FS1Ch5 XMQs and MS

(Total: 74 marks)

1. FS1_2019 Q3 . 6 marks - FS1ch5 Central limit theorem
2. FS1_2020 Q7 . 15 marks - FS1ch5 Central limit theorem
3. FS1_2021 Q3 . 4 marks - FS1ch5 Central limit theorem
4. FS1_2022 Q5 . 5 marks - FS1ch5 Central limit theorem
5. FS1_2022 Q7 . 11 marks - FS1ch5 Central limit theorem
6. FS1_2023 Q4 . 6 marks - FS1ch5 Central limit theorem
7. FS1_2024 Q4 . 12 marks - FS1ch5 Central limit theorem
8. FS1_Sample Q4 . 4 marks - FS1ch5 Central limit theorem
9. FS1_Specimen Q7 . 11 marks - FS1ch3 Geometric and negative binomial distributions

3. A biased spinner can land on the numbers 1, 2, 3, 4 or 5 with the following probabilities.

Number on spinner	1	2	3	4	5
Probability	0.3	0.1	0.2	0.1	0.3

The spinner will be spun 80 times and the mean of the numbers it lands on will be calculated. Find an estimate of the probability that this mean will be greater than 3.25

(6)

Qu	Scheme	Marks	AO
3.	{ Let X = the number when the spinner is spun} $\mu = \underline{\mathbf{3}}$	B1	1.1b
	$\left[\mathrm{E}(X^2) =\right] 0.3 + 4\times0.1 + 9\times0.2 + 16\times0.1 + 25\times0.3$ [= 11.6 or $\frac{58}{5}$]	M1	1.1b
	$\sigma^2 \left[= 11.6 - 3^2 =\right]$ $\underline{\mathbf{2.6}}$	A1	1.1b
	$\bar{X} \approx \mathrm{N}\left("3", \sqrt{\frac{"2.6"}{80}}^{\,2}\right)$	M1	2.1
		A1ft	1.1b
	$\mathrm{P}(\bar{X} > 3.25) = [\mathrm{P}(Z > 1.3867...) =] 0.0827589...$ (calc) awrt $\underline{\mathbf{0.0828}}$	A1	3.4
		(6 marks)	

	Notes
	B1 for stating or using mean = 3
	1st M1 for using the given model to attempt $\mathrm{E}(X^2)$ with at least 3 correct products seen
	1st A1 for Var(X) = 2.6 or $\sigma = \sqrt{2.6} = 1.6124...$ (awrt 1.61)
ALT	**Use of pgf** (B1 when mean = 3 seen) (M1 when correct $\mathrm{G}''(t)$ seen with attempt at $\mathrm{G}''(1)$)
	$\mathrm{G}(t) = 0.3t + 0.1t^2 + 0.2t^3 + 0.1t^4 + 0.3t^5$
	$\mathrm{G}'(t) = 0.3 + 0.2t + 0.6t^2 + 0.4t^3 + 1.5t^4$
	$\mathrm{G}''(t) = 0.2 + 1.2t + 1.2t^2 + 6t^3$ leading to $\mathrm{G}''(1) = 8.6$
	2nd M1 for use of CLT – must use $\bar{X}$ and normal <u>or</u> sight of $\mathrm{N}\left("3", \sqrt{\frac{"2.6"}{80}}^{\,2}\right)$ with any letter
	2nd A1ft for a correct mean and variance, ft their 3 and their 2.6
	This M1A1ft may be implied by sight of correct st. dev. used in a standardisation leading to P(Z > 1.39) Must see correct use of Z
	NB $\frac{2.6}{80} = 0.0325$ and $\sqrt{\frac{2.6}{80}} = 0.18027...$ so allow e.g. N(3, awrt $(0.180)^2$)
	3rd A1 for using the normal model to find probability awrt 0.0828
ALT	**Use of** $\sum X$ (If see clear attempt at P(ΣX > 260) condone P(ΣX > 260.5) then:
	2nd M1 for $\Sigma X \sim$ N(…) <u>or</u> any letter ~N("240", $\sqrt{"2.6"\times80}^{\,2}$)
	2nd A1ft for mean = "3" × 80 = 240 <u>and</u> variance = "2.6" × 80 = 208
	May see P(ΣX > 260.5) = 0.077597… but it will only score 2nd M1 2nd A1ft and **3rd A0**

7. A six-sided die has sides labelled 1, 2, 3, 4, 5 and 6

The random variable S represents the score when the die is rolled.

Alicia rolls the die 45 times and the mean score, $\overline{S}$, is calculated.

Assuming the die is fair and using a suitable approximation,

(a) find, to 3 significant figures, the value of k such that $P(\overline{S} < k) = 0.05$

(8)

(b) Explain the relevance of the Central Limit Theorem in part (a).

