FS1Ch8 XMQs and MS

(Total: 89 marks)

1. FS1_2019 Q5 . 12 marks - FS1ch8 Quality of tests
2. FS1_2021 Q5 . 18 marks - FS1ch4 Hypothesis testing
3. FS1_2021 Q7 . 8 marks - FS1ch8 Quality of tests
4. FS1_2023 Q5 . 8 marks - FS1ch8 Quality of tests
5. FS1_2024 Q5 . 10 marks - FS1ch8 Quality of tests
6. FS1_Sample Q7 . 18 marks - FS1ch8 Quality of tests
7. FS1_Specimen Q6 . 15 marks - FS1ch8 Quality of tests

5. Information was collected about accidents on the *Seapron* bypass. It was found that the number of accidents per month could be modelled by a Poisson distribution with mean 2.5

Following some work on the bypass, the numbers of accidents during a series of 3-month periods were recorded. The data were used to test whether or not there was a change in the mean number of accidents per month.

(a) Stating your hypotheses clearly and using a 5% level of significance, find the critical region for this test. You should state the probability in each tail.

(5)

(b) State P(Type I error) using this test.

(1)

Data from the series of 3-month periods are recorded for 2 years.

(c) Find the probability that at least 2 of these 3-month periods give a significant result.

(3)

Given that the number of accidents per month on the bypass, after the work is completed, is actually 2.1 per month,

(d) find P(Type II error) for the test in part (a)

(3)

Qu	Scheme	Marks	AO
5(a)	$H_0 : \lambda = 2.5$ (or $\mu = 7.5$) $H_1 : \lambda \neq 2.5$ (or $\mu \neq 7.5$)	B1	2.5
	[*X* = no. of accidents in a 3-month period] *X* ~Po(7.5)	M1	3.3
	P(*X* „ 2) = 0.0203 (calc: 0.020256…) { or P(*X* „ 3) = 0.0591}		
	P(*X* „ 13) = 0.9784 so P(*X* …14) = 0.0216 (calc: 0.0215646…)	M1	3.4
	{or P(*X* …15) =0.0103}		
	Giving Critical region of: ***X* „ 2**	A1	1.1b
	***X* …14**	A1	1.1b
		(5)	
(b)	[0.0203 +0.0216] =awrt **0.0419** or (calc: 0.041821366… awrt **0.0418**)	B1ft	1.2
		(1)	
(c)	[Let *M* = no of 3-month periods with a significant result]		
	M ~ B(8, "0.0419")	M1	3.3
	[P(*M* …2)] = 1 – P(*M* „ 1)	M1	1.1b
	[= 1 – 0.9584…]		
	=0.04153…(calc: 0.041394…) [**0.04139~ 0.04154]**	A1cso	1.1b
		(3)	
(d)	*Y* ~Po(6.3)	M1	3.3
	P(Type II error) = P(3 „ *Y* „ 13) or P(*Y* „ 13) – P(*Y* „ 2)	M1	3.4
	[= 0.9945147… – 0.049846…]		
	= 0.9446… awrt **0.945**	A1	1.1b
		(3)	
		(12 marks)	

	Notes
(a)	B1 for both hypotheses in terms of λ or μ (either way around)
	1st M1 for selecting the correct Po model. Sight or use of Po(7.5) may be implied by 2nd M1
	2nd M1 for using the correct model to find one of these probs with correct label (2sf or better)
	1st A1 for one end correct — Allow any letter, even CR „ 2 or set notation but **not** P(*X*„ 2)
	2nd A1 for a fully correct CR — Can have *X* < 3 and *X* > 13 etc
(b)	B1ft for awrt 0.0419 or awrt 0.0418
	or ft addition of their **two** probs provided both are 0 < prob < 0.025 (awrt 3sf)
(c)	1st M1 for selecting a correct binomial model, ft their answer to part (b)
	2nd M1 for a correct probability statement of 1 – P(*M* „ 1) **dep on** a binomial selected
	A1cso for answer in range [0.04139, 0.04154] **dep on** use of B(8, "0.0419") or better
(d)	1st M1 for selecting a Po(6.3) model
	2nd M1 for a correct probability statement using their Poisson model and their CR in (a) which may have just one tail.
	A1 for awrt 0.945

5. Asha, Davinda and Jerry each have a bag containing a large number of counters, some of which are white and the rest are red.
Each person draws counters from their bag one at a time, notes the colour of the counter and returns it to their bag.

The probability of Asha getting a red counter on any one draw is 0.07

(a) Find the probability that Asha will draw at least 3 white counters before a red counter is drawn.
(2)

(b) Find the probability that Asha gets a red counter for the second time on her 9th draw.
(2)

The probability of Davinda getting a red counter on any one draw is p.
Davinda draws counters until she gets n red counters. The random variable D is the number of counters Davinda draws.

Given that the mean and the standard deviation of D are 4400 and 660 respectively,

(c) find the value of p.
(4)

Jerry believes that his bag contains a smaller proportion of red counters than Asha's bag. To test his belief, Jerry draws counters from his bag until he gets a red counter. Jerry defines the random variable J to be the number of counters drawn up to and including the first red counter.

(d) Stating your hypotheses clearly and using a 10% level of significance, find the critical region for this test.
(5)

Jerry gets a red counter for the first time on his 34th draw.

(e) Giving a reason for your answer, state whether or not there is evidence that Jerry's bag contains a smaller proportion of red counters than Asha's bag.
(2)

Given that the probability of Jerry getting a red counter on any one draw is 0.011

(f) show that the power of the test is 0.702 to 3 significant figures.
(3)

Question	Scheme	Marks	AOs
5(a)	P(at least 3 whites) = $(1-0.07)^3$ **or** $1-0.07-0.93\times0.07-0.93^2\times0.07$	M1	1.1b
	= 0.8043… awrt 0.804	A1	1.1b
		(2)	
(b)	P(2nd red on 9th draw) = $\binom{8}{1}0.93^7\times0.07^2$	M1	3.3
	= 0.02358… awrt 0.0236	A1	1.1b
		(2)	
(c)	$\frac{n}{p}=4400$ and $\frac{n(1-p)}{p^2}=660^2$	M1 A1	3.1b 1.1b
	$1-p=99p$ oe	M1	1.1b
	$p=0.01$	A1	1.1b
		(4)	
(d)	H_0: p = 0.07 H_1: p < 0.07	B1	2.5
	J ~ Geo(0.07)	M1	3.3
	$P(J\geq c)<0.1\Rightarrow(1-0.07)^{c-1}<0.1$	M1	3.4
	$c-1>\frac{\log 0.1}{\log 0.93}$	M1	1.1b
	$c>32.72...$ $\therefore$ CR $J\geq 33$	A1	1.1b
		(5)	
(e)	34 is in the Critical region	M1	1.1b
	There is evidence to suggest that Jerry**'s bag** contains a smaller **proportion** of **red counters** than Asha**'s** bag.	A1	2.2b
		(2)	
(f)	Power of test = $P(J\geq 33 \mid p=0.011)$	M1	2.1
	= $(1-0.011)^{32}$ oe	M1	1.1b
	= 0.7019…*	A1*	1.1b
		(3)	
			(18 marks)

