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Y1P10 XMQs and MS

(Total: 51 marks)

1. P2_Sample Q2 . 3 marks - Y1P10 Trigonometric identities and equations
2. P2_Sample Q12. 8 marks - Y1P10 Trigonometric identities and equations
3. P1(AS)_2018 Q12. 8 marks - Y1P10 Trigonometric identities and equations
4. P1(AS)_2019 Q12. 7 marks - Y1P10 Trigonometric identities and equations
5. P1(AS)_2020 Q9 . 8 marks - Y1P10 Trigonometric identities and equations
6. P1(AS)_2021 Q12. 9 marks - Y1P10 Trigonometric identities and equations
7. P1(AS)_2022 Q13. 8 marks - Y1P10 Trigonometric identities and equations

2. Some A level students were given the following question.

Solve, for $-90^\circ < \theta < 90^\circ$, the equation

$$\cos \theta = 2 \sin \theta$$

The attempts of two of the students are shown below.

Student A

$$\cos \theta = 2 \sin \theta$$

$$\tan \theta = 2$$

$$\theta = 63.4^\circ$$

Student B

$$\cos \theta = 2 \sin \theta$$

$$\cos^2 \theta = 4 \sin^2 \theta$$

$$1 - \sin^2 \theta = 4 \sin^2 \theta$$

$$\sin^2 \theta = \frac{1}{5}$$

$$\sin \theta = \pm \frac{1}{\sqrt{5}}$$

$$\theta = \pm 26.6^\circ$$

- (a) Identify an error made by student A.

(1)

Student B gives $\theta = -26.6^\circ$ as one of the answers to $\cos \theta = 2 \sin \theta$.

- (b) (i) Explain why this answer is incorrect.

- (ii) Explain how this incorrect answer arose.

(2)

(Total for Question 2 is 3 marks)

Paper 2: Pure Mathematics 2 Mark Scheme

Question	Scheme	Marks	AOs
1	Sets $f(-2) = 0 \Rightarrow 2 \times (-2)^3 - 5 \times (-2)^2 + a \times -2 + a = 0$	M1	3.1a
	Solves linear equation $2a - a = -36 \Rightarrow a =$	dM1	1.1b
	$\Rightarrow a = -36$	A1	1.1b
(3 marks)			
Notes:			
M1: Selects a suitable method given that $(x + 2)$ is a factor of $f(x)$ Accept either setting $f(-2) = 0$ or attempted division of $f(x)$ by $(x + 2)$			
dM1: Solves linear equation in a . Minimum requirement is that there are two terms in ' a ' which must be collected to get $..a = .. \Rightarrow a =$			
A1: $a = -36$			

Question	Scheme	Marks	AOs
2(a)	Identifies an error for student A: They use $\frac{\cos \theta}{\sin \theta} = \tan \theta$ It should be $\frac{\sin \theta}{\cos \theta} = \tan \theta$	B1	2.3
		(1)	
(b)	(i) Shows $\cos(-26.6^\circ) \neq 2 \sin(-26.6^\circ)$, so cannot be a solution	B1	2.4
	(ii) Explains that the incorrect answer was introduced by squaring	B1	2.4
		(2)	
(3 marks)			
Notes:			
(a)			
B1: Accept a response of the type 'They use $\frac{\cos \theta}{\sin \theta} = \tan \theta$. This is incorrect as $\frac{\sin \theta}{\cos \theta} = \tan \theta$ ' It can be implied by a response such as 'They should get $\tan \theta = \frac{1}{2}$ not $\tan \theta = 2$ ' Accept also statements such as 'it should be $\cot \theta = 2$ '			
(b)			
B1: Accept a response where the candidate shows that -26.6° is not a solution of $\cos \theta = 2 \sin \theta$. This can be shown by, for example, finding both $\cos(-26.6^\circ)$ and $2 \sin(-26.6^\circ)$ and stating that they are not equal. An acceptable alternative is to state that $\cos(-26.6^\circ) = +ve$ and $2 \sin(-26.6^\circ) = -ve$ and stating that they therefore cannot be equal.			
B1: Explains that the incorrect answer was introduced by squaring Accept an example showing this. For example $x = 5$ squared gives $x^2 = 25$ which has answers ± 5			