(2)

Alicia considers the following hypotheses:

H_0: The die is fair

H_1: The die is not fair

If $\overline{S} < 3.1$ or $\overline{S} > 3.9$, then H_0 will be rejected.

Given that the true distribution of S has mean 4 and variance 3

(c) find the power of this test.

(3)

(d) Describe what would happen to the power of this test if Alicia were to increase the number of rolls of the die.
Give a reason for your answer.

(2)

Qu.	Scheme	Marks	AOs
7(a)	Realising S has a discrete uniform distribution over {1,…6}	M1	3.3
	$E(S) = 1\times\frac{1}{6}+2\times\frac{1}{6}+3\times\frac{1}{6}+4\times\frac{1}{6}+5\times\frac{1}{6}+6\times\frac{1}{6}$	M1	1.1b
	$\text{Var}(S) = \frac{6^2-1}{12}$ or $1^2\times\frac{1}{6}+2^2\times\frac{1}{6}+3^2\times\frac{1}{6}+4^2\times\frac{1}{6}+5^2\times\frac{1}{6}+6^2\times\frac{1}{6}-3.5^2$	M1	1.1b
	$E(S) = 3.5$ and $\text{Var}(S) = \frac{35}{12}$	A1	1.1b
	$\bar{S} \sim N(3.5, ...)$	M1	3.1a
	$\text{Var}(\bar{S}) = \frac{\frac{35}{12}}{45} = \frac{7}{108}$, $\bar{S} \sim N(3.5, 0.0648...)$	A1	1.1b
	$P(\bar{S} < k) = 0.05 \to \frac{k-3.5}{\sqrt{\frac{7}{108}}} = -1.6449$	M1	3.4
	$k = 3.08122\ldots$ awrt **<u>3.08</u>**	A1	1.1b
		(8)	
(b)	CLT applies since the sample size is large	B1	3.5b
	CLT states that the sample mean/ $\bar{S}$ is (approximately) normally distributed	B1	3.5b
		(2)	
(c)	True $\bar{S} \sim N(4, \frac{3}{45})$	M1	3.3
	$P(\bar{S} < 3.1) + P(\bar{S} > 3.9)$ or $1 - P(3.1 < \bar{S} < 3.9)$	dM1	3.4
	Power = awrt **<u>0.651</u>**	A1	1.1b
		(3)	
(d)	E.g. The increase in sample size would decrease the variance of $\bar{S}$ [leading to an increase in $P(\bar{S} > 3.9)$ and the decrease in $P(\bar{S} < 3.1)$ would be negligible]	B1	2.4
	So the power would increase.	dB1	2.2a
		(2)	
			(15 marks)

Notes

(a)	**M1:** Setting up model for S **M1**: Attempt at expression for E(S) **M1:** Attempt at expression for Var(S) **A1:** Correct mean and variance for S (may be implied by a correct distribution for $\bar{S}$) **M1:** Use of CLT to find distribution for $\bar{S} \sim$ N('3.5', …) f.t. their 3.5 but variance $\neq \frac{35}{12}$ **A1:** Correct distribution with correct variance, allow $\sigma^2 =$ awrt 0.0648 or $\sigma =$ awrt 0.255 **M1:** Standardising using their model **and** equating to a z-value $1 < \lvert z \rvert < 2$ **A1:** awrt 3.08
(b)	**B1:** Correct explanation about appropriateness of the CLT given large sample size (allow > 30) **B1:** Requires both <u>sample</u> and <u>mean</u> or $\bar{S}$
(c)	**M1:** Writing or using $\bar{S} \sim N(4, \frac{3}{45})$ allow $\sigma^2 =$ awrt 0.0667 or $\sigma =$ awrt 0.258 **dM1:** (dep on 1st M1) correct probability statement for power **A1:** awrt 0.651
(d)	**B1:** Correct reasoning which refers to decrease in variance **dB1:** (dep on 1st B1) Correct deduction with no incorrect reasoning

3. A courier delivers parcels. The random variable X represents the number of parcels delivered successfully each day by the courier where $X \sim \text{B}(400, 0.64)$

A random sample $X_1, X_2, \ldots X_{100}$ is taken.

Estimate the probability that the mean number of parcels delivered each day by the courier is greater than 257

(4)

Question	Scheme	Marks	AOs
3	$\overline{X} \approx \mathrm{N}(256,...)$ oe	M1	3.1a
	$\overline{X} \approx \mathrm{N}(256, 0.9216)$	A1	1.1b
	$\mathrm{P}(\overline{X} > 257) = \mathrm{P}\left(Z > \dfrac{257-256}{\sqrt{"0.9216"}}\right)$ [= awrt 1.04]	dM1	3.4
	$p = 0.1492...$	A1	1.1b
		(4)	
			(4 marks)

Notes:

	M1:	For realising the need to use the CLT with correct mean
	A1:	For a correct normal stated
	dM1:	Dep on previous Method mark. Use of the normal model to find $\mathrm{P}(\overline{X} > 257)$ If final answer is incorrect then we need to see the standardisation using their σ.
	A1:	awrt 0.149 (0.14878… from calculator)
		NB Allow awrt 0.148 if a continuity correction is used.