Notes:		
(a)	M1:	A correct method to find $P(X \geqslant 3)$
	A1:	awrt 0.804
(b)	M1:	For selecting the appropriate model negative binomial or binomial with an extra trial
	A1:	awrt 0.0236
(c)	M1:	Forming an equation for the mean and variance. At least one correct.
	A1:	Both equations correct
		Allow M1 A1 if both equations correct with the same number subst for n
	M1:	Solving the 2 equations leading to $1-p=99p$ oe Allow $p-p^2=99p^2$ ft their 4400 and 660 Allow $1-p=0.15p$
	A1:	0.01
(d)	M1:	Both hypotheses correct using correct notation allow eg $p > 0.93$
	M1:	Realising the need to use Geo(0.07) ft their Hypotheses
	M1:	Using the model to find $P(J \geqslant c)$ Condone $(1-0.07)^c < 0.1$ ft their 0.07 $\neq 0.93$ **ALT** $P(J \geqslant 32) = 0.1[054...\}]$ or $P(J \geqslant 33) = 0.09[8...]$ Implied by correct CR
	M1:	For a valid method to solve the inequality or $P(J \geqslant 32) = 0.1[054]$ and $P(J \geqslant 33) = 0.09[81]$ Implied by correct CR
	A1:	Correct CR(any letter) A0 if given as a probability statement. Must be integer
(e)	M1:	Comparing 34 with their CR eg $34 > 33$ $34 \geqslant 33$ or $P(J \geqslant 34) = 0.09[12]$
	A1:	Fully correct conclusion in context. Allow Jerry's belief is true. Allow probability for proportion
(f)	M1:	Realising they need to find P(their CR in (d)) Allow $1-P(J \leqslant 32)$
	M1:	For a Correct method. Allow 1 – 0.2981… May be implied by 0.7019… If the CR is incorrect $(1-0.011)^{"CR"-1}$ or 1 – $\{1-(1-0.011)^{"CR"-1}\}$ must be seen
	A1*:	Only award if both method marks awarded.

7. A manufacturer has a machine that produces lollipop sticks.
The length of a lollipop stick produced by the machine is normally distributed with unknown mean μ and standard deviation 0.2

Farhan believes that the machine is not working properly and the mean length of the lollipop sticks has decreased.
He takes a random sample of size n to test, at the 1% level of significance, the hypotheses

$$H_0: \mu = 15 \qquad H_1: \mu < 15$$

(a) Write down the size of this test.

(1)

Given that the actual value of μ is 14.9

(b) (i) calculate the minimum value of n such that the probability of a Type II error is less than 0.05
Show your working clearly.

(6)

(ii) Farhan uses the same sample size, n, but now carries out the test at a 5% level of significance. Without doing any further calculations, state how this would affect the probability of a Type II error.

(1)

Question	Scheme	Marks	AOs
7(a)	Size of the test = 0.01	B1	1.2
		(1)	
(b)(i)	Let CR be $\overline{L} < k$		
	$\dfrac{k-15}{\frac{0.2}{\sqrt{n}}} = -2.3263$	M1	3.4
	$k = 15 - \dfrac{0.46526}{\sqrt{n}}$	A1	1.1b
	$\dfrac{"15 - \frac{0.46526}{\sqrt{n}}" - 14.9}{\frac{0.2}{\sqrt{n}}} > 1.6449$	M1d A1ft	3.4 1.1b
	$\dfrac{0.79424}{\sqrt{n}} < 0.1 \quad \sqrt{n} > 7.9424$ oe	M1d	1.1b
	$n = 64$	A1cso	2.1
		(6)	
(ii)	The probability of a Type II error would decrease.	B1	2.2a
		(1)	
		(8 marks)	

Notes

(a)	**B1:**	0.01
(b)(i)	**M1:**	Finding the CR using the Normal distribution must have $1.5 < \lvert z \rvert < 3.5$
	A1:	A correct equation in the form $k = \ldots$ and for use of awrt 2.326 (implied by awrt 0.46526 or awrt 0.46527)
	M1d:	Dependent on previous M being awarded. Standardising using their *k* and equating to a *z* value $1.5 < \lvert z \rvert < 3$ to form an equation to able *n* to be found. May use = rather than $>$
	A1ft:	Ft their *k* for a correct equation with awrt 1.645
	M1d:	Dependent on previous M being awarded. Isolating $\sqrt{n}$ or squaring both sides leading to a value for *n*. Condone *n* = 7.9424
	A1cso:	64 with correct working
(ii)	**B1:**	Suitable comment

ALT (b)(i)	$\dfrac{k-14.9}{\frac{0.2}{\sqrt{n}}}=1.6449$	M1	3.4
	$k = 14.9+\dfrac{0.32898}{\sqrt{n}}$	A1	1.1b
	$\dfrac{"14.9+\frac{0.32898}{\sqrt{n}}"-15}{\frac{0.2}{\sqrt{n}}}>-2.3263$	M1d A1ft	3.4 1.1b
	$\dfrac{0.79424}{\sqrt{n}}<0.1 \quad \sqrt{n}>7.9424$ oe	M1d	1.1b
	n = 64	A1cso	2.1
		(6)	

5. A machine fills cartons with juice.

The amount of juice in a carton is normally distributed with mean μ ml and standard deviation 8 ml.

A manager wants to test whether or not the amount of juice in the cartons, X ml, is less than 330 ml. The manager takes a random sample of 25 cartons of juice and calculates the mean amount of juice $\bar{x}$ ml.

(a) Using a 5% level of significance, find the critical region of $\bar{X}$ for this test. State your hypotheses clearly.

(4)

The Director is concerned about the machine filling the cartons with more than 330 ml of juice as well as less than 330 ml of juice. The Director takes a sample of 55 cartons, records the mean amount of juice $\bar{y}$ ml and uses a test with a critical region of

$$\left\{\bar{Y} < 328\right\} \cup \left\{\bar{Y} > 332\right\}$$

(b) Find P(Type I error) for the Director's test.

(2)

When $\mu = 325$ ml

(c) find P(Type II error) for the test in part (a)

(2)

Qu 5	Scheme	Marks	AO
(a)	$H_0 : \mu = 330 \qquad H_1 : \mu < 330$	B1	2.5
	$[\bar{X} \sim]\text{N}\left(330, \left(\frac{8}{\sqrt{25}}\right)^2\right)$	M1	3.3
	$\text{P}\left(\bar{X} < C\right) = 0.05 \quad \Rightarrow \quad \frac{C - 330}{8/\sqrt{25}} = -1.6449$	M1	3.4
	So C = 327.368… and critical region is: $\bar{X}$ < awrt **327**	A1	1.1b
		(4)	
(b)	$\bar{Y} \sim \text{N}\left(330, \left(\frac{8}{\sqrt{55}}\right)^2\right)$ and require $2 \times \text{P}\left(\bar{Y} < 328\right)$ (o.e.)	M1	3.3
	= 0.063732… awrt **0.0637**	A1	1.1b
		(2)	
(c)	P($\bar{X}$ > "327.368…"| μ = 325) or 1 – P($\bar{X}$ < "327.368…"| μ = 325)	M1	3.4
	= 0.0694233… awrt **0.0694**	A1	1.1b
		(2)	
		(8 marks)	

	Notes
(a)	B1 for both hypotheses in terms of μ
	1st M1 for stating or using the correct model – may be implied by use in later line. Condone X or any letter for $\bar{X}$
	2nd M1 for a correct equation for C Allow any z value that satisfies 1.6 < |z| < 1.7 If standardisation equation not seen, this mark may be implied by CV = awrt 327 or CR: <awrt 327
	A1 for a correct CR allow just "< awrt 327" Condone e.g. X < 327 rather than $\bar{X}$ < 327 Condone $\leqslant$
(b)	M1 for sight of correct model and attempt at $\text{P}\left(\bar{Y} < 328\right)$ (o.e.) Condone missing $2\times$
	A1 for awrt 0.0637 (correct answer scores 2 out of 2)
(c)	M1 for a correct (ft) statement may be implied by sight of e.g. $Z > \frac{\text{"327.36..."} - 325}{8/5} = 1.48..$ For μ = 325 allow $\bar{X}$ ~N(325, ….) Allow ft from a 2-tailed test in part (a)
	A1 for awrt 0.0694 (correct answer scores 2 out of 2)
SC	Sight of P(328 < $\bar{X}$ < 332| μ = 325) or 1 – P($\bar{X}$ < 328 $\cup$ $\bar{X}$ > 332| μ = 325) scores M1A0

5. Some of the components produced by a factory are defective. The management requires that no more than 3% of the components produced are defective.
Niluki monitors the production process and takes a random sample of n components.

(a) Write down the hypotheses Niluki should use in a test to assess whether or not the proportion of defective components is greater than 0.03

(1)

Niluki defines the random variable D_n to represent the number of defective components in a sample of size n. She considers two tests **A** and **B**

In test **A**, Niluki uses $n = 100$ and if $D_{100} \geqslant 5$ she rejects H_0

(b) Find the size of test **A**

(2)

In test **B**, Niluki uses $n = 80$ and

- if $D_{80} \geqslant 5$ she rejects H_0
- if $D_{80} \leqslant 3$ she does not reject H_0
- if $D_{80} = 4$ she takes a second random sample of size 80 and if $D_{80} \geqslant 1$ in this second sample then she rejects H_0 otherwise she does not reject H_0

(c) Find the size of test **B**

(3)

Given that the actual proportion of defective components is 0.06

(d) (i) find the power of test **A**

(ii) find the expected number of components sampled using test **B**

(3)

Given also that, when the actual proportion of defective components is 0.06, the power of test **B** is 0.713

(e) suggest, giving your reasons, which test Niluki should use.

(1)

Qu	Scheme	Mark	AO
5 (a)	$H_0: p = 0.03 \quad H_1: p > 0.03$	B1	2.5
		(1)	
(b)	$D_{100} \sim B(100, 0.03)$ $[P(D_{100} \geqslant 5) = 1 - P(D_{100} \leqslant 4)]$	M1	3.3
	$= 0.18214\ldots$ awrt **<u>0.182</u>**	A1	1.1b
		(2)	
(c)	$P(D_{80} \geqslant 5) + P(D_{80} = 4) \times P(D_{80} \geqslant 1)$	M1	2.1
	$= 0.09279\ldots + 0.12654\ldots \times 0.91255\ldots$	A1	1.1b
	$= 0.20826\ldots$ awrt **<u>0.208</u>**	A1	1.1b
		(3)	
(d)(i)	Test **A**: $[X \sim B(100, 0.06)\ P(X \geqslant 5) =]\ 0.72322\ldots$ awrt **<u>0.723</u>**	B1	1.2
(ii)	Expected number $= 80 + 80 \times P(Y = 4)$ with $Y \sim B(80, 0.06)$	M1	3.4
	$= 94.87\ldots =$ **<u>95</u>**	A1	1.1b
		(3)	
(e)	Tests of similar size and power but Test **B** involves sampling fewer components so would advise to use test **B**.	B1	2.4
		(1)	
		(10 marks)	

	Notes
(a)	B1 for both hypotheses in terms of p or π
(b)	M1 for sight or use of the correct model. Allow for 1 – 0.81785… or 1 – 0.91916.. A1 for awrt 0.182
(c)	M1 for a correct expression for required probability. May be implied by 1st A1 1st A1 for a correct numerical expression – values to 2 s.f. or better 2nd A1 for awrt 0.208
(d)(i)	B1 for power of test **A** = awrt 0.723
(ii)	M1 Correct expression with $Y \sim B(80, 0.06)$ **or** for use of B(80, 0.06) to obtain a probability of 0.814 or 0.186 Implied by sight of 95 or better A1 for 95 (accept awrt 94.9) **NB:** May see $0.814 \times 80 + 0.186 \times 160 =$ awrt 94.9 or 95, which is M1A1
(e)	B1 for a choice backed up by a suitable reason: If concluding **B**, they need to mention <u>similar power</u> and a <u>smaller sample size</u> If concluding **A**, they need to mention <u>smaller size</u> and <u>greater power</u> **NB:** We do not ft the candidate's incorrect values for size or power
SC	Use of Poisson approximation
(b)	Allow M1 for Po(3) and A1 for answer of 0.1847 or better
(c)	Allow M1 and 1st A1 for awrt 0.21 but 2nd A0 (Po(2.4) gives 0.2099…)
(d)(i)	Allow B1 for 0.7149 or better from Po(6)
(ii)	Allow M1 for expression (answer should be 94.56…) A0 for answer.

7. Sam and Tessa are testing a spinner to see if the probability, p, of it landing on red is less than $\frac{1}{5}$.
They both use a 10% significance level.

Sam decides to spin the spinner 20 times and record the number of times it lands on red.

(a) Find the critical region for Sam's test.
(2)

(b) Write down the size of Sam's test.
(1)

Tessa decides to spin the spinner until it lands on red and she records the number of spins.

(c) Find the critical region for Tessa's test.
(6)

(d) Find the size of Tessa's test.
(1)

(e) (i) Show that the power function for Sam's test is given by

$$(1 - p)^{19}(1 + 19p)$$

(ii) Find the power function for Tessa's test.
(4)

(f) With reference to parts (b), (d) and (e), state, giving your reasons, whether you would recommend Sam's test or Tessa's test when $p = 0.15$
(4)

Question	Scheme	Marks	AOs
Q7(a)	$X \sim B(20, 0.2)$ and seek c such that $P(X \leqslant c) < 0.10$	M1	3.3
	$[P(X \leqslant 1) = 0.0692]$ CR is $X \leqslant 1$	A1	1.1b
		(2)	
(b)	Size = **<u>0.0692</u>**	B1ft	1.2
		(1)	
(c)	Y = no. of spins until red obtained so $Y \sim \text{Geo}(0.2)$	M1	3.3
	$\mu = \frac{1}{p}$ so if $p < 0.2$ then mean is <u>larger</u> so seek d so that $P(Y \geqslant d) < 0.10$	M1	2.4
	$P(Y \geqslant d) = (0.8)^{d-1}$	M1	3.4
	$(0.8)^{d-1} < 0.10 \Rightarrow d - 1 > \frac{\log(0.1)}{\log(0.8)}$	M1	1.1b
	$d > 11.3..$	A1	1.1b
	CR is $\boldsymbol{Y \geqslant 12}$	A1	2.2b
		(6)	
(d)	Size = $[0.8^{11} = 0.085899\ldots]$ = **<u>0.0859</u>**	B1	1.1b
		(1)	
(e)(i)	Power = P(reject H_0 when it is false) = $P(X \leqslant 1 \mid X \sim B(20, p))$	M1	2.1
	$= (1-p)^{20} + 20(1-p)^{19} p$	M1	1.1b
	$= (1-p)^{19}(1+19p)$ *	A1*cso	1.1b
(ii)	Power = $(1-p)^{11}$	B1	1.1b
		(4)	
(f)	Sam's test has smaller P(Type I error) (or size) so is better	B1	2.2a
	Power of Sam's test = 0.1755…	B1	1.1b
	Power of Tessa's test = 0.85^{11} = 0.1673…	B1	1.1b
	So for $p = 0.15$ **Sam's test** is recommended	B1	2.2b
		(4)	
			(18 marks)

Notes:
(a) **M1:** Realising the need to use the model Using B(20,0.2) with method for finding the CR or implied by a correct CR **A1:** $X \leqslant 1$ or $X < 2$
(b) **B1:** awrt 0.0692
(c) **M1:** Realising that the model Geo(0.2)is needed. This may be written or used **M1:** Realising the key step that they need to find $P(Y \geqslant d) < 0.10$ **M1:** **U**sing the model $(0.8)^{d-1}$ **M1:** Using the model $(0.8)^{d-1} < 0.10$ and finding a method to solve leading to a value/range of values for d **A1:** For $d > 11.3..$ **A1:** For $Y \geqslant 12$ or $Y > 11$ (a correct inference)
(d) **B1ft:** awrt 0.0692. ft their answer to part (c)
(e)(i) **M1:** Using B(20, p) and realizing they need to find $P(X \leqslant 1)$ o.e. This may be used or written **M1:** Using $P(X = 0) + P(X = 1)$ **A1*:** Fully correct proof (no errors) cso
(ii) **B1:** For $(1 - p)^{11}$
(f) **B1:** Making a deduction about the tests using the answers to part(b) and (d) **B1:** awrt 0.0176 **B1:** awrt 0.167 **B1:** A correct inference about which test is recommended

6. The weights of bars of soap are known to be normally distributed.

The standard deviation of the weights is 3 grams.

A label on the bars of soap states that the mean weight is 120 grams.

Gizel believes that the mean weight of the bars of soap is greater than 120 grams.
She takes a random sample of 10 bars of soap and finds the mean weight of her sample.

Gizel then tests, at the 5% level of significance, whether there is evidence that the bars of soap weigh more than 120 grams.

(a) Write down the probability of a Type I error.

(1)

Given that the true mean weight of the bars of soap is 122 grams,

(b) show that the power of Gizel's test is 0.68 to 2 significant figures.

(5)

Alex decides to carry out the same test at the 1% level of significance.

(c) Without carrying out the test, compare the power of Alex's test with the power of Gizel's test.
Give a reason for your answer.

(2)

Joseph decides to increase the sample size and carry out the test at the 5% level of significance.

Assuming that the true mean weight of the bars of soap is 122 grams,

(d) calculate the smallest sample size that will produce a power of at least 0.9

(5)

(e) State which of Gizel's and Joseph's tests should be used.
Give a reason for your answer.

(2)

Question	Scheme	Marks	AOs
6(a)	P(Type I error) = **0.05**	B1	1.2
		(1)	
(b)	$\bar{X} \sim \text{N}(120, \frac{3^2}{10})$ $\quad$ $\text{P}(\bar{X} > c) < 0.05$	M1	3.1b
	$\frac{c-120}{\frac{3}{\sqrt{10}}} > 1.6449$	M1	3.4
	$c > 121.56\ldots$	A1	1.1b
	$\text{P}(\bar{X} > 121.56 \mid \mu = 122)$		
	$\text{P}\left(Z > \frac{121.56-122}{\frac{3}{\sqrt{10}}}\right) = \text{P}(Z > -0.4638...)$	M1	2.1
	$= 0.6786\ldots$ $\quad$ $= 0.68^*(2\text{sf})$	A1*cso	1.1b
		(5)	
(c)	Power of Alex's test is smaller than power of Gizel's test since the null hypothesis is less likely to be rejected/Type II error has increased.	B1 B1	2.2a 2.4
		(2)	
(d)	$\frac{c-120}{\frac{3}{\sqrt{n}}} > 1.6449$	M1	3.4
	$c > 120 + 1.6449 \times \frac{3}{\sqrt{n}}$	A1	1.1b
	$\text{P}(\bar{X} > c \mid \mu = 122) > 0.9$		
	$\frac{(120 + 1.6449 \times \frac{3}{\sqrt{n}}) - 122}{\frac{3}{\sqrt{n}}} < -1.2816$	M1	2.1
	$2.9265\frac{3}{\sqrt{n}} < 2 \quad \rightarrow \quad \sqrt{n} > 4.38...$	M1	1.1b
	$n > 19.26\ldots$ $\quad$ $n = \underline{\mathbf{20}}$	A1	1.1b
		(5)	
(e)	(As they both have the same size/Type I error and) Joseph's test has a higher power, so Joseph's test is recommended.	M1 A1	2.4 2.2b
		(2)	
			(15 marks)

Notes

(a)	B1 0.05oe
(b)	1st M1 Selecting correct normal model 2nd M1 Using model to standardise and set up inequality 1st A1 Correct critical region 3rd M1 Correct probability statement to find power 2nd A1*cso awrt 0.68 with no errors seen.
(c)	B1 Correct deduction about the size of the two tests B1 Correct explanation

(d)	1^{st} M1 Using normal model to find critical region 1^{st} A1 Correct critical region in terms of n 2^{nd} M1 Setting up comparison with $\|1.2816\|$ to find n 3^{rd} M1 Solving equation to $\sqrt{n} > ...$ 2^{nd} A1 20cao
(e)	M1 Comparison of powers A1 Correct conclusion based on power