Question	Scheme	Marks	AOs
12(a)	Uses $\cos^2 x = 1 - \sin^2 x \Rightarrow 3\sin^2 x + \sin x + 8 = 9(1 - \sin^2 x)$	M1	3.1a
	$\Rightarrow 12\sin^2 x + \sin x - 1 = 0$	A1	1.1b
	$\Rightarrow (4\sin x - 1)(3\sin x + 1) = 0$	M1	1.1b
	$\Rightarrow \sin x = \frac{1}{4}, -\frac{1}{3}$	A1	1.1b
	Uses arcsin to obtain two correct values	M1	1.1b
	All four of $x = 14.48^\circ, 165.52^\circ, -19.47^\circ, -160.53^\circ$	A1	1.1b
		(6)	
(b)	Attempts $2\theta - 30^\circ = -19.47^\circ$	M1	3.1a
	$\Rightarrow \theta = 5.26^\circ$	A1ft	1.1b
		(2)	
(8 marks)			
Notes:			
(a)			
M1:	Substitutes $\cos^2 x = 1 - \sin^2 x$ into $3\sin^2 x + \sin x + 8 = 9\cos^2 x$ to create a quadratic equation in just $\sin x$		
A1:	$12\sin^2 x + \sin x - 1 = 0$ or exact equivalent		
M1:	Attempts to solve their quadratic equation in $\sin x$ by a suitable method. These could include factorisation, formula or completing the square.		
A1:	$\sin x = \frac{1}{4}, -\frac{1}{3}$		
M1:	Obtains two correct values for their $\sin x = k$		
A1:	All four of $x = 14.48^\circ, 165.52^\circ, -19.47^\circ, -160.53^\circ$		
(b)			
M1:	For setting $2\theta - 30^\circ = \text{their } '-19.47^\circ'$		
A1ft:	$\theta = 5.26^\circ$ but allow a follow through on their $'-19.47^\circ'$		

Question	Scheme	Marks	AOs
12 (a)	$4\cos\theta - 1 = 2\sin\theta \tan\theta \Rightarrow 4\cos\theta - 1 = 2\sin\theta \times \frac{\sin\theta}{\cos\theta}$	M1	1.2
	$\Rightarrow 4\cos^2\theta - \cos\theta = 2\sin^2\theta$ oe	A1	1.1b
	$\Rightarrow 4\cos^2\theta - \cos\theta = 2(1 - \cos^2\theta)$	M1	1.1b
	$6\cos^2\theta - \cos\theta - 2 = 0$ *	A1*	2.1
		(4)	
(b)	For attempting to solve given quadratic	M1	1.1b
	$(\cos 3x) = \frac{2}{3}, -\frac{1}{2}$	B1	1.1b
	$x = \frac{1}{3} \arccos\left(\frac{2}{3}\right)$ or $\frac{1}{3} \arccos\left(-\frac{1}{2}\right)$	M1	1.1b
	$x = 40^\circ, 80^\circ, \text{awrt } 16.1^\circ$	A1	2.2a
		(4)	
(8 marks)			
Notes			
(a)			
M1: Recall and use the identity $\tan\theta = \frac{\sin\theta}{\cos\theta}$ Note that it cannot just be stated.			
A1: $4\cos^2\theta - \cos\theta = 2\sin^2\theta$ oe.			
This is scored for a correct line that does not contain any fractional terms.			
It may be awarded later in the solution after the identity $1 - \cos^2\theta = \sin^2\theta$ has been used Eg for $\cos\theta(4\cos\theta - 1) = 2(1 - \cos^2\theta)$ or equivalent			
M1: Attempts to use the correct identity $1 - \cos^2\theta = \sin^2\theta$ to form an equation in just $\cos\theta$			
A1*: Proceeds to correct answer through rigorous and clear reasoning. No errors in notation or bracketing. For example $\sin^2\theta = \sin\theta^2$ is an error in notation			
(b)			
M1: For attempting to solve the given quadratic " $6y^2 - y - 2 = 0$ " where y could be $\cos 3x$, $\cos x$, or even just y . When factoring look for $(ay + b)(cy + d)$ where $ac = \pm 6$ and $bd = \pm 2$			
This may be implied by the correct roots (even award for $\left(y \pm \frac{2}{3}\right)\left(y \pm \frac{1}{2}\right)$), an attempt at factorising, an attempt at the quadratic formula, an attempt at completing the square and even \pm the correct roots.			
B1: For the roots $\frac{2}{3}, -\frac{1}{2}$ oe			
M1: Finds at least one solution for x from $\cos 3x$ within the given range for their $\frac{2}{3}, -\frac{1}{2}$			
A1: $x = 40^\circ, 80^\circ, \text{awrt } 16.1^\circ$ only Withhold this mark if there are any other values even if they are outside the range. Condone 40 and 80 appearing as 40.0 and 80.0			

Question	Scheme	Marks	AOs
12(a)	$\frac{10\sin^2 \theta - 7\cos \theta + 2}{3 + 2\cos \theta} \equiv \frac{10(1 - \cos^2 \theta) - 7\cos \theta + 2}{3 + 2\cos \theta}$	M1	1.1b
	$\equiv \frac{12 - 7\cos \theta - 10\cos^2 \theta}{3 + 2\cos \theta}$	A1	1.1b
	$\equiv \frac{(3 + 2\cos \theta)(4 - 5\cos \theta)}{3 + 2\cos \theta}$	M1	1.1b
	$\equiv 4 - 5\cos \theta *$	A1*	2.1
		(4)	
(b)	$4 + 3\sin x = 4 - 5\cos x \Rightarrow \tan x = -\frac{5}{3}$	M1	2.1
	$x = \text{awrt } 121^\circ, 301^\circ$	A1 A1	1.1b 1.1b
		(3)	
(7 marks)			

Notes

(a)

M1: Uses the identity $\sin^2 \theta = 1 - \cos^2 \theta$ within the fraction

A1: Correct (simplified) expression in just $\cos \theta$ $\frac{12 - 7\cos \theta - 10\cos^2 \theta}{3 + 2\cos \theta}$ or exact equivalent such as $\frac{(3 + 2\cos \theta)(4 - 5\cos \theta)}{3 + 2\cos \theta}$ Allow for $\frac{12 - 7u - 10u^2}{3 + 2u}$ where they introduce $u = \cos \theta$

We would condone mixed variables here.

M1: A correct attempt to factorise the numerator, usual rules. Allow candidates to use $u = \cos \theta$ oe**A1*:** A fully correct proof with correct notation and no errors.Only withhold the last mark for (1) Mixed variable e.g. θ and x 's (2) Poor notation $\cos \theta^2 \leftrightarrow \cos^2 \theta$ or $\sin^2 = 1 - \cos^2$ within the solution.

Don't penalise incomplete lines if it is obvious that it is just part of their working

E.g. $\frac{10\sin^2 \theta - 7\cos \theta + 2}{3 + 2\cos \theta} \equiv \frac{10(1 - \cos^2 \theta) - 7\cos \theta + 2}{3 + 2\cos \theta} = \frac{12 - 7\cos \theta - 10\cos^2 \theta}{3 + 2\cos \theta}$

(b)

M1: Attempts to use part (a) and proceeds to an equation of the form $\tan x = k$, $k \neq 0$ Condone $\theta \leftrightarrow x$ Do not condone $a \tan x = 0 \Rightarrow \tan x = b \Rightarrow x = \dots$ Alternatively squares $3\sin x = -5\cos x$ and uses $\sin^2 x = 1 - \cos^2 x$ oe to reach $\sin x = A, -1 < A < 1$ or $\cos x = B, -1 < B < 1$ **A1:** Either $x = \text{awrt } 121^\circ$ or 301° . Condone awrt 2.11 or 5.25 which are the radian solutions**A1:** Both $x = \text{awrt } 121^\circ$ and 301° and no other solutions.

Answers without working, or with no incorrect working in (b).

Question states hence or otherwise so allow

For 3 marks both $x = \text{awrt } 121^\circ$ and 301° and no other solutions.For 1 marks scored SC 100 for either $x = \text{awrt } 121^\circ$ or 301°

Alternative proof in part (a):

M1: Multiplies across and forms 3TQ in $\cos \theta$ on rhs

$$10\sin^2 \theta - 7\cos \theta + 2 = (4 - 5\cos \theta)(3 + 2\cos \theta) \Rightarrow 10\sin^2 \theta - 7\cos \theta + 2 = A\cos^2 \theta + B\cos \theta + C$$

A1: Correct identity formed $10\sin^2 \theta - 7\cos \theta + 2 = -10\cos^2 \theta - 7\cos \theta + 12$

dM1: Uses $\cos^2 \theta = 1 - \sin^2 \theta$ on the rhs or $\sin^2 \theta = 1 - \cos^2 \theta$ on the lhs

Alternatively proceeds to $10\sin^2 \theta + 10\cos^2 \theta = 10$ and makes a statement about

$$\sin^2 \theta + \cos^2 \theta = 1 \text{ oe}$$

A1*: Shows that $(4 - 5\cos \theta)(3 + 2\cos \theta) \equiv 10\sin^2 \theta - 7\cos \theta + 2$ oe AND makes a minimal statement "hence true"

9.

