FS1Ch6 XMQs and MS

(Total: 91 marks)

1. FS1_2019 Q4 . 19 marks - FS1ch6 Chi-squared tests
2. FS1_2020 Q5 . 13 marks - FS1ch6 Chi-squared tests
3. FS1_2021 Q1 . 7 marks - FS1ch6 Chi-squared tests
4. FS1_2022 Q1 . 9 marks - FS1ch6 Chi-squared tests
5. FS1_2023 Q3 . 15 marks - FS1ch6 Chi-squared tests
6. FS1_2024 Q3 . 6 marks - FS1ch6 Chi-squared tests
7. FS1_Sample Q3 . 14 marks - FS1ch6 Chi-squared tests
8. FS1_Specimen Q1 . 8 marks - FS1ch6 Chi-squared tests

4. Liam and Simone are studying the distribution of oak trees in some woodland. They divided the woodland into 80 equal squares and recorded the number of oak trees in each square. The results are summarised in Table 1 below.

Number of oak trees in a square	0	1	2	3	4	5	6	7 or more
Frequency	1	4	21	23	13	11	7	0

Table 1

Liam believes that the oak trees were deliberately planted, with 6 oak trees per square and that a constant proportion p of the oak trees survived.

(a) Suggest the model Liam should use to describe the number of oak trees per square.
(2)

Liam decides to test whether or not his model is suitable and calculates the expected frequencies given in Table 2.

Number of oak trees in a square	0 or 1	2	3	4	5	6
Expected frequency	5.53	14.89	24.26	22.24	10.87	2.21

Table 2

(b) Showing your working clearly, complete the test using a 5% level of significance. You should state your critical value and conclusion clearly.
(7)

Simone believes that a Poisson distribution could be used to model the number of oak trees per square. She calculates the expected frequencies given in Table 3.

Number of oak trees in a square	0 or 1	2	3	4	5	6 or more
Expected frequency	12.69	16.07	s	14.58	t	9.37

Table 3

(c) Find the value of s and the value of t, giving your answers to 2 decimal places.
(4)

(d) Write down hypotheses to test the suitability of Simone's model.
(1)

The test statistic for this test is 8.749

(e) Complete the test. Use a 5% level of significance and state your critical value and conclusion clearly.
(3)

(f) Using the results of these tests, explain whether the origin of this woodland is likely to be cultivated or wild.
(2)

Qu	Scheme	Marks	AO
4(a)	[T = no. of oak trees in a square] T ~ Binomial	M1	3.3
	$T \sim B(6, p)$	A1	1.1b
		(2)	
(b)	Expected frequency for 6 is less than 5 so pool: new $E_i = 13.08$	M1	2.1

$\frac{(O_i - E_i)^2}{E_i}$	0.051	2.51	0.0654	3.84	1.85
$\frac{O_i^2}{E_i}$	4.521	29.617	21.805	7.599	24.771

Qu	Scheme	Marks	AO
	$\sum \frac{(O_i - E_i)^2}{E_i} = 8.313$	M1,A1	1.1b x2
	p needed estimating ($\hat{p} = 0.55$) so $\nu = 5 - 2 = 3$; cv 7.815	B1,B1ft	1.1b x2
	Significant result, so Liam's <u>model is not suitable</u>	M1,A1	1.1b2.2b
		(7)	
(c)	[R = no. of oak trees in a square for Simone's model] $R \sim Po(3.3)$	M1	3.3
	Correct expression for s or t using Poisson	M1	3.4
	s = **<u>17.67</u>** and t = **<u>9.62</u>**	A1,A1	1.1b x2
		(4)	
(d)	H_0: Poisson is a good fit (for no. of oak trees per square) H_1: Poisson is not a good fit (for no. of oak trees per square)	B1	2.5
		(1)	
(e)	No pooling needed so degrees of freedom is 6 – 2 = 4	B1	1.1b
	Critical value is 9.488 (accept 9.49)	B1	1.1a
	Not significant so Poisson (or Simone's) model is suitable	B1	2.2b
		(3)	
(f)	Poisson model has better fit so suggests that oak trees occur at random <u>Or</u> binomial suggests deliberately planted or cultivated	B1	2.2b
	Therefore the forest is likely to be wild not cultivated	B1	3.5a
		(2)	
		(19 marks)	