5. A random sample of 150 observations is taken from a geometric distribution with parameter 0.3

Estimate the probability that the mean of the sample is less than 3.45

(5)

Question	Scheme	Marks	AOs
5	Geo (0.3) $\mu = \frac{1}{0.3}\left[\text{ or exact equivalent e.g. } \frac{10}{3}\right]$	B1	1.1b
	$\sigma^2 = \frac{1-0.3}{0.3^2}\left[\text{ or exact equivalent e.g. } \frac{70}{9}\right]$	B1	1.1b
	CLT $\Rightarrow \bar{X} \approx \mathrm{N}\left(\frac{10}{3}, \ldots\right)$ oe	M1	2.1
	$\Rightarrow \bar{X} \approx \mathrm{N}\left(\frac{10}{3}, \frac{7}{135}\right)$ and attempt (sight of) $\mathrm{P}\left(\bar{X} < 3.45\right)$	M1	3.4
	$= 0.69579\ldots$ awrt **<u>0.696</u>**	A1	1.1b
		(5)	
		Total 5	

1st B1	correct mean
2nd B1	correct Var may be implied by sight of $\frac{7}{135}$ in distribution of $\bar{X}$
1st M1	For use of CLT (must see $\bar{X}$ and Normal with mean correct ft) <u>or</u> sight of $\mathrm{N}\left(\frac{10}{3}, \frac{7}{135}\right)$ <u>or</u> $\mathrm{N}\left(\text{"}\frac{10}{3}\text{"}, \frac{\text{"70"}}{\text{"9"}\times 150}\right)$ with any letter Allow 3.33 or better for $\frac{10}{3}$ and 7.78 or better for $\frac{70}{9}$ May be implied by 2nd M1
2nd M1	Using the normal distribution to find $\mathrm{P}\left(\bar{X} < 3.45\right)$ ft their "$\frac{10}{3}$"and $\frac{\text{"}\frac{70}{9}\text{"}}{150}$ May be implied by correct answer.
A1	awrt 0.696 Correct answer with no incorrect working scores 5/5
	Alternative (Use of $Y = \sum X$)
	$\mu = \frac{150}{0.3}[=500]$ — B1
	$\sigma^2 = \frac{150\times 0.7}{0.3^2}\left[\frac{3500}{3}\right] = 1166.\dot{6}$ — B1
	$\Rightarrow Y \approx \mathrm{N}\left(500, \frac{3500}{3}\right)$ — M1
	$\mathrm{P}\left(Y < 517.5\right)$ — M1
	$= 0.69579...$ — A1

7. A machine fills bags with flour. The weight of flour delivered by the machine into a bag, X grams, is normally distributed with mean μ grams and standard deviation 30 grams. To check if there is any change to the mean weight of flour delivered by the machine into each bag, Olaf takes a random sample of 10 bags. The weight of flour, x grams, in each bag is recorded and $\bar{x} = 1020$

(a) Test, at the 5% level of significance, $H_0 : \mu = 1000$ against $H_1 : \mu \neq 1000$

(4)

Olaf decides to alter the test so that the hypotheses are $H_0 : \mu = 1000$ and $H_1 : \mu > 1000$ but keeps the level of significance at 5%

He takes a second sample of size n and finds the critical region, $\bar{X} > c$

(b) Find an equation for c in terms of n

(2)

When the true value of μ is 1020 grams, the probability of making a Type II error is 0.0050, to 2 significant figures.

(c) Calculate the value of n and the value of c

(5)

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Question	Scheme	Marks	Aos
7(a)	$\overline{X} \sim \mathrm{N}(1000, 90)$ (May be implied by correct prob or z value seen)	M1	3.3
	$\mathrm{P}(\overline{X} > 1020) = 0.0175\ldots$ or $z = 2.108$	A1	3.4
	$0.0175\ldots < 0.025$ or $z = 2.108\ldots > 1.96$ therefore reject H_0.	M1	1.1b
	There is evidence that the mean weight of the flour in a bag is not 1000 g **or** evidence of a change in mean weight of flour in a bag	A1 cso	2.2b
		(4)	
(b)	$\left[\overline{Y} \sim \mathrm{N}\left(1000, \frac{900}{n}\right) \Rightarrow\right] \frac{c-1000}{30/\sqrt{n}} = 1.6449$	M1	3.4
	$c = 1000 + \frac{49.347}{\sqrt{n}}$	A1	1.1b
		(2)	
(c)	$\frac{\text{"}1000 + \frac{49.347}{\sqrt{n}}\text{"} - 1020}{30/\sqrt{n}} = -2.5758$	M1 A1ft	3.4 1.1b
	$\frac{126.621}{\sqrt{n}} = 20$ or $\frac{49.34...}{c-1000} = \frac{-77.274}{c-1020}$ (Allow 2sf accuracy)	dM1	1.1b
	$n =$ **40**	A1	2.1
	$c = 1007.8\ldots$ awrt **1010**	A1	1.1b
		(5)	
		(11 marks)	

