FS1Ch7 XMQs and MS

(Total: 98 marks)

1. FS1_2020 Q6 . 13 marks - FS1ch7 Probability generating functions

2. FS1_2021 Q6 . 14 marks - FS1ch7 Probability generating functions

3. FS1_2022 Q6 . 14 marks - FS1ch7 Probability generating functions

4. FS1_2023 Q6 . 13 marks - FS1ch7 Probability generating functions

5. FS1_2024 Q6 . 16 marks - FS1ch7 Probability generating functions

6. FS1_Sample Q6 . 14 marks - FS1ch7 Probability generating functions

7. FS1_Specimen Q4 . 14 marks - FS1ch7 Probability generating functions

6. A discrete random variable X has probability generating function given by

$$G_X(t) = \frac{1}{64}\,(a + bt^2)^2$$

where a and b are positive constants.

(a) Write down the value of $P(X = 3)$

(1)

Given that $P(X = 4) = \dfrac{25}{64}$

(b) (i) find $P(X = 2)$

(7)

(ii) find $E(X)$

(3)

The random variable $Y = 3X + 2$

(c) Find the probability generating function of Y

(2)

Question	Scheme	Marks	AOs
6(a)	P(X = 3) = **0**	B1	1.1b
		(1)	
(b)(i)	Coefficient of $t^4 = \frac{1}{64}b^2$	M1	2.1
	$\frac{1}{64}b^2 = \frac{25}{64}$	M1	1.1b
	$b = 5$ (reject $b = -5$ since b > 0)	A1	2.3
	$G_X(1) = 1$ $\frac{1}{64}(a + "5")^2 = 1$	M1	2.1
	$a = 3$ (reject $a = -13$ since a > 0)	A1	1.1b
	P(X = 2) = coefficient of $t^2 = \frac{1}{64}(2ab)$	M1	3.4
	$= \underline{\frac{15}{32}}$	A1	1.1b
		(7)	
(ii)	$E(X) = G'_X(1)$	M1	2.1
	$G'_X(t) = \frac{2}{64}("3" + "5"t^2) \times "10"t$ or $G'_X(t) = \frac{1}{64}("60"t + "100"t^3)$	M1	1.1b
	$G'_X(1) = 2.5$	A1ft	1.1b
		(3)	
(c)	$G_Y(t) = t^2 G_X(t^3) [= \frac{t^2}{64}(a + b(t^3)^2)^2]$	M1	3.1a
	$G_Y(t) = \frac{t^2}{64}("3" + "5"t^6)^2$	A1ft	1.1b
		(2)	
			(13 marks)

Notes

(a)	**B1:** 0 (Since there is no term in t^3)
(b)(i)	**M1:** Realising that $\frac{1}{64}b^2$, the coefficient of t^4, is needed **M1:** Equating their coefficient of t^4 to $\frac{25}{64}$ with an attempt to find b **A1:** b = 5 only **M1:** Realising that $G_X(1) = 1$ is required **A1:** a = 3 only **M1:** Finding coefficient of t^2 with their a > 0 and b > 0 **A1:** $\frac{15}{32}$ (condone awrt 0.469)
(b)(ii)	**M1:** Realising $G'_X(1)$ is needed **M1:** Attempt to differentiate $G_X(t)$ with their values of a and b **A1ft:** 2.5 (ft (3sf) their values of a and b, a > 0 and b > 0) $E(X) = \frac{ab+b^2}{16}$ Alternative: **M1:** Realising X = 0, 2 and 4 only **M1:** [0 ×P(X = 0)] + 2×P(X = 2) + 4×P(X = 4)
(c)	**M1:** either $G_X(t^3)$ or $\times t^2$ or using Y = 2, 8, 14 **A1ft:** ft their values of a and b, a > 0 and b > 0 $G_Y(t) = \frac{t^2}{64}("3" + "5"t^6)^2$ or $G_Y(t) = \frac{t^2}{64}("9" + "30"t^6 + "25"t^{12})$ or $G_Y(t) = \frac{1}{64}("9t^2" + "30"t^8 + "25"t^{14})$

6. The probability generating function of the random variable X is

$$G_X(t) = k(1 + 2t)^5$$

where k is a constant.

(a) Show that $k = \dfrac{1}{243}$ **(2)**

(b) Find $P(X = 2)$ **(2)**

(c) Find the probability generating function of $W = 2X + 3$ **(2)**

The probability generating function of the random variable Y is

$$G_Y(t) = \frac{t(1 + 2t)^2}{9}$$

Given that X and Y are independent,

(d) find the probability generating function of $U = X + Y$ in its simplest form. **(2)**

(e) Use calculus to find the value of $\mathrm{Var}(U)$ **(6)**

Question	Scheme	Marks	AOs
6(a)	$G_X(1) = 1$	M1	2.1
	$k \times 3^5 = 1 \quad \therefore k = \frac{1}{243}$ *	A1*cso (2)	1.1b
(b)	$P(X=2)$ is coefficient of t^2 so $G_X(t) = k\left(\ldots + {}^5C_2(2t)^2 + \ldots\right)$	M1	1.1b
	$P(X=2) = \frac{40}{243}$	A1 (2)	1.1b
(c)	$G_W(t) = \frac{t^3}{243}\left(1+2\left(t^2\right)\right)^5$	M1	3.1a
	$G_W(t) = \frac{t^3}{243}\left(1+2t^2\right)^5$	A1 (2)	1.1b
(d)	$G_U(t) = \frac{1}{243}(1+2t)^5 \times \frac{t(1+2t)^2}{9}$	M1	3.1a
	$= \frac{t(1+2t)^7}{2187}$	A1 (2)	1.1b
(e)	$G_U'(t) = \frac{14t(1+2t)^6}{2187} + \frac{(1+2t)^7}{2187}$	M1	2.1
	$G_U'(1) = \frac{17}{3}$	A1ft	1.1b
	$G_U''(t) = \frac{168t(1+2t)^5}{2187} + \frac{14(1+2t)^6}{2187} + \frac{14(1+2t)^6}{2187}$	M1	2.1
	$G_U''(1) = 28$	A1	1.1b
	$\text{Var}(U) = \text{"}28\text{"} + \text{"}\frac{17}{3}\text{"} - \left(\text{"}\frac{17}{3}\text{"}\right)^2$	M1	2.1
	$= \frac{14}{9}$	A1 (6)	1.1b
ALT(e)	$G_X''(t) = A(1+2t)^3$	M1	
	$G_X'(1) = \frac{10}{3}$ and $G_X''(1) = \frac{80}{9}$	A1ft	
	$G_Y''(t) = H(8+24t)$	M1	
	$G_Y'(1) = \frac{7}{3}$ and $G_Y''(1) = \frac{32}{9}$	A1	
	Using $G_U''(1) + G_U'(1) - \left(G_U'(1)\right)^2$ to find Var(X), Var Y and Var U	M1	
	$\frac{14}{9}$ or awrt1.56	A1	
			(14 marks)

Notes:		
(a)	**M1:**	Stating $G_X(1) = 1$ eg $G_X(1) = k(1+2)^5 = 1$ $k(1+2)^5 = 1$ Allow Verification $\frac{1}{243}\times 3^5 = 1$
	A1*:	Fully correct proof with no errors Substituting $t = 1$ Verification need therefore $G_X(1) = 1$
(b)	**M1:**	Attempting to find the coefficient of t^2
	A1:	$\frac{40}{243}$ or awrt 0.165
(c)	**M1:**	Realising the need to multiply through by t^3 or subst t^2 for t
	A1:	$\frac{t^3}{243}\left(1+2t^2\right)^5$ oe eg $\frac{t^3}{243}\left(1+10t^2+40t^4+80t^6+80t^8+32t^{10}\right)$
(d)	**M1:**	Realising the need to use $G_U(t) = G_X(t)\times G_Y(t)$
	A1:	$\frac{t(1+2t)^7}{2187}$ oe
(e)	**M1:**	For an attempt to differentiate G (u) e.g $G_U'(t) = At(1+2t)^6 + B(1+2t)^7$ ft their part(d) if in the form $kt(1+2t)^n$ where $n \geqslant 5$
	A1ft:	$\frac{17}{3}$ or awrt 5.67
	M1:	For attempting second derivative eg $G_U''(t) = Ct(1+2t)^5 + D(1+2t)^6$ ft their part(d) if in the form $kt(1+2t)^n$ where $n \geqslant 5$
	A1	28
	M1:	Using $G_U''(1) + G_U'(1) - \left(G_U'(1)\right)^2$ ft their values
	A1:	$\frac{14}{9}$ or awrt1.56

6. The discrete random variable V has probability distribution

v	2	3	4
$P(V = v)$	$\frac{9}{25}$	$\frac{12}{25}$	$\frac{4}{25}$

(a) Show that the probability generating function of V is

$$G_V(t) = t^2\left(\frac{2}{5}t + \frac{3}{5}\right)^2$$

(2)

The discrete random variable W has probability generating function

$$G_W(t) = t\left(\frac{2}{5}t + \frac{3}{5}\right)^5$$

(b) Use calculus to find

(i) $E(W)$

(2)

(ii) $Var(W)$

(4)

Given that V and W are independent,

(c) find the probability generating function of $X = V + W$ in its simplest form.

(2)

The discrete random variable $Y = 2X + 3$

(d) Find the probability generating function of Y

(2)

(e) Find $P(Y = 15)$

(2)

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Question	Scheme	Marks	AOs
6(a)	$\mathrm{G}_V(t)=\frac{9}{25}t^2+\frac{12}{25}t^3+\frac{4}{25}t^4$ <u>or</u> $t^2\left(\frac{9}{25}+\frac{12}{25}t+\frac{4}{25}t^2\right)$	M1	1.1b
	$=t^2\left(\frac{2}{5}t+\frac{3}{5}\right)^2$ *	A1* cso	2.1
		(2)	
(b)(i)	$\mathrm{G}_W'(t)=2t\left(\frac{2}{5}t+\frac{3}{5}\right)^4+\left(\frac{2}{5}t+\frac{3}{5}\right)^5$	M1	2.1
	$\left[\mathrm{G}_W'(1)=\right]$ $\underline{\mathbf{3}}$	A1	1.1b
(ii)	$\mathrm{G}_W''(t)=2\left(\frac{2}{5}t+\frac{3}{5}\right)^4+\frac{16}{5}t\left(\frac{2}{5}t+\frac{3}{5}\right)^3+2\left(\frac{2}{5}t+\frac{3}{5}\right)^4$ oe	M1	2.1
	$\mathrm{G}_W''(1)=\frac{36}{5}$	A1	1.1b
	$\mathrm{Var}(W)="\frac{36}{5}"+"3"-("3")^2$	M1	2.1
	$=\frac{6}{5}$	A1	1.1b
		(6)	
(c)	$\mathrm{G}_X(t)=t^2\left(\frac{2}{5}t+\frac{3}{5}\right)^2\times t\left(\frac{2}{5}t+\frac{3}{5}\right)^5$	M1	3.1a
	$=t^3\left(\frac{2}{5}t+\frac{3}{5}\right)^7$	A1	1.1b
		(2)	
(d)	$\mathrm{G}_Y(t)=t^3\times(t^2)^3\times\left(\frac{2}{5}t^2+\frac{3}{5}\right)^7$	M1	3.1a
	$=t^9\left(\frac{2}{5}t^2+\frac{3}{5}\right)^7$	A1	1.1b
		(2)	
(e)	$\mathrm{P}(Y=15)$ is coefficient of t^{15} ie $...+t^9\times{}^7C_3\left(\frac{2}{5}t^2\right)^3\left(\frac{3}{5}\right)^4+...$ <u>or</u> P(X = 6) need coefficient of t^6 i.e. $...+t^3\times{}^7C_3\left(\frac{2}{5}t\right)^3\left(\frac{3}{5}\right)^4+...$	M1	1.1b
	$\left[\mathrm{P}(Y=15)=\right]\frac{22680}{78125}=\frac{4536}{15625}=0.290304$	A1	1.1b
		(2)	
			(14 marks)

Notes:		
(a)	**M1**	A correct un-simplified pgf based on $\sum t^v \mathrm{P}(V=v)$
	A1*	cso must see an un-simplified version i.e. M1 scored and no incorrect working seen
(b) (i)	**M1**	Differentiating using the product rule to find $\mathrm{G}_W{}'(t)$ Allow un-simplified e.g. $5\times\frac{2}{5}t$ Need two terms added and at least one correct. If they expand we need 3 correct.
	A1	3 from a correct derivative
(ii)	**1stM1**	Attempt $\mathrm{G}_W{}''(t)$ ft their $\mathrm{G}_W{}'(t)$[must be at least 2 terms or a product], one correct ft term, same rule for differentiating a product
	1stA1	$\frac{36}{5}$ or 7.2 from a correct derivative
	2ndM1	$\mathrm{G}_W{}''(1)+\mathrm{G}_W{}'(1)-\left(\mathrm{G}_W{}'(1)\right)^2$ ft their $\mathrm{G}_W{}''(t)$ if different from $\mathrm{G}_W{}'(t)$ and $\mathrm{G}_W(t)$
	2ndA1	**Dep on M3A2** $\frac{6}{5}$ or 1.2
(c)	**M1**	Realising the need to use $\mathrm{G}_X(t) = \mathrm{G}_V(t)\times\mathrm{G}_W(t)$
	A1	$t^3\left(\frac{2}{5}t+\frac{3}{5}\right)^7$
(d)	**M1**	Realising the need to multiply through by t^3 <u>or</u> substitute t^2 for t <u>or</u> sight of $t^3\mathrm{G}_X(t^2)$
	A1	$t^9\left(\frac{2}{5}t^2+\frac{3}{5}\right)^7$ oe Need not be in its simplest form
(e)	**M1**	Attempting to find correct coefficient of t^n <u>or</u> identify $Y = 2J + 9$ where $J \sim \mathrm{B}(7, 0.4)$ Need an expression can ft their $\mathrm{G}_Y(t)$ or $\mathrm{G}_X(t)$ of the form $t^n(at^m+b)^k$ Allow a statement that $\mathrm{P}(Y = 15) = 0$ if it follows from their pgf
	A1	For a correct exact answer or allow awrt 0.2903 Allow 0.29 from correct expression

Alternative for (b)

(b)	$W = P+1$ where $P \sim \mathrm{B}(5, 0.4)$ so $\mathrm{Var}(W) = \mathrm{Var}(P)$		
(i)	$\mathrm{G}_P{}'(t) = 2\left(\frac{2}{5}t+\frac{3}{5}\right)^4$	M1	2.1
	$\mathrm{G}_W{}'(1) = 2+1 = 3$	A1	1.1b
(ii)	$\mathrm{G}_P{}''(t) = \frac{16}{5}\left(\frac{2}{5}t+\frac{3}{5}\right)^3$; $\mathrm{G}_P{}''(1) = \frac{16}{5}$	M1; A1	2.1 1.1b
	$\mathrm{Var}(W) = "\frac{16}{5}"+"2"-("2")^2$; $=\frac{6}{5}$	M1; A1	2.1 1.1b
SC	**MR** They use $\mathrm{G}_V(t)$ instead of $\mathrm{G}_W(t)$ Provided some correct differentiation seen: Award B1 for $\mathrm{E}(V) = \frac{14}{5}$ and B1 for $\mathrm{Var}(V) = \frac{12}{25}$ score as M0A1M0A0M0A1		

6. The discrete random variable X has probability generating function

$$G_X(t) = \frac{t^2}{(3-2t)^2}$$

(a) Specify the distribution of X

(2)

A fair die is rolled repeatedly.

(b) Describe an outcome that could be modelled by the random variable X

(1)

(c) Use calculus and $G_X(t)$ to find

(i) E(X)

(ii) Var(X)

(7)

The discrete random variable Y has probability generating function

$$G_Y(t) = \frac{t^{10}}{(3-2t^3)^2}$$

(d) Find the exact value of P(Y = 19)

(3)

Qu6	Scheme	Marks	AO
(a)	NegBin(r, p) has pgf $\left[\dfrac{pt}{1-(1-p)t}\right]^r$ and identify the connection	M1	2.1
	NegBin(2, $\frac{1}{3}$)	A1	2.2a
		(2)	
(b)	e.g. no. of rolls to achieve 5 or 6 (so that $p = \frac{1}{3}$) twice (oe)	B1ft	3.3
		(1)	
(c)(i)	$G'_X(t) = \dfrac{2t(3-2t)^2 - (-2)\times 2(3-2t)t^2}{(3-2t)^4}$ or $\dfrac{6t}{(3-2t)^3}$	M1 A1	2.1 1.1b
	E(X) = $G'_X(1)$ = **6**	A1	1.1b
(ii)	$G''_X(t) = \dfrac{6(3-2t)^3 - (-2)\times 3(3-2t)^2 \times 6t}{(3-2t)^6}$ or $\dfrac{18+24t}{(3-2t)^4}$	M1	2.1
	$G''_X(1) = 42$	A1	1.1b
	Var(X) = "42" + "6" − "6"^{2}	M1	1.1b
	= **12**	A1	1.1b
		(7)	
(d)	$G_Y(t) = t^{10} \times \dfrac{1}{9}\left[1 - \dfrac{2}{3}t^3\right]^{-2} = \dfrac{t^{10}}{9}\left[1 + \ldots \dfrac{(-2)(-3)(-4)}{3!}\left(-\dfrac{2}{3}\right)^3 t^9 \ldots\right]$	M1 A1	2.1 1.1b
	P(Y = 19) = $\dfrac{32}{243}$	A1	1.1b
		(3)	
ALT	Identify that $Y = 3X + 4$	M1	
	(Y = 19 requires X = 5 so) P(X = 5) = $\binom{4}{1}\left(\dfrac{1}{3}\right)\left(\dfrac{2}{3}\right)^3\left(\dfrac{1}{3}\right)$	A1	
		(13 marks)	

	Notes
(a)	M1 for identifying the NegBin distribution (allow NB for NegBin) A1 for $r = 2$ and $p = \frac{1}{3}$
(b)	B1ft for identifying a suitable definition for X using a (fair) die, with $p = \frac{1}{3}$ and the second occurrence of the event, only ft their NegBin distribution in (a). A finite number of rolls is B0
(c)(i)	1st M1 for attempt to differentiate quotient or product. At least one uv' style term correct. 1st A1 for a fully correct first derivative (needn't be simplified) 2nd A1 for E(X) = 6 NB this A1 depends on M1 only but M1A0A1 is possible
(ii)	2nd M1 for attempt to diff' quotient or product again. At least one uv' style term correct. 3rd A1 for 42 (may be given for incorrect G'' provided their $G''(1)$ gives 42 and M1 scored) Note all powers of (3 – 2t) equal 1 when t = 1 is substituted so can be used as a check 3rd M1 for correct use of pgf to find Var(X) 4th A1 dep on M3 for 12
(d)	M1 for writing pgf in suitable form to carry out binomial expansion 1st A1 for a correct expression for coefficient of t^{19} 2nd A1 for $\frac{32}{243}$ or exact equivalent

ALT	M1 for identifying connection $Y = 3X + 4$ 1st A1 for a correct numerical probability expression for $\mathrm{P}(X = 5)$

Qu7	**Scheme**	**Marks**	**AO**
(a)(i)	$X \sim \mathrm{Geo}(0.2)$ or $\mathrm{P}(X = 4) = 0.8^3 \times 0.2$	M1	3.3
	$= \underline{\mathbf{0.1024}}$	A1	1.1b
		(2)	
(ii)	$T \sim \mathrm{NegBin}(3, 0.2)$ or $\mathrm{P}(T = 8) = \binom{7}{2} 0.2^2 \times 0.8^5 \times 0.2$	M1	3.3
	$= 0.05505\ldots$ awrt $\underline{\mathbf{0.0551}}$	A1	1.1b
		(2)	
(iii)	$F \sim \mathrm{B}(10, 0.2)$ or $\mathrm{P}(F = 4) = \binom{10}{4} 0.2^4 \times 0.8^6$	M1	3.3
	$\mathrm{P}(F = 4) = 0.088080\ldots$ awrt $\underline{\mathbf{0.0881}}$	A1	1.1b
		(2)	
(b)	$\mathrm{P}(R) = \mathrm{P}(X \leqslant 4)$ <u>and</u> $X \sim \mathrm{Geo}(0.2)$ | $\mathrm{P}(X \geqslant 1)$, $X \sim \mathrm{B}(4, 0.2)$	M1	3.1b
	$= 1 - \mathrm{P}(X > 4) = 1 - 0.8^4$ | $= 1 - \mathrm{P}(Y = 0) = 1 - 0.8^4$	M1	3.4
	$= \underline{0.59(04)}$	A1	1.1b
	$\mathrm{P}(Y) = \mathrm{P}(N \leqslant 7)$ <u>and</u> $N \sim \mathrm{NegBin}(3, 0.4)$	M1	3.1b
	$0.4^3 + \binom{3}{2} 0.4^3 0.6^1 + \binom{4}{2} 0.4^3 0.6^2 + \binom{5}{2} 0.4^3 0.6^3 + \binom{6}{2} 0.4^3 0.6^4$ | $1 - \left(\binom{7}{2} 0.4^2 0.6^5 + \binom{7}{1} 0.4^1 0.6^6 + \binom{7}{0} 0.6^7 \right)$	M1	3.4
ALT	$\mathrm{P}(Y) = \mathrm{P}(W > 2)$ where $W \sim \mathrm{B}(7, 0.4)$	M1	
	$= 1 - \mathrm{P}(W \leqslant 2)$ $[= 1 - 0.419904]$	M1	
	$= \underline{0.58(0096)}$	A1	1.1b
	R (has the greater probability)	A1	3.2b
		(7)	
		(13 marks)	

	Notes
(a)(i)	M1 for selecting the correct model. Stated or used which may be implied by ans. A1 for 0.1024 or $\frac{64}{625}$ (accept 0.102) (correct answer scores 2 out of 2)
(ii)	M1 for selecting the correct model. Stated or used which may be implied by ans. Allow $0.2 \times \mathrm{P}(V = 2)$ from $V \sim \mathrm{B}(7, 0.2)$ A1 for awrt 0.0551 (correct answer scores 2 out of 2)
(iii)	M1 for selecting the correct model. Stated or used may be implied by ans of 0.967(2) A1 for awrt 0.0881 (correct answer scores 2 out of 2)
(b)	1st M1 for a correct distribution and prob. expression for $\mathrm{P}(R)$ (may be implied by 2nd M1) 2nd M1 for a correct numerical expression for $\mathrm{P}(R)$ (allow any equivalent expression) 1st A1 for awrt 0.590 or $\frac{369}{625}$ (accept 0.59 or better) awrt 0.590 implies M1M1A1 3rd M1 for a correct distribution and prob. expression for $\mathrm{P}(Y)$ (may be implied by 4th M1) 4th M1 for a correct numerical expression for $\mathrm{P}(Y)$ (allow any equivalent expression) 2nd A1 for awrt 0.580 or (accept 0.58 or better) awrt 0.580 implies M1M1A1

6. The random variable X has probability generating function $G_X(t)$ where

$$G_X(t) = \frac{1}{\sqrt{4-3t}}$$

(a) Use calculus to find $\text{Var}(X)$
Show your working clearly.

(6)

(b) Find the exact value of $P(X \leqslant 2)$

(4)

The independent random variables X_1 and X_2 each have the same distribution as X

The random variable $Y = X_1 + X_2 + 1$

(c) By finding the probability generating function of Y, state the name of the distribution of Y

(4)

(d) Hence, or otherwise, find $P(X_1 + X_2 > 5)$

(2)

Qu	Scheme	Mark	AO
6 (a)	$[G_X(t) = (4-3t)^{-\frac{1}{2}} \Rightarrow]$ $G'_X(t) = \frac{3}{2}(4-3t)^{-\frac{3}{2}}$; [So E($X$) =] $G'_X(t) = \frac{3}{2}$	M1; A1	2.1; 1.1b
	$G''_X(t) = \frac{27}{4}(4-3t)^{-\frac{5}{2}}$; so $G''_X(1) = \frac{27}{4}$	M1; A1ft	2.1 1.1b
	Var(X) $=$ "$\frac{27}{4}$" + "$\frac{3}{2}$" − ("$\frac{3}{2}$")^2; = **6**	M1; A1	1.1b 1.1b
		(6)	
(b)	Using Maclaurin: $\frac{G''_X(0)}{2!} = \left\{\frac{1}{2}\right\} \times$ "$\frac{27}{4}$" $\times \frac{1}{32}$ / Using Binomial: $[G_X(t) =]\ \frac{1}{2}\left(1-\frac{3}{4}t\right)^{-\frac{1}{2}}$	M1	2.1
	Maclaurin: $[P(X=2) =]\ \frac{1}{2} \times \frac{27}{4} \times \frac{1}{32}\left[= \frac{27}{256}\right]$ / Binomial: $\frac{1}{2}\left(\ldots + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}\left(-\frac{3}{4}t\right)^2 + \ldots\right) = \left[\ldots + \frac{27}{256}t^2 + \ldots\right]$	A1	1.1b
	Maclaurin: $[P(X=0) + P(X=1) =]$ "$\frac{3}{2}$" $\times \frac{1}{8} + \frac{1}{2}$ / Binomial: $\frac{1}{2}\left(1 + \frac{3}{8}t + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}\left(-\frac{3}{4}t\right)^2 + \ldots\right)$	M1	2.1
	Maclaurin: P($X \leqslant 2$) = $\frac{203}{256}$ / Binomial: P($X \leqslant 2$) = $\frac{203}{256}$	A1	1.1b
		(4)	
(c)	$\left[G_W(t) = G_{X_1}(t) \times G_{X_2}(t) \times t =\right] \frac{1}{\sqrt{4-3t}} \times \frac{1}{\sqrt{4-3t}} \times t$; $G_Y(t) = = \frac{t}{4-3t}$	M1; A1	3.1a/2.1 1.1b
	$G_Y(t) = = \frac{\frac{1}{4}}{1-\frac{3}{4}t}$ or $G_Y(t) = = \frac{\frac{1}{4}t}{1-\frac{3}{4}t}$; ***Y* ~ Geo($\frac{1}{4}$)**	M1 A1	2.1 3.2a/2.2a
		(4)	
(d)	P(Y > 6) = $\left(1-\frac{1}{4}\right)^6$; $= \frac{729}{4096}$ = 0.177978… awrt **<u>0.178</u>**	M1 A1	3.4,1.1b
		(2)	
	Notes	**(16 marks)**	
(a)	1st M1 for attempt to differentiate leading to $k(4-3t)^{-1.5}$; 1st A1 for E(X) = 1.5 or exact equivalent 2nd M1 for attempting to differentiate again leading to $m(4-3t)^{-2.5}$ 2nd A1ft for $\frac{27}{4}$ or correct ft from their k provided both Ms are scored 3rd M1 for a correct method for finding Var(X); can ft their $\frac{3}{2}$ and their $\frac{27}{4}$ 3rd A1 for 6		
(b)	1st M1 for Maclaurin to find P(X = 2) condone "$\frac{27}{4}$" $\times \frac{1}{32}\left[= \frac{27}{128} \text{ or } 0.2109375\right]$ **or** putting in the form $a(1-0.75t)^{-0.5}$ 1st A1 for a correct unsimplified prob for P(X = 2) (may be in binomial expansion) Allow 0.105(468…) 2nd M1 for use of pgf to find P(X = 1) and P(X = 0) <u>or</u> attempt 1st 3 terms of bin expansion 2nd A1 for $\frac{203}{256}$ or exact equivalent.		
(c)	1st M1 for using product of pgf or multiplication by t 1st A1 for correct unsimplified form of pgf 2nd dM1 (dep. on 1st M1) for attempting to convert pgf to form given in the formula book **or** for stating Geometric alongside a correct PGF for Y (may be unsimplified) 2nd A1 for correctly deducing the distribution of Y as geometric with p = 0.25 [may be seen in (d)] **NB:** Final A1 dependent on previous marks being awarded.		
(d)	M1 for attempting to use geometric formula or their pgf (using correct coefficients) ; A1 for awrt 0.178		

6. The probability generating function of the discrete random variable X is given by

$$G_X(t) = k(3 + t + 2t^2)^2$$

(a) Show that $k = \frac{1}{36}$ **(2)**

(b) Find $P(X = 3)$ **(2)**

(c) Show that $\text{Var}(X) = \frac{29}{18}$ **(8)**

(d) Find the probability generating function of $2X + 1$ **(2)**

Question	Scheme	Marks	AOs
Q6(a)	$G_X(1)=1$ gives	M1	2.1
	$k \times 6^2 = 1$ so $k = \frac{1}{36}$ *	A1*cso	1.1b
		(2)	
(b)	$P(X=3)$ = coefficient of t^3 so $G_X(t) = k\left(...+4t^3...\right)$	M1	1.1b
	[$P(X=3)$ =] $\underline{\frac{1}{9}}$	A1	1.1b
		(2)	
(c)	$G'_X(t) = 2k\left(3+t+2t^2\right)\times\left(1+4t\right)$	M1	2.1
	$E(X) = G'_X(1) = 2k\left(3+1+2\right)\times\left(1+4\right)$	M1	1.1b
	$= \frac{5}{3}$	A1	1.1b
	$G''_X(t) = 2k\left[\left(3+t+2t^2\right)\times 4 + \left(1+4t\right)^2\right]$	M1 A1	2.1 1.1b
	$G''_X(1) = 2k[6\times 4 + 5^2]$ $\left\{= \frac{49}{18}\right\}$	M1	1.1b
	$\text{Var}(X) = G''_X(1) + G'_X(1) - \left[G'_X(1)\right]^2 = \frac{49}{18} + \frac{5}{3} - \frac{25}{9}$	M1	2.1
	$= \underline{\frac{29}{18}}$ *	A1*cso	1.1b
		(8)	
(d)	$G_{2X+1}(t) = \frac{t}{36}\left(3+t^2+2\left(t^2\right)^2\right)^2$ [$\times t$ <u>or</u> sub t^2 for t]	M1	3.1a
	$= G_{2X+1}(t) = \frac{t}{36}\left(3+t^2+2t^4\right)^2$	A1	1.1b
		(2)	
		(14 marks)	

Notes:

(a)

M1: Stating $G_X(1)=1$

A1*: Fully correct proof with no errors cso

(b)

M1: Attempting to find the coefficient of t^3. May be implied by obtaining $\frac{1}{9}$ or awrt 0.11

A1: $\frac{1}{9}$, allow awrt 0.111

Question 6 notes continued:	
(c)	
M1:	Attempting to find $G'_X(t)$. Allow Chain rule or multiplying out the brackets and differentiating
M1:	Substituting $t = 1$ into $G'_X(t)$
A1:	$\frac{5}{3}$, allow awrt 1.67
M1:	Attempting to find $G''_X(t)$
A1:	$2k\left[\left(3+t+2t^2\right)\times 4+\left(1+4t\right)^2\right]$ **or** $k(48t^2+24t+26)$ o.e.
A1:	$2k[6\times 4+5^2]$ o.e.
M1:	Using $G''_X(1)+G'_X(1)-\left[G'_X(1)\right]^2$ to find the Variance
A1*:	$\frac{29}{18}$ cso
(d)	
M1:	Realising the need to $\times t$ **or** sub t^2 for t
A1:	$\frac{t}{36}\left(3+t^2+2t^4\right)^2$, or $\frac{t}{36}\left(9+6t^2+13t^4+4t^6+4t^8\right)$ o.e.

4. The discrete random variable $X \sim B(n, p)$ has probability generating function given by

$$G_X(t) = (0.4 + 0.6t)^2$$

(a) Write down the value of n and the value of p

(2)

Using the probability generating function, find

(b) (i) $P(X = 1)$

(2)

(ii) $E(X)$

(3)

Two independent observations, X_1 and X_2, are taken from the distribution of X
The random variable $Y = X_1 + X_2$

(c) Use calculus to show that $E(Y^2) = 6.72$

(7)

Question	Scheme	Marks	AOs
4(a)	$n = 2$ and $p = 0.6$	B1 B1	1.1b 1.1b
		(2)	
(b)(i)	P(X=1) = coefficient of t $\quad G_X(t) = 0.16 + 0.48t + 0.36t^2$	M1	1.1b
	P(X=1) = **<u>0.48</u>**	A1	1.1b
		(2)	
(ii)	$E(X) = G'_X(1)$	M1	2.1
	$G'_X(t) = 2(0.4 + 0.6t) \times 0.6$	M1	1.1b
	$G'_X(1) = 1.2$	A1	1.1b
		(3)	
(c)	$G_Y(t) = G_X(t) \times G_X(t)$		
	$G_Y(t) = (0.4 + 0.6t)^4$	B1	3.1a
	$G'_Y(t) = 4(0.4 + 0.6t)^3 \times 0.6$ $G'_Y(1) = 2.4$	M1 A1	2.1 1.1b
	$G''_Y(t) = 7.2(0.4 + 0.6t)^2 \times 0.6$ $G''_Y(1) = 4.32$	M1 A1	2.1 1.1b
	$E(Y^2)[= Var(Y) + [E(Y)]^2] = G'_Y(1) + G''_Y(1)$	M1	1.1b
	$E(Y^2) = 2.4 + 4.32 = 6.72$ *	A1*cso	1.1b
		(7)	
			(14 marks)

Notes

(a)	1st B1 $n = 2$ 2nd B1 $p = 0.6$
(b)(i) **(b)(ii)**	M1 Finding coefficient of t A1 0.48oe 1st M1 Realising $G'_X(1)$ is needed 2nd M1 Differentiation A1 1.2cao
(c)	B1 Correct use of $G_Y(t) = G_X(t) \times G_X(t)$ 1st M1 Differentiation to find $G'_Y(t)$ 1st A1 $G'_Y(1) = 2.4$ 2nd M1 Differentiation to find $G''_Y(t)$ 2nd A1 $G''_Y(1) = 4.32$ 3rd M1 Realising $E(Y^2) = G'_Y(1) + G''_Y(1)$ 3rd A1*cso 6.72