	Notes
(a)	M1 for choosing binomial A1 for B(6, p) can be in words and allow B(6, 0.55)
(b)	1st M1 for pooling last 2 classes ($E_i = 13.08$ but accept 13.1) 2nd M1 for at least 3 correct values or expressions. Either row to at least 2 sf 1st A1 for awrt 8.31 (8.31 gets 3/3) [NB no pooling gives awrt 16.8458.. and implies M0M1A0] 1st B1 for 3 degrees of freedom 2nd B1ft for critical value of 7.815 (e.g. $\nu = 4$ use 9.488) 3rd M1 for a correct conclusion (non-contextual ignore any contradictory contextual comments for this mark) based on their cv and their test statistic This mark can be implied by a fully correct solution ending with correct contextual conclusion 2nd A1 for correct conclusion in context with **all other marks scored**
(c)	1st M1 for selecting a correct model Po(3.3) [Allow Po(awrt 3.3)] 2nd M1 for use of the model with an expression or correct value for s or t 1st A1 for one correct 2nd A1 for both correct (allow awrt 2dp)
(d)	B1 for correct hypotheses must mention Poisson: use of Po(3.3) is B0
(e)	1st B1 for correct degrees of freedom $\nu = 4$ only 2nd B1 for selecting correct critical value (9.488 only) 3rd B1 for <u>not significant</u> conclusion based on 8.749 vs their cv (condone use of Po(3.3) here)
(f)	1st B1 for choosing Poisson as better <u>or</u> stating Poisson implies wild <u>or</u> bino'l implies cultivated 2nd B1 (dep on rejecting bin and accepting Poisson) for clearly stating woodland is wild If the tests give the same results then 2nd B0 automatically

5. A factory produces pins.

An engineer selects 40 independent random samples of 6 pins produced at the factory and records the number of defective pins in each sample.

Number of defective pins	0	1	2	3	4	5	6
Observed frequency	19	11	7	2	0	1	0

(a) Show that the proportion of defective pins in the 40 samples is 0.15

(2)

The engineer suggests that the number of defective pins in a sample of 6 can be modelled using a binomial distribution. Using the information from the sample above, a test is to be carried out at the 10% significance level, to see whether the data are consistent with the engineer's suggested model.

The value of the test statistic for this test is 2.689

(b) Justifying the degrees of freedom used, carry out the test, at the 10% significance level, to see whether the data are consistent with the engineer's suggested model. State your hypotheses clearly.

(8)

The engineer later discovers that the previously recorded information was incorrect. The data should have been as follows.

Number of defective pins	0	1	2	3	4	5	6
Observed frequency	19	11	6	3	1	0	0

(c) Describe the effect this would have on the value of the test statistic that should be used for the hypothesis test.
Give reasons for your answer.

(3)

<table>
<tr><th>Qu.</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>5(a)</td><td>$p = \dfrac{(0)+11+14+6+(0)+5+(0)}{6\times 40}$</td><td>M1</td><td>2.1</td></tr>
<tr><td></td><td>$p =$ <u>0.15</u> *</td><td>A1*cso</td><td>1.1b</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
<tr><td>(b)</td><td>X~B(6, 0.15)
<table><tr><td>x</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>$40\times \text{P}(X=x)$</td><td>7.04…</td><td>1.65…</td><td>0.219...</td><td>0.015…</td><td>0.00…</td></tr></table></td><td>M1</td><td>3.4</td></tr>
<tr><td></td><td>Require 40 × P($X \geq k$) > 5
Exp. frequency for $X \geq 2$ = 8.94…/ $X \geq 3$ = 1.89...</td><td>M1</td><td>1.1b</td></tr>
<tr><td></td><td>Combine last 5 cells / only 3 cells in total</td><td>A1</td><td>2.2a</td></tr>
<tr><td></td><td>2 is subtracted (as there are 2 restrictions) and the proportion used from data (and 1 equal totals)</td><td>B1</td><td>2.4</td></tr>
<tr><td></td><td>3 – 2 = 1 degree of freedom</td><td>A1</td><td>1.1b</td></tr>
<tr><td></td><td>H_0 : Binomial distribution is a suitable model
H_1 : Binomial distribution is not a suitable model</td><td>B1</td><td>3.4</td></tr>
<tr><td></td><td>Critical value $\chi^2_{(1,0.10)}$ = 2.705 or 2.706</td><td>B1ft</td><td>1.1b</td></tr>
<tr><td></td><td>Test statistic is not in the critical region, insufficient evidence to reject H_0 (2.689 < 2.705/6)
Data are consistent with binomial/engineer’s/suggested model.</td><td>B1ft</td><td>3.5a</td></tr>
<tr><td></td><td></td><td>(8)</td><td></td></tr>
<tr><td>(c)</td><td>The total amount/proportion of defective pins remains the same.</td><td>M1</td><td>2.4</td></tr>
<tr><td></td><td>The cells for $X \geq 2$ are still combined in the test.</td><td>M1</td><td>1.1b</td></tr>
<tr><td></td><td>So there is no change to the value of the test statistic.</td><td>A1</td><td>2.2a</td></tr>
<tr><td></td><td></td><td>(3)</td><td></td></tr>
<tr><td colspan="4" align="right">(13 marks)</td></tr>
</table>

	Notes
(a)	**M1:** Correct expression for p (may be seen in stages). Allow $\frac{36}{240}$ but not $\frac{6}{40}$ on its own **A1*cso:** p = 0.15 stated and no incorrect working seen
(b)	**M1:** Attempting to find expected frequencies, at least 2 correct trunc. or rounded 1dp **M1:** Recognising need to combine cells (Sight of awrt 8.94 implies M1M1) **A1:** Combining cells for $X \geq 2$ (to make 3 cells) **B1:** Justifying why 2 is subtracted with p being calculated from data **A1:** 1 degree of freedom **B1:** Correct hypotheses (0.15 must **not** be included) Allow engineer’s model. **B1ft:** Correct critical value (ft their df) May see $\chi^2_{(2,0.10)} = 4.605$ or $\chi^2_{(3,0.10)} = 6.251$ **B1ft:** Correct inference (ft comparison of their CV with 2.689). Condone p = 0.15 included here. Do not allow contradictory statements to score here. Hypotheses must be correct way round.
(c)	**M1**: Determining the number (N=36)/proportion (p=0.15) of defective pins has not changed. e.g. 11 + 12 + 9 + 4 = 36. But not 7 + 2 + 1 = 6 + 3 + 1 **M1**: Understanding the cells for $X \geq 2$ are still combined in the test **A1**: (dep on both M1s) Concluding that there is no change to the value of the test statistic.

1. Kelly throws a tetrahedral die n times and records the number on which it lands for each throw.

She calculates the expected frequency for each number to be 43 if the die was unbiased.

The table below shows three of the frequencies Kelly records but the fourth one is missing.

Number	1	2	3	4
Frequency	47	34	36	x

(a) Show that $x = 55$

(1)

Kelly wishes to test, at the 5% level of significance, whether or not there is evidence that the tetrahedral die is unbiased.

(b) Explain why there are 3 degrees of freedom for this test.

(1)

(c) Stating your hypotheses clearly and the critical value used, carry out the test.

(5)

Paper 3B/ 2021: Statistics 1 Mark scheme

Question	Scheme	Marks	AOs
1(a)	$x = 4\times 43 - 47 - 34 - 36 = 55$*	B1*	3.4
		(1)	
(b)	$\nu = 4 - 1 = 3$ since the only constraint is that the totals agree	B1	2.4
		(1)	
(c)	H_0: The die is unbiased H_1: The die is biased	B1	2.1
	Test Statistic = $\frac{(47-43)^2}{43}+\frac{(34-43)^2}{43}+\frac{(36-43)^2}{43}+\frac{(55-43)^2}{43}$	M1	1.1b
	$= 6.744\ldots$	A1	1.1b
	$\chi^2_{(3,0.05)} = 7.815$	B1	1.1b
	Not in the critical region since 7.815 > "6.74…" therefore insufficient evidence to reject H_0 Inconclusive test - consistent with the **die** being unbiased.	A1	3.5a
		(5)	
			(7 marks)

Notes:

(a)	**B1*:**	Using the uniform model to show the missing observed value eg $x = \frac{43 - 0.25\times(47+34+36)}{0.25} = 55$
(b)	**B1:**	$4 - 1 = 3$ (may be in words) and explanation of what the constraint is
(c)	**B1:**	Both hypotheses correct. eg The data fits a discrete uniform distribution
	M1:	Attempting to find $\sum\frac{(O-E)^2}{E}$ or $\sum\frac{O^2}{E} - N$ May be implied by awrt 6.74 or p value of 0.0805…
	A1:	awrt 6.74 or $\frac{290}{43}$ oe May be implied by p value of 0.0805…
	B1:	awrt 7.82 (Calc 7.8147…)
	A1:	Drawing correct inference in context. Need the word die or tetrahedral

1. A researcher is investigating the number of female cubs present in litters of size 4
He believes that the number of female cubs in a litter can be modelled by B(4, 0.5)
He randomly selects 100 litters each of size 4 and records the number of female cubs.
The results are recorded in the table below.

Number of female cubs	0	1	2	3	4
Observed number of litters	10	33	33	15	9

He calculated the expected frequencies as follows

Number of female cubs	0	1	2	3	4
Expected number of litters	6.25	r	s	r	6.25

(a) Find the value of r and the value of s

(3)

(b) Carry out a suitable test, at the 5% level of significance, to determine whether or not the number of female cubs in a litter can be modelled by B(4, 0.5)
You should clearly state your hypotheses and the critical value used.

(6)

Qu	Scheme	Marks	AOs
1(a)	$r = \mathrm{P}(X=3)\times 100$ or $r = \mathrm{P}(X=1)\times 100$ or $s = \mathrm{P}(X=2)\times 100$	M1	3.4
	r = **<u>25</u>** (value may be in table)	A1	1.1b
	s = **<u>37.5</u>** (value may be in table)	A1	1.1b
		(3)	
(b)	H_0: B(4,0.5) is a suitable model (o.e.) Condone B(0.5, 4) H_1: B(4,0.5) is not a suitable model (o.e.)	B1	2.5
	$\frac{(O_i-E_i)^2}{E_i}$: 2.25, 2.56, 0.54, 4, 1.21 $\frac{O_i^2}{E_i}$: 16, 43.56, 29.04, 9, 12.96	M1	1.1b
	$\sum\frac{(O_i-E_i)^2}{E_i} = 10.56$ <u>or</u> $\sum\frac{O_i^2}{E_i} - N = 110.56 - 100 = 10.56$ $\left(=\frac{264}{25}\right)$	A1	1.1b
	$\nu = 5-1 = 4$	B1	1.1b
	CV = 9.488 (Calc 9.487729035…)	B1ft	1.1b
	Significant so there is evidence that the researcher's **model is not suitable**	A1	2.2b
		(6)	
			Total 9

(a)	M1	Using the Binomial model to expected value. Allow if <u>both</u> probs 0.25 and 0.375 seen May be implied by a correct value of r or s. Alternatives $r = 6.25\times 4$ or $s = 6.25\times 6$
	1st A1	for r = 25
	2nd A1	for s = 37.5
SC	"B1"	If M0 scored but their values of r and s satisfy $2r + s = 87.5$ score as **M0A0A1**
(b)	1st B1	Both hypotheses correct using the correct notation in at least one <u>or</u> written in full e.g. binomial with n = 4 and p = 0.5
	M1	Calculating either $\frac{(O_i-E_i)^2}{E_i}$ or $\frac{O_i^2}{E_i}$ at least 4 correct. Implied by sight of awrt 10.6
	1st A1	Allow 10.6 (from correct working)
	2nd B1	Correct dof May be implied by CV of 9.48 or 9.49 or better
	3rd B1ft	For 9.488 or better. Can ft their dof NB $\chi_3^2(5\%)$ = 7.815 (allow awrt 7.815)
	2nd A1	**Indep'of hypotheses but dep on 1st A1** Evaluating the outcome by drawing a correct inference. Compatible with comparison of 10.56 or 10.6 with their CV (which must be > 1) They must say **model not suitable** (o.e.) They do not need to state the comparison or say reject H_0 etc No need to explicitly see B(4, 0.5) mentioned here

Question	Scheme	Marks	AOs

3. In a class experiment, each day for 170 days, a child is chosen at random and spins a large cardboard coin 5 times and the number of heads is recorded.
The results are summarised in the following table.

Number of heads	0	1	2	3	4	5
Frequency	3	10	45	62	38	12

Marcus believes that a B(5, 0.5) distribution can be used to model these data and he calculates expected frequencies, to 2 decimal places, as follows

Number of heads	0	1	2	3	4	5
Expected frequency	r	26.56	s	s	26.56	r

(a) Find the value of r and the value of s

(3)

(b) Carry out a suitable test, at the 5% level of significance, to determine whether or not the B(5, 0.5) distribution is a good model for these data.
You should state clearly your hypotheses, the test statistic and the critical value used.

(6)

Nima believes that a better model for these data would be B(5, p)

(c) Find a suitable estimate for p

(1)

To test her model, Nima uses this value of p, to calculate expected frequencies as follows

Number of heads	0	1	2	3	4	5
Expected frequency	2.07	14.65	41.44	58.63	41.47	11.74

The test statistic for Nima's test is 1.62 (to 3 significant figures)

(d) State,
(i) giving your reasons, the degrees of freedom
(ii) the critical value
that Nima should use for a test at the 5% significance level.

(3)

(e) With reference to Marcus' and Nima's test results, comment on
(i) the probability of the coin landing on heads,
(ii) the independence of the spins of the coin.
Give reasons for your answers.

(2)

Qu 3	Scheme	Mark	AO
(a)	$[X \sim B(5, 0.5)]$ $P(X=0) = P(X=5) = 0.03125$ <u>or</u> $P(X=2)$ or $P(X=3) = 0.3125$	M1	1.1b
	[multiply by 170 to get] $r =$ **<u>5.31</u>**(25) ; $s =$ **<u>53.1</u>**(25)	A1;A1	1.1b(x2)
		(3)	
(b)	$H_0: B(5, 0.5)$ is a suitable model $H_1: B(5, 0.5)$ is NOT a …	B1	2.5

$\frac{(O_i-E_i)^2}{E_i}$	$\frac{(3-'5.31')^2}{'5.31'}$ $=1.00...$	$\frac{(10-26.56)^2}{26.56}$ $=10.3...$	$\frac{(45-'53.1')^2}{'53.1'}$ $=1.23...$	$\frac{(62-'53.1')^2}{'53.1'}$ $=1.48...$	$\frac{(38-26.56)^2}{26.56}$ $=4.92...$	$\frac{(12-'5.31')^2}{'5.31'}$ $=8.41...$
$\frac{O_i^2}{E_i}$	$\frac{3^2}{'5.31'}$ $=1.69...$	$\frac{10^2}{26.56}$ $=3.76...$	$\frac{45^2}{'53.1'}$ $=38.1...$	$\frac{62^2}{'53.1'}$ $=72.3...$	$\frac{38^2}{26.56}$ $=54.3...$	$\frac{12^2}{'5.31'}$ $=27.1...$

(M1, 1.1b)

	$\sum\frac{(O_i-E_i)^2}{E_i}$ or $\sum\frac{O_i^2}{E_i}-170 = 27.4 \ldots$ awrt **<u>27.4</u>** or awrt **<u>27.5</u>**	A1	1.1b
	Degrees of freedom is $6 - 1 =$ **<u>5</u>** , and critical value is **<u>11.07(0)</u>**	B1ft B1ft	1.1b(x2)
	[Significant result] <u>Marcus' model/B(5, 0.5)</u> is not a good fit. (o.e.)	A1	2.2b
		(6)	
(c)	$\hat{p} = \left[\frac{0\times3+1\times10+...+5\times12}{170\times5}\right] = 0.58588\ldots$ awrt **<u>0.586</u>**	B1	1.1b
		(1)	
(d)(i)	Need to pool (first 2) cells (0 and 1 since $E(0) < 5$) <u>and</u> use of $\hat{p}$	M1	2.4
	Degrees of freedom: 5 groups – 2 constraints = **<u>3</u>**	A1	1.1b
(ii)	Critical value is **<u>7.815</u>**	B1ft	1.1b
		(3)	
(e)(i)	Nima's model is a good fit (since 1.62 < '7.815')/Marcus' is not <u>and</u> this suggests coin is biased/probability of head approx. 0.6	B1	2.4
(ii)	Nima's test suggests binomial is a good model <u>and</u> therefore independence of spins is a reasonable assumption	B1	2.2b
		(2)	**(15 marks)**

	Notes
(a)	M1 for 1 correct probability which may be embedded (0.03125 or 0.3125 or 0.5^5 or 5C2 $0.5^2 \times 0.5^3$)
	1st A1 for r = awrt 5.31 (condone $\frac{85}{16}$)
	2nd A1 for s = awrt 53.1 (condone $\frac{425}{8}$)
(b)	1st B1 for both hypotheses mentioning B(5, 0.5) or Marcus' distribution at least once
	M1 for at least one correct (ft) term or expression of the test statistic (accept 2sf)
	1st A1 for awrt 27.4 or awrt 27.5 (correct value here scores M1A1)
	2nd B1 for 5 <u>or</u> ft if 'their r' < 5, then df (= 4 – 1) = 3
	3rd B1 for 11.07(0) (or better) for ft df = 4 $\rightarrow$ 9.488 or df = 3 $\rightarrow$ 7.815
	A1 dep on 1st M1 for a suitable conclusion in context rejecting <u>B(5, 0.5)/Marcus' model</u> Must be compatible with their test statistic and their CV. Just 'Bin is not a good fit' is A0 A0 if inconsistent comments are seen e.g. "do not reject H_0, B(5, 0.5) is not a good fit"
(c)	B1 for awrt 0.586 allow $\frac{498}{850}$ o.e.
(d)(i)	M1 for both reasons, must mention pooling or show pooling or mention exp. value < 5 <u>and</u> use of estimated parameter
	A1 for df = 3 (must have scored the M1 for this mark)
(ii)	B1 for 7.815 (or better) allow this independent of the M1 only allow ft on df=4 $\rightarrow$ 9.488
(e)(i)	1st B1 for stating Nima's (binomial) model is a good fit/do not reject H_0 for Nima's model/Marcus' model is not a good fit <u>and</u> suggest that coin is probably biased/ $p > 0.5$ (p closer to 'their (c)') Only comparing 1.62 with '27.4' to reach $p > 0.5$ is incorrect and scores B0

(ii)	2^{nd} B1 for mention of Nima's test suggests Binomial distribution is suitable and that spins are independent (ignore reference to Marcus' test for 2^{nd} B1)		
Qu 4	**Scheme**	**Mark**	**AO**
	[X = no. of rolls to 4 sixes] X~NegBin(4, $\frac{1}{6}$)	M1	3.3
	$\mu\left[=\frac{r}{p}\right]=$ **24** , $\sigma^2\left[=\frac{r(1-p)}{p^2}\right]=\frac{4\times\frac{5}{6}}{\frac{1}{36}}=$ **120**	A1, A1	1.1b(x2)
	$[\bar{X}\approx\sim]\text{N}\left("24", \sqrt{\frac{"120"}{32}}^2\right)$	M1 M1	2.1,3.4
	$\text{P}(\bar{X}<27.2)=0.95078\ldots$ awrt **0.951**	A1	1.1b
		(6 marks)	
	Notes		
	1^{st} M1 for selecting the correct negative binomial model. May be implied by correct mean or variance NegBin on its own is M0 1^{st} A1 for mean = 24 2^{nd} A1 for variance = 120 $\sigma = 120$ is A0 unless recovered 2^{nd} M1 for writing or using of normal with mean 24 (may be implied by correct answer) ft their mean which may come from any distribution 3^{rd} M1 for writing or using normal with standard deviation $=\sqrt{\frac{120}{32}}\left[=\sqrt{3.75}\right]$ ft $\frac{\text{their } \sigma}{\sqrt{32}}$ where σ may come from any distribution (may be implied by correct answer) 2^{nd} A1 for awrt 0.951 (correct answer scores 6 out of 6)		

3. Tisam took a survey of students' favourite colours. The results are summarised in the table below.

		Colour					
		Red	**Blue**	**Green**	**Yellow**	**Black**	**Total**
Year group	**1–5**	34	15	14	22	3	88
	6–9	23	32	12	9	8	84
	10–12	5	28	19	8	8	68
	Total	62	75	45	39	19	240

Tisam carries out a suitable test to see if there is any association between favourite colour and year group.

(a) Write down the hypotheses for a suitable test.

(1)

For her table, Tisam only needs to check one cell to show that none of the expected frequencies are less than 5

(b) (i) Identify this cell, giving your reason.

(ii) Calculate the expected frequency for this cell.

(2)

The test statistic for Tisam's test is 38.449

(c) Using a 1% level of significance, complete the test.
You should state your critical value and conclusion clearly.

(3)

Qu	Scheme	Mark	AO
3 (a)	H_0: There is no association between the colour chosen and year group (allow use of "independence" instead of association) H_1: There is some association between colour and year group	B1 **(1)**	2.5
(b)(i)	Need lowest row total and column total so "Black" and "10-12"	B1	2.2a
(ii)	With expected frequency $\frac{68 \times 19}{240} = 5.3833\ldots$	B1 **(2)**	1.1b
(c)	$\upsilon = (5-1) \times (3-1) = \underline{\mathbf{8}}$	B1	3.4
	cv of $\chi_8^2(1\%) = \underline{\mathbf{20.090}}$	B1ft	1.1b
	(significant) evidence of an association between colour chosen and year group	B1ft **(3)**	2.2b
		(6 marks)	

	Notes
(a)	B1 for both hypotheses in context. Must mention colour and year group at least once **NB:** condone use of related/correlated/linked etc if recovered with independent/associated
(b)(i)	B1 for a choosing "Black" and "10-12" **with** some correct reasoning mentioning row and column totals (allow clear equivalent, e.g. 'lowest total frequencies')
(ii)	B1 for a correct expression or awrt 5.38
(c)	1st B1 for a correct calculation or answer of 8. May be implied by sight of cv of 20.09 2nd B1ft for 20.090 (accept 20.09) or a correct ft 1% cv using their df and correct to 2 d.p. 3rd B1ft for a correct conclusion in context (ft their cv) E.g. "students in different years (tend to) have different favourite colours" Do not allow contradictory statements. Condone 'related/relationship' in part (c) **NB:** We ignore their hypotheses when marking (c)

3. Bags of £1 coins are paid into a bank. Each bag contains 20 coins.

The bank manager believes that 5% of the £1 coins paid into the bank are fakes. He decides to use the distribution $X \sim \mathrm{B}(20, 0.05)$ to model the random variable X, the number of fake £1 coins in each bag.

The bank manager checks a random sample of 150 bags of £1 coins and records the number of fake coins found in each bag. His results are summarised in Table 1. He then calculates some of the expected frequencies, correct to 1 decimal place.

Number of fake coins in each bag	0	1	2	3	4 or more
Observed frequency	43	62	26	13	6
Expected frequency	53.8	56.6		8.9	

Table 1

(a) Carry out a hypothesis test, at the 5% significance level, to see if the data supports the bank manager's statistical model. State your hypotheses clearly.

(10)

The assistant manager thinks that a binomial distribution is a good model but suggests that the proportion of fake coins is higher than 5%. She calculates the actual proportion of fake coins in the sample and uses this value to carry out a new hypothesis test on the data. Her expected frequencies are shown in Table 2.

Number of fake coins in each bag	0	1	2	3	4 or more
Observed frequency	43	62	26	13	6
Expected frequency	44.5	55.7	33.2	12.5	4.1

Table 2

(b) Explain why there are 2 degrees of freedom in this case.

(2)

(c) Given that she obtains a χ^2 test statistic of 2.67, test the assistant manager's hypothesis that the binomial distribution is a good model for the number of fake coins in each bag. Use a 5% level of significance and state your hypotheses clearly.

(2)

Question	Scheme	Marks	AOs
Q3(a)	Expected value for 2 = $150 \times P(X = 2)$	M1	3.4
	= 28.3015…	A1	1.1b
	Expected value for 4 or more = 150 – (53.8 + 56.6 + 28.3 + 8.9) = 2.4	A1ft	1.1b
	H_0: Bin(20, 0.05) is a suitable model H_1: Bin(20, 0.05) is not a suitable model	B1	2.5
	Combining last two groups | | $\geqslant$ **3** | | **Observed frequency** | 19 | | **Expected frequency** | 11.3 |	M1	2.1
	$\nu = 4 - 1 = 3$	B1	1.1b
	Critical value, χ^2 (0.05) = 7.815	B1	1.1a
	Test statistic = $\frac{(43-53.8)^2}{53.8} + \frac{(62-56.6)^2}{56.6} + ...$	M1	1.1b
	= 8.117	A1	1.1b
	In critical region, sufficient evidence to reject H_0, accept H_1 Significant evidence at 5% level to reject the manager's model	A1	3.5a
		(10)	
(b)	$\nu = 4 - 2 = 2$		
	4 classes due to pooling	B1	2.4
	2 restrictions (equal total and mean/proportion)	B1	2.4
		(2)	
(c)	H_0: Binomial distribution is a good model H_1: Binomial distribution is not a good model	B1	3.4
	Critical value, χ^2 (0.05) = 5.991 Test statistic is not in critical region, insufficient evidence to reject H_0 There is evidence that the Binomial distribution is a good model	B1	3.5a
		(2)	
			(14 marks)

Notes:
(a) **M1:** Using the binomial model $150\times p^2\times(1-p)^{18}$ may be implied by 28.3 **A1:** awrt 28.3 **A1:** awrt 2.4 or ft their "28.3" **B1:** Both hypotheses correct using the correct notation or written out in full **M1:** For recognising the need to combine groups **B1:** Number of degrees of freedom = 3 may be implied by a correct CV **B1:** awrt 7.82 **M1:** Attempting to find $\sum\frac{(O_i-E_i)^2}{E_i}$ or $\sum\frac{O_i^2}{E_i}-N$ may be implied by awrt 8.12 **A1:** awrt 8.12 **A1:** Evaluating the outcome of a model by drawing a correct inference in context
(b) **B1:** Explaining why there are 4 classes **B1:** Explanation of why 2 is subtracted
(c) **B1:** Correct hypotheses for the refined model **B1:** The CV awrt 5.99 and drawing the correct inference for the refined model

Answer ALL questions. Write your answers in the spaces provided.

1. In a randomly selected week, a camera recorded the number of speeding drivers on a particular stretch of motorway.

	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Frequency	35	30	28	24	40	51	37

Jeremy believes drivers are equally likely to be recorded speeding on any day of the week.

Carry out a hypothesis test, at the 5% level of significance, to see if the data support Jeremy's belief.

You should state your hypotheses, the degrees of freedom and the critical value used for this test.

(8)

Specimen Paper 9FM0/3B: Further Statistics 1 Mark scheme

Question	Scheme	Marks	AOs
1	H_0 : Drivers are equally likely to be recorded speeding on any day of the week H_1 : Drivers are not equally likely to be recorded speeding on any day of the week	B1	2.1
	Expected frequency = $\left[\dfrac{35+30+28+24+40+51+37}{7}\right]$	M1	3.4
	= 35	A1	1.1b
	Test statistic = $\dfrac{(35-35)^2}{35}+\dfrac{(30-35)^2}{35}+\dfrac{(28-35)^2}{35}+\ldots$	M1	1.1b
	= 13.714…	A1	1.1b
	$\nu = 7 - 1 = 6$	B1	1.1b
	$\chi^2_{(6,0.05)} = 12.592$	B1	1.1a
	In critical region, sufficient evidence to reject H_0, Significant evidence at 5% level of significance to reject Jeremy's belief.	A1	3.5a
			(8 marks)

Notes

1st B1 Both hypotheses correct (condone reference to discrete uniform distribution)
1st M1 Using uniform model to calculate expected frequencies
1st A1 35
2nd M1 Attempting to find $\sum\dfrac{(O_i-E_i)^2}{E_i}$ or $\sum\dfrac{O_i^2}{E_i}-N$ may be implied by awrt 13.7
2nd A1 awrt 13.7
2nd B1 Degrees of freedom = 6 may be implied by a correct CV
3rd B1 awrt 12.6
3rd A1 Evaluating the outcome of a model by drawing correct inference in context