Notes:

(a)	**1st M1**	Setting up the correct model. Normal with $\mu = 1000, \sigma^2 = 90$ or $\sigma = \sqrt{90}$ or awrt 9.49
	1st A1	Using the model to find the correct z value or $\mathrm{P}(\overline{X} > 1020) =$ awrt 0.0175 Allow CR $\overline{C} \ldots$ 1018.59.. awrt 1019 [> is OK] Ignore lower CR provided < 1000
	2nd M1	Correct comparison or non-contextual conclusion. Allow comparison of 1020 with critical region. **Dep on** $\mathrm{P}(\overline{X} > 1020)$ M0 if there are contradictory statements.
	2nd A1	cso **dep on M1A1M1** for a correct conclusion in context with underlined words Do NOT accept "mean weight has increased"
(b)	**M1**	For Finding the CR using the Normal distribution. Condone $\sigma = \sqrt{\frac{30}{n}}$ to score M1 $\frac{c-1000}{30/\sqrt{n}} = z$ where $\lvert z\rvert > 1.5$ **Allow any inequality or = for M1 in (b) and M1 A1ft M1 in (c)**
	A1	A correct equation in the form $c = \ldots$ and for use of awrt 1.6449 (implied by awrt 49.3[4]) Condone $\overline{X}$ used for c (o.e.)
(c)	**1st M1**	Standardising using their c (letter or expression) and equating to z ($\lvert z\rvert > 2$) to form an equation in n or n and c. Can ft their σ **used** in (b) for M1A1ft here
	1st A1ft	Ft their "c" for a correct equation with -2.58 (or 1.64 or 1.65 used in (b))
	2nd dM1	**Dependent 1st M1**. Isolating or eliminating either $\sqrt{n}$ or n or eliminating c leading to an equation for n or c
	2nd A1	For 40 (allow 41) Must be an integer. With correct working. — e.g. Check correct σ has been used
	3rd A1	For awrt 1010 from correct working

4. There are 32 students in a class.

Each student rolls a fair die repeatedly, stopping when their total number of sixes is 4
Each student records the total number of times they rolled the die.

Estimate the probability that the mean number of rolls for the class is less than 27.2

(6)

(ii)	2nd B1 for mention of Nima's test suggests Binomial distribution is suitable and that spins are independent (ignore reference to Marcus' test for 2nd B1)		
Qu 4	**Scheme**	**Mark**	**AO**
	[X = no. of rolls to 4 sixes] X~NegBin(4, $\frac{1}{6}$)	M1	3.3
	$\mu\left[=\frac{r}{p}\right]=$ **24** , $\sigma^2\left[=\frac{r(1-p)}{p^2}\right]=\frac{4\times\frac{5}{6}}{\frac{1}{36}}=$ **120**	A1, A1	1.1b(x2)
	$[\bar{X}\approx\sim]\text{N}\left(\text{"24"},\sqrt{\frac{\text{"120"}}{32}}^{\,2}\right)$	M1 M1	2.1,3.4
	P($\bar{X}$ < 27.2) = 0.95078… awrt **0.951**	A1	1.1b
		(6 marks)	
	Notes		
	1st M1 for selecting the correct negative binomial model. May be implied by correct mean or variance NegBin on its own is M0 1st A1 for mean = 24 2nd A1 for variance = 120 $\sigma = 120$ is A0 unless recovered 2nd M1 for writing or using of normal with mean 24 (may be implied by correct answer) ft their mean which may come from any distribution 3rd M1 for writing or using normal with standard deviation $=\sqrt{\frac{120}{32}}\left[=\sqrt{3.75}\right]$ ft $\frac{\text{their }\sigma}{\sqrt{32}}$ where σ may come from any distribution (may be implied by correct answer) 2nd A1 for awrt 0.951 (correct answer scores 6 out of 6)		

4. Every morning Geethaka repeatedly rolls a fair, six-sided die until he rolls a 3 and then he stops. The random variable X represents the number of times he rolls the die each morning.

(a) Suggest a suