

Y1M11 XMQs and MS

(Total: 52 marks)

1. P3_Sample Q6 . 6 marks - Y1M11 Variable acceleration
2. P32(AS)_2018 Q8 . 10 marks - Y1M11 Variable acceleration
3. P32(AS)_2019 Q3 . 8 marks - Y1M11 Variable acceleration
4. P32(AS)_2020 Q3 . 9 marks - Y1M11 Variable acceleration
5. P32(AS)_2021 Q2 . 10 marks - Y1M11 Variable acceleration
6. P32(AS)_2022 Q3 . 9 marks - Y1M11 Variable acceleration

SECTION B: MECHANICS

Answer ALL questions. Write your answers in the spaces provided.

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

6. At time t seconds, where $t \geq 0$, a particle P moves so that its acceleration $\mathbf{a} \text{ m s}^{-2}$ is given by

$$\mathbf{a} = 5t\mathbf{i} - 15t^{\frac{1}{2}}\mathbf{j}$$

When $t = 0$, the velocity of P is $20\mathbf{i} \text{ m s}^{-1}$

Find the speed of P when $t = 4$

(6)

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Question	Scheme	Marks	AOs
6	Integrate \mathbf{a} w.r.t. time	M1	1.1a
	$\mathbf{v} = \frac{5t^2}{2}\mathbf{i} - 10t^{\frac{3}{2}}\mathbf{j} + \mathbf{C}$ (allow omission of \mathbf{C})	A1	1.1b
	$\mathbf{v} = \frac{5t^2}{2}\mathbf{i} - 10t^{\frac{3}{2}}\mathbf{j} + 20\mathbf{i}$	A1	1.1b
	When $t = 4$, $\mathbf{v} = 60\mathbf{i} - 80\mathbf{j}$	M1	1.1b
	Attempt to find magnitude: $\sqrt{(60^2 + 80^2)}$	M1	3.1a
	Speed = 100 m s^{-1}	A1ft	1.1b
			(6 marks)
Notes:			
<p>1st M1: for integrating \mathbf{a} w.r.t. time (powers of t increasing by 1)</p> <p>1st A1: for a correct \mathbf{v} expression without \mathbf{C}</p> <p>2nd A1: for a correct \mathbf{v} expression including \mathbf{C}</p> <p>2nd M1: for putting $t = 4$ into their \mathbf{v} expression</p> <p>3rd M1: for finding magnitude of their \mathbf{v}</p> <p>3rd A1: ft for 100 m s^{-1}, follow through on an incorrect \mathbf{v}</p>			

Question	Scheme	Marks	AOs
8(a)	Multiply out and differentiate wrt to time (or use of product rule i.e. must have two terms with correct structure)	M1	1.1a
	$v = 2t^3 - 3t^2 + t$	A1	1.1b
	$2t^3 - 3t^2 + t = 0$ and solve: $t(2t - 1)(t - 1) = 0$	DM1	1.1b
	$t = 0$ or $t = \frac{1}{2}$ or $t = 1$; any two	A1	1.1b
	All three	A1	1.1b
		(5)	
(b)	Find x when $t = 0, \frac{1}{2}, 1$ and 2 : $(0, \frac{1}{32}, 0, 2)$	M1	2.1
	Distance = $\frac{1}{32} + \frac{1}{32} + 2$	M1	2.1
	$2\frac{1}{16}$ (m) oe or 2.06 or better	A1	1.1b
		(3)	
(c)	$x = \frac{1}{2}t^2(t - 1)^2$	M1	3.1a
	$\frac{1}{2}$ perfect square so $x \geq 0$ i.e. never negative	A1 cso	2.4
		(2)	
			(10 marks)
Notes:			
(a)			
M1: Must have 3 terms and at least two powers going down by 1			
A1: A correct expression			
DM1: Dependent on first M, for equating to zero and attempting to solve a <u>cubic</u>			
A1: Any two of the three values (Two correct answers can imply a correct method)			
A1: The third value			
(b)			
M1: For attempting to find the values of x (at least two) at their t values found in (a) or at $t = 2$ or equivalent e.g. they may integrate their v and sub in at least two of their t values			
M1: Using a correct strategy to combine their distances (must have at least 3 distances)			

A1: $2\frac{1}{16}$ (m) oe or 2.06 or better

(c)

M1: Identify strategy to solve the problem such as:

- (i) writing x as $\frac{1}{2} \times$ perfect square
- (ii) or using x values identified in (b).
- (iii) or using calculus i.e. identifying min points on $x-t$ graph.
- (iv) or using $x-t$ graph.

A1 cso : Fully correct explanation to show that $x \geq 0$ i.e. never negative

Question	Scheme	Marks	AOs	Notes
3(a)	$v = 12 + 4t - t^2 = 0$ and solving	M1	3.1a	Equating v to 0 and solving the quadratic If no evidence of solving, and at least one answer wrong, M0
	$t = 6$ (or -2)	A1	1.1b	6 but allow -2 as well at this stage
	Differentiate v wrt t	M1	1.1a	For differentiation (both powers decreasing by 1)
	$(a = \frac{dv}{dt} \Rightarrow) 4 - 2t$	A1	1.1b	Cao; only need RHS
	When $t = 6$, $a = -8$; Magnitude is $8 \text{ (m s}^{-2}\text{)}$	A1	1.1b	Substitute in $t = 6$ and get $8 \text{ (m s}^{-2}\text{)}$ as the answer . Must be positive . (A0 if two answers given)
		(5)		
(b)	Integrate v wrt t	M1	3.1a	For integration (at least two powers increasing by 1)
	$(s \Rightarrow) 12t + 2t^2 - \frac{1}{3}t^3 (+C)$	A1	1.1b	Correct expression (ignore C) only need RHS Must be used in part (b)
	$t = 3 \Rightarrow \text{distance} = 45 \text{ (m)}$	A1	1.1b	Correct distance. Ignore units
		(3)		
(8 marks)				

Question	Scheme		Marks	AOs
3(a)	$v = 3t - 2t^2 + 14$ and differentiate		M1	3.1a
	$a = \frac{dv}{dt} = 3 - 4t$ or $(7 - 2t) - 2(t + 2)$ using product rule		A1	1.1b
	$3 - 4t = 0$ and solve for t		M1	1.1b
	$t = \frac{3}{4}$ oe		A1	1.1b
			(4)	
3(b)	Solve problem using $v = 0$ to find a value of t $\left(t = \frac{7}{2}\right)$		M1	3.1a
	$v = 3t - 2t^2 + 14$ and integrate		M1	1.1b
	$s = \frac{3t^2}{2} - \frac{2t^3}{3} + 14t$		A1	1.1b
	Substitute $t = \frac{7}{2}$ into their s expression (M0 if using <i>suvat</i>)		M1	1.1b
	$s = \frac{931}{24} = 38\frac{19}{24} = 38.79166..(m)$ Accept 39 or better		A1	1.1b
			(5)	
(9 marks)				
Notes:				
(a)	M1	Multiply out and attempt to differentiate, with at least one power decreasing		
	A1	Correct expression		
	M1	Equate their a to 0 and solve for t		
	A1	cao		
(b)	M1	Uses $v = 0$ to obtain a value of t		
	M1	Attempt to integrate, with at least one power increasing		
	A1	Correct expression		
	M1	Substitute in their value of t , which must have come from using $v = 0$, into their s (must have integrated)		
	A1	39 or better		

Question	Scheme		Marks	AOs
2(a)	Differentiate v w.r.t. t		M1	3.1a
	$a = \frac{dv}{dt} = 10 - 2t$ isw		A1	1.1b
			(2)	
2(b)	Solve problem using $v = 0$ when $t = 6$		M1	3.1a
	$0 = 10t - t^2 - 24$		A1	1.1b
	Solve quadratic oe to find other value of t		M1	1.1b
	$t = 4$		A1	1.1b
			(4)	
2(c)	Integrate v or $-v$ w.r.t. t		M1	3.1a
	$5t^2 - \frac{1}{3}t^3 - 24t$		A1	1.1b
	Total distance = $-\left[5t^2 - \frac{1}{3}t^3 - 24t\right]_0^4 + \left[5t^2 - \frac{1}{3}t^3 - 24t\right]_4^6$		M1	2.1
	$\frac{116}{3}$ (m)		A1	1.1b
			(4)	
(10 marks)				
Notes:				
2a	M1	Differentiate, with both powers decreasing by 1		
	A1	Correct expression		
2b	M1	Put $t = 6$ OR use $(t-6)(t-x) = t^2 - 10t + k$ oe		
	A1	Correct expression (unsimplified) for v OR $v = (t-6)(t-4)$		
	M1	Put $v = 0$ to give quadratic in t and solve for other value of t		
	A1	$t = 4$		
2c	M1	Integrate, with at least two powers increasing by 1 (allow if only two terms integrated)		
	A1	Correct expression		
	M1	Complete method to find the total distance		
	A1	Accept 39(m) or better		

Question	Scheme	Marks	AOs
3(a)	Differentiate s wrt t	M1	3.1a
	$(v \Rightarrow) t^2 - 5t + 6$	A1	1.1b
	Equate their v to 0 and solve	M1	1.1b
	$t = 2$ or 3	A1	1.1b
	$(a \Rightarrow) 2t - 5$	B1ft	2.1
	$a = 1$ and -1 (m s^{-2}) isw (A0 if extras)	A1	1.1b
		(6)	
(b)	Attempt to find values of s for $t = 2, 3$ and 4 oe Correct values are $\left(s_2 = \frac{14}{3}, s_3 = \frac{9}{2} \text{ and } s_4 = \frac{16}{3} \right)$ Could be implied by correct values for: $s_2, (s_3 - s_2)$ and $(s_4 - s_3)$ which are $\frac{14}{3}, \left(-\frac{1}{6}\right)$ and $\frac{5}{6}$	DM1	1.1b
	Total distance travelled $= s_2 + (s_2 - s_3) + s_4 - s_3$ OR $s_2 - (s_3 - s_2) + s_4 - s_3$ OR $\left[\frac{1}{3}t^3 - \frac{5}{2}t^2 + 6t \right]_0^2 - \left[\frac{1}{3}t^3 - \frac{5}{2}t^2 + 6t \right]_2^3 + \left[\frac{1}{3}t^3 - \frac{5}{2}t^2 + 6t \right]_3^4$ OR $\frac{14}{3} - \left(-\frac{1}{6}\right) + \frac{5}{6}$ OR $s_2 + 2(s_2 - s_3) + s_4 - s_2$ $(= 2s_2 - 2s_3 + s_4)$ oe	M1	2.1
	$5\frac{2}{3}$ oe (m) Accept 5.7 or better	A1	1.1b
		(3)	
(9 marks)			
Notes:			
3a	M1	Differentiate, with at least 2 powers decreasing by 1	
	A1	Correct expression	
	M1	Must have attempted to differentiate s to find v and be solving a 3 term quadratic	
	A1	Both values needed	
	B1ft	Follow their v (must be differentiating)	

	A1	cao
3b	DM 1	<u>This mark is dependent on the 2nd M1 in part (a) and their t values are between 0 and 4.</u> Clear attempt to find all three s values (may integrate their v incorrectly) N.B. No penalty for extra values.
	M1	Complete method using their s values Do NOT condone sign errors.
	A1	Any equivalent fraction, 5.7 or better.
		S.C. Correct answer, with no working, scores all 3 marks, since $\int_0^4 t^2 - 5t + 6 dt$ entered on a calculator will give $\frac{17}{3}$