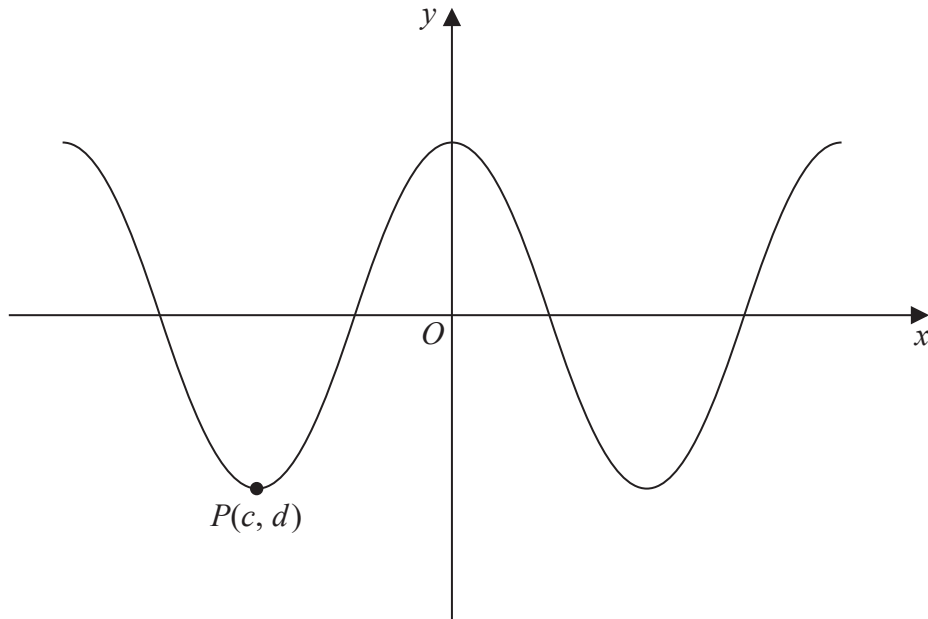


Figure 3

Figure 3 shows part of the curve with equation $y = 3 \cos x^\circ$.

The point $P(c, d)$ is a minimum point on the curve with c being the smallest negative value of x at which a minimum occurs.

(a) State the value of c and the value of d . (1)

(b) State the coordinates of the point to which P is mapped by the transformation which transforms the curve with equation $y = 3 \cos x^\circ$ to the curve with equation

(i) $y = 3 \cos \left(\frac{x^\circ}{4} \right)$

(ii) $y = 3 \cos (x - 36)^\circ$ (2)

(c) Solve, for $450^\circ \leq \theta < 720^\circ$,

$$3 \cos \theta = 8 \tan \theta$$

giving your solution to one decimal place.

In part (c) you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable. (5)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

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Question	Scheme	Marks	AOs
9 (a)	$(-180^\circ, -3)$	B1	1.1b
		(1)	
(b)	(i) $(-720^\circ, -3)$	B1ft	2.2a
	(ii) $(-144^\circ, -3)$	B1 ft	2.2a
		(2)	
(c)	Attempts to use both $\tan \theta = \frac{\sin \theta}{\cos \theta}$, $\sin^2 \theta + \cos^2 \theta = 1$ and solves a quadratic equation in $\sin \theta$ to find at least one value of θ	M1	3.1a
	$3 \cos \theta = 8 \tan \theta \Rightarrow 3 \cos^2 \theta = 8 \sin \theta$	B1	1.1b
	$3 \sin^2 \theta + 8 \sin \theta - 3 = 0$ $(3 \sin \theta - 1)(\sin \theta + 3) = 0$	M1	1.1b
	$\sin \theta = \frac{1}{3}$	A1	2.2a
	awrt 520.5° only	A1	2.1
		(5)	
(8 marks)			

(a)

B1: Deduces that $P(-180^\circ, -3)$ or $c = -180^{(o)}, d = -3$

(b)(i)

B1ft: Deduces that $P'(-720^\circ, -3)$ Follow through on their $(c, d) \rightarrow (4c, d)$ where d is negative

(b)(ii)

B1ft: Deduces that $P'(-144^\circ, -3)$ Follow through on their $(c, d) \rightarrow (c + 36^\circ, d)$ where d is negative

(c)

M1: An overall problem solving mark, condoning slips, for an attempt to

- use $\tan \theta = \frac{\sin \theta}{\cos \theta}$,
- use $\pm \sin^2 \theta \pm \cos^2 \theta = \pm 1$
- find at least one value of θ from a quadratic equation in $\sin \theta$

B1: Uses the correct identity and multiplies across to give $3 \cos \theta = 8 \tan \theta \Rightarrow 3 \cos^2 \theta = 8 \sin \theta$ oe**M1:** Uses the correct identity $\sin^2 \theta + \cos^2 \theta = 1$ to form a 3TQ in $\sin \theta$ which they attempt to solve using an appropriate method. It is OK to use a calculator to solve this**A1:** $\sin \theta = \frac{1}{3}$ Accept sight of $\frac{1}{3}$. Ignore any reference to the other root even if it is "used"**A1:** Full method with all identities correct leading to the answer of awrt 520.5° and no other values.

12. In this question you should show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 < \theta \leq 450^\circ$, the equation

$$5 \cos^2 \theta = 6 \sin \theta$$

giving your answers to one decimal place.

(5)

(ii) (a) A student's attempt to solve the question

“Solve, for $-90^\circ < x < 90^\circ$, the equation $3 \tan x - 5 \sin x = 0$ ”

is set out below.

$$\begin{aligned} 3 \tan x - 5 \sin x &= 0 \\ 3 \frac{\sin x}{\cos x} - 5 \sin x &= 0 \\ 3 \sin x - 5 \sin x \cos x &= 0 \\ 3 - 5 \cos x &= 0 \\ \cos x &= \frac{3}{5} \\ x &= 53.1^\circ \end{aligned}$$

Identify two errors or omissions made by this student, giving a brief explanation of each.

(2)

The first four positive solutions, in order of size, of the equation

$$\cos(5\alpha + 40^\circ) = \frac{3}{5}$$

are $\alpha_1, \alpha_2, \alpha_3$ and α_4

(b) Find, to the nearest degree, the value of α_4

(2)

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Question	Scheme	Marks	AOs
12 (i)	Uses $\cos^2 \theta = 1 - \sin^2 \theta$ $5 \cos^2 \theta = 6 \sin \theta \Rightarrow 5 \sin^2 \theta + 6 \sin \theta - 5 = 0$	M1 A1	1.2 1.1b
	$\Rightarrow \sin \theta = \frac{-3 + \sqrt{34}}{5} \Rightarrow \theta = \dots$	dM1	3.1a
	$\Rightarrow \theta = 34.5^\circ, 145.5^\circ, 394.5^\circ$	A1 A1	1.1b 1.1b
		(5)	
(ii) (a)	One of <ul style="list-style-type: none"> • They cancel by $\sin x$ (and hence they miss the solution $\sin x = 0 \Rightarrow x = 0$) • They do not find all the solutions of $\cos x = \frac{3}{5}$ (in the given range) or they missed the solution $x = -53.1^\circ$ 	B1	2.3
	Both of the above	B1	2.3
		(2)	
(ii) (b)	Sets $5\alpha + 40^\circ = 720^\circ - 53.1^\circ$	M1	3.1a
	$\alpha = 125^\circ$	A1	1.1b
		(2)	

(9 marks)

Notes

(i)

M1: Uses $\cos^2 \theta = 1 - \sin^2 \theta$ to form a 3TQ in $\sin \theta$

A1: Correct 3TQ=0 $5 \sin^2 \theta + 6 \sin \theta - 5 = 0$

dM1: Solves their 3TQ in $\sin \theta$ to produce one value for θ . It is dependent upon having used $\cos^2 \theta = \pm 1 \pm \sin^2 \theta$

A1: Two of awrt $\theta = 34.5^\circ, 145.5^\circ, 394.5^\circ$ (or if in radians two of awrt 0.60, 2.54, 6.89)

A1: All three of awrt $\theta = 34.5^\circ, 145.5^\circ, 394.5^\circ$ and no other values

(i) (a)

See scheme

(ii)(b)

M1: Sets $5\alpha + 40^\circ = 666.9^\circ$ o.e.

A1: awrt $\alpha = 125^\circ$

Question	Scheme	Marks	AOs	
13(a)	$\frac{1}{\cos \theta} + \tan \theta = \frac{1 + \sin \theta}{\cos \theta} \text{ or } \frac{(1 + \sin \theta) \cos \theta}{\cos^2 \theta}$	M1	1.1b	
	$= \frac{1 + \sin \theta}{\cos \theta} \times \frac{1 - \sin \theta}{1 - \sin \theta} = \frac{1 - \sin^2 \theta}{\cos \theta (1 - \sin \theta)} = \frac{\cos^2 \theta}{\cos \theta (1 - \sin \theta)}$ or $\frac{(1 + \sin \theta) \cos \theta}{\cos^2 \theta} = \frac{(1 + \sin \theta) \cos \theta}{1 - \sin^2 \theta} = \frac{(1 + \sin \theta) \cos \theta}{(1 + \sin \theta)(1 - \sin \theta)}$	dM1	2.1	
	$= \frac{\cos \theta}{1 - \sin \theta} *$	A1*	1.1b	
		(3)		
(b)	$\frac{1}{\cos 2x} + \tan 2x = 3 \cos 2x$ $\Rightarrow 1 + \sin 2x = 3 \cos^2 2x = 3(1 - \sin^2 2x)$	$\frac{\cos 2x}{1 - \sin 2x} = 3 \cos 2x$ $\Rightarrow \cos 2x = 3 \cos 2x (1 - \sin 2x)$	M1	2.1
	$\Rightarrow 3 \sin^2 2x + \sin 2x - 2 = 0$	$\Rightarrow \cos 2x (2 - 3 \sin 2x) = 0$	A1	1.1b
	$\sin 2x = \frac{2}{3}, (-1) \Rightarrow 2x = \dots \Rightarrow x = \dots$		M1	1.1b
	$x = 20.9^\circ, 69.1^\circ$		A1	1.1b
			A1	1.1b
			(5)	
(8 marks)				
Notes				
(a) If starting with the LHS: Condone if another variable for θ is used except for the final mark				
M1: Combines terms with a common denominator. The numerator must be correct for their common denominator.				
dM1: Either:				
<ul style="list-style-type: none"> $\frac{1 + \sin \theta}{\cos \theta}$: Multiplies numerator and denominator by $1 - \sin \theta$, uses the difference of two squares and applies $\cos^2 \theta = 1 - \sin^2 \theta$ $\frac{(1 + \sin \theta) \cos \theta}{\cos^2 \theta}$: Uses $\cos^2 \theta = 1 - \sin^2 \theta$ on the denominator, applies the difference of two squares 				
It is dependent on the previous method mark.				
A1*: Fully correct proof with correct notation and no errors in the main body of their work. Withhold this mark for writing eg sin instead of $\sin \theta$ anywhere in the solution and for eg $\sin \theta^2$ instead of $\sin^2 \theta$				

Alt(a) If starting with the RHS: Condone if another variable is used for θ except for the final mark

M1: Multiplies by $\frac{1 + \sin \theta}{1 + \sin \theta}$ leading to $\frac{\cos \theta(1 + \sin \theta)}{1 - \sin^2 \theta}$ or

Multiplies by $\frac{\cos \theta}{\cos \theta}$ leading to $\frac{\cos^2 \theta}{\cos \theta(1 - \sin \theta)}$

dM1: Applies $\cos^2 \theta = 1 - \sin^2 \theta$ and cancels the $\cos \theta$ factor from the numerator and denominator leading to $\frac{1 + \sin \theta}{\cos \theta}$ or

Applies $\cos^2 \theta = 1 - \sin^2 \theta$ and uses the difference of two squares leading to $\frac{(1 + \sin \theta)(1 - \sin \theta)}{\cos \theta(1 - \sin \theta)}$

It is dependent on the previous method mark.

A1*: Fully correct proof with correct notation and no errors in the main body of their work. If they work from both the LHS and the RHS and meet in the middle with both sides the same then they need to conclude at the end by stating the original equation.

(b) ***Be aware that this can be done entirely on their calculator which is not acceptable***

M1: Either multiplies through by $\cos 2x$ and applies $\cos^2 2x = 1 - \sin^2 2x$ to obtain an equation in $\sin 2x$ only or alternatively sets $\frac{\cos 2x}{1 - \sin 2x} = 3 \cos 2x$ and multiplies by $1 - \sin 2x$

A1: Correct equation or equivalent. The $= 0$ may be implied by their later work (Condone notational slips in their working)

M1: Solves for $\sin 2x$, uses arcsin to obtain at least one value for $2x$ and divides by 2 to obtain at least one value for x . The roots of the quadratic can be found using a calculator. They cannot just write down values for x from their quadratic in $\sin 2x$

A1: For 1 of the required angles. Accept awrt 21 or awrt 69. Also accept awrt 0.36 rad or awrt 1.21 rad

A1: For both angles (awrt 20.9 and awrt 69.1) and no others inside the range. If they find $x = 45$ it must be rejected. (Condone notational slips in their working)