Rightarrow 2x^2 - 4x + k^2 = 0^*$	A1*	2.1	
	(2)			
(c)	Attempts to set $b^2 - 4ac = 0$	M1	3.1a	
	$8k^2 = 16$	A1	1.1b	
	$k = \pm\sqrt{2}$	A1	1.1b	
	(3)			
(8 marks)				
Notes				
(a)	<p>M1: For the shape of a $\frac{1}{x}$ type curve in Quadrant 1. It must not cross either axis and have acceptable curvature. Look for a negative gradient changing from $-\infty$ to 0 condoning "slips of the pencil". (See Practice and Qualification for clarification)</p> <p>A1: Correct shape and position for both branches. It must lie in Quadrants 1, 2 and 3 and have the correct curvature including asymptotic behaviour</p> <p>B1: Asymptote given as $y = 1$. This could appear on the diagram or within the text. Note that the curve does not need to be asymptotic at $y = 1$ but this must be the only horizontal asymptote offered by the candidate.</p>			
(b)	<p>M1: Attempts to combine $y = \frac{k^2}{x} + 1$ with $y = -2x + 5$ to form an equation in just x</p> <p>A1*: Multiplies by x (the processed line must be seen) and proceeds to given answer with no slips. Condone if the order of the terms are different $2x^2 + k^2 - 4x = 0$</p>			
(c)	<p>M1: Deduces that $b^2 - 4ac = 0$ or equivalent for the given equation. If a, b and c are stated only accept $a = 2, b = \pm 4, c = k^2$ so $4^2 - 4 \times 2 \times k^2 = 0$</p> <p>Alternatively completes the square $x^2 - 2x + \frac{1}{2}k^2 = 0 \Rightarrow (x-1)^2 = 1 - \frac{1}{2}k^2 \Rightarrow "1 - \frac{1}{2}k^2" = 0$</p> <p>A1: $8k^2 = 16$ or exact simplified equivalent. Eg $8k^2 - 16 = 0$</p>			

If a , b and c are stated they must be correct. Note that b^2 appearing as 4^2 is correct

Note on Question 7 continue

A1: $k = \pm\sqrt{2}$ and following correct a , b and c if stated

A solution via differentiation would be awarded as follows

M1: Sets the gradient of the curve $= -2 \Rightarrow -\frac{k^2}{x^2} = -2 \Rightarrow x = (\pm)\frac{k}{\sqrt{2}}$ oe and attempts to

substitute into $2x^2 - 4x + k^2 = 0$

A1: $2k^2 = (\pm)2\sqrt{2}k$ oe

A1: $k = \pm\sqrt{2}$

Question	Scheme	Marks	AOs
11 (a)	Attempts $f(4) = 2 \times 4^3 - 13 \times 4^2 + 8 \times 4 + 48$	M1	1.1b
	$f(4) = 0 \Rightarrow (x-4)$ is a factor	A1	1.1b
		(2)	
(b)	$2x^3 - 13x^2 + 8x + 48 = (x-4)(2x^2 \dots x - 12)$	M1	2.1
	$= (x-4)(2x^2 - 5x - 12)$	A1	1.1b
	Attempts to factorise quadratic factor or solve quadratic eqn	dM1	1.1b
	$f(x) = (x-4)^2(2x+3) \Rightarrow f(x) = 0$ has only two roots, 4 and -1.5	A1	2.4
		(4)	
(c)	Deduces either three roots or deduces that $f(x)$ is moved down two units	M1	2.2a
	States three roots, as when $f(x)$ is moved down two units there will be three points of intersection (with the x - axis)	A1	2.4
		(2)	
(d)	For sight of $k = \pm 4, \pm \frac{3}{2}$	M1	1.1b
	$k = 4, -\frac{3}{2}$	A1ft	1.1b
		(2)	

(10 marks)

Notes

(a)**M1:** Attempts to calculate $f(4)$.Do not accept $f(4) = 0$ without sight of embedded values or calculations.If values are not embedded look for two correct terms from $f(4) = 128 - 208 + 32 + 48$ Alternatively attempts to divide by $(x-4)$. Accept via long division or inspection.

See below for awarding these marks.

A1: Correct reason with conclusion. Accept $f(4) = 0$, hence factor as long as M1 has been scored.This should really be stated on one line after having performed a correct calculation. It could appear as a preamble if the candidate states "If $f(4) = 0$, then $(x-4)$ is a factor before doing the calculation and then writing hence proven or \checkmark oe.If division/inspection is attempted it must be correct and there must be some attempt to explain why they have shown that $(x-4)$ is a factor. Eg Via division they must state that there is no remainder, hence factor**(b)****M1:** Attempts to find the quadratic factor by inspection (correct first and last terms) or by division (correct first two terms)

So for inspection award for $2x^3 - 13x^2 + 8x + 48 = (x-4)(2x^2 \dots x \pm 12)$

$$\begin{array}{r} 2x^2 - 5x \\ x-4 \overline{) 2x^3 - 13x^2 + 8x + 48} \end{array}$$

For division look for
$$\frac{2x^3 - 8x^2}{-5x^2}$$

A1: Correct quadratic factor $(2x^2 - 5x - 12)$ For division award for sight of this "in the correct place" You don't have to see it paired with the $(x-4)$ for this mark.

If a student has used division in part (a) they can score the M1 A1 in (b) as soon as they start attempting to factorise their $(2x^2 - 5x - 12)$.

dM1: Correct attempt to solve or factorise their $(2x^2 - 5x - 12)$ including use of formula

Apply the usual rules $(2x^2 - 5x - 12) = (ax+b)(cx+d)$ where $ac = \pm 2$ and $bd = \pm 12$

Allow the candidate to move from $(x-4)(2x^2 - 5x - 12)$ to $(x-4)^2(2x+3)$ for this mark.

A1: Via factorisation

Factorises twice to $f(x) = (x-4)(2x+3)(x-4)$ or $f(x) = (x-4)^2(2x+3)$ or

$f(x) = 2(x-4)^2\left(x + \frac{3}{2}\right)$ followed by a valid explanation why there are only two roots.

The explanation can be as simple as

- hence $x=4$ and $-\frac{3}{2}$ (only). The roots must be correct
- only two distinct roots as 4 is a repeated root

There must be some understanding between roots and factors.

E.g. $f(x) = (x-4)^2(2x+3)$

only two distinct roots is insufficient.

This would require two distinct factors, so there are two distinct roots.

Via solving.

Factorises to $(x-4)(2x^2 - 5x - 12)$ and solves $2x^2 - 5x - 12 = 0 \Rightarrow x = 4, -\frac{3}{2}$ followed

by an explanation that the roots are $4, 4, -\frac{3}{2}$ so only two distinct roots.

Note that this question asks the candidate to use algebra so you cannot accept any attempt to use their calculators to produce the answers.

(c)

M1: For a valid **deduction**.

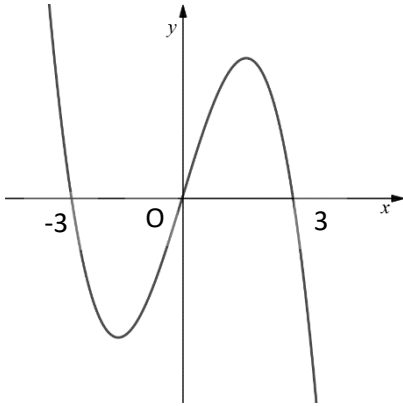
Accept **either** there are 3 roots **or** states that it is a solution of $f(x) = 2$ or $f(x) - 2 = 0$

A1: Fully explains:

Eg. States three roots, as $f(x)$ is moved down by **two** units (giving three points of intersection with the x -axis)

Eg. States three roots, as it is where $f(x) = 2$ (You may see $y = 2$ drawn on the diagram)

(d)**M1:** For sight of ± 4 **and** $\pm \frac{3}{2}$ Follow through on \pm their roots.**A1ft:** $k = 4, -\frac{3}{2}$ Follow through on their roots. Accept $4, -\frac{3}{2}$ but not $x = 4, -\frac{3}{2}$

Question	Scheme	Marks	AOs	
7(a)	$9x - x^3 = x(9 - x^2)$	M1	1.1b	
	$9x - x^3 = x(3 - x)(3 + x)$ oe	A1	1.1b	
		(2)		
(b)		A cubic with correct orientation	B1	1.1b
		Passes through origin, (3, 0) and (-3, 0)	B1	1.1b
			(2)	
(c)	$y = 9x - x^3 \Rightarrow \frac{dy}{dx} = 9 - 3x^2 = 0 \Rightarrow x = (\pm)\sqrt{3} \Rightarrow y = \dots$	M1	3.1a	
	$y = (\pm)6\sqrt{3}$	A1	1.1b	
	$\{k \in \mathbb{R} : -6\sqrt{3} < k < 6\sqrt{3}\}$ oe	A1ft	2.5	
		(3)		

(7 marks)

Notes

(a)

M1: Takes out a factor of x or $-x$. Scored for $\pm x(\pm 9 \pm x^2)$ May be implied by the correct answer or $\pm x(\pm x \pm 3)(\pm x \pm 3)$.

Also allow if they attempt to take out a factor of $(\pm x \pm 3)$ so score for $(\pm x \pm 3)(\pm 3x \pm x^2)$

A1: Correct factorisation. $x(3-x)(3+x)$ on its own scores M1A1.

Allow eg $-x(x-3)(x+3)$, $x(x-3)(-x-3)$ or other equivalent expressions

Condone an = 0 appearing on the end and condone eg x written as $(x+0)$.

(b)

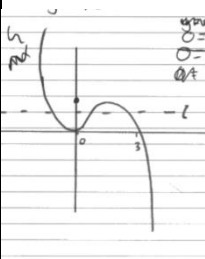
B1: Correct shape (negative cubic) appearing anywhere on a set of axes. It must have a minimum to the left and maximum to the right. Be tolerant of pen slips. Judge the intent of the shape. (see examples)

B1: Passes **through** each of the origin, (3, 0) and (-3, 0) and no other points on the x axis. (The graph should not turn on any of these points).

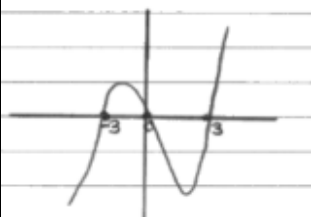
The points may be indicated as just 3 and -3 on the axes. Condone x and y to be the wrong way round eg (0, -3) for (-3, 0) as long as it is on the correct axis but do not allow (-3, 0) to be labelled as (3, 0).

Examples

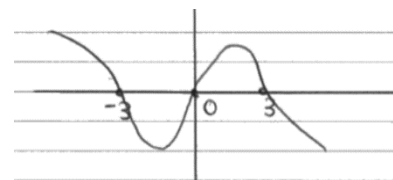
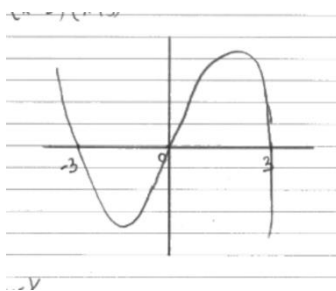
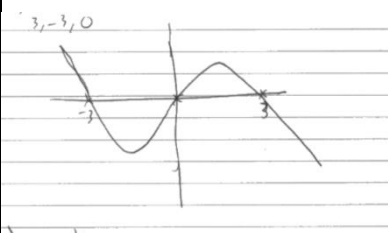
B1B0



B0B1



B1B1



(c) ***Be aware the value of y can be solved directly using a calculator which is not acceptable***

M1: Uses a correct strategy for the y value of either the maximum or minimum. E.g. differentiates to achieve a quadratic, solves $\frac{dy}{dx} = 0$ and uses their x to find y

A1: Either or both of the values $(\pm)6\sqrt{3}$.

Cannot be scored for an answer without any working seen.

A1ft: Correct answer in any acceptable set notation following through their $6\sqrt{3}$.

Condone $\{-6\sqrt{3} < k < 6\sqrt{3}\}$ or $\{-6\sqrt{3} < k\} \cap \{k < 6\sqrt{3}\}$ but not

$\{-6\sqrt{3} < k\} \cup \{k < 6\sqrt{3}\}$

Note: If there is a contradiction of their solution on different lines of working do not penalise intermediate working and mark what appears to be their final answer.

Must be in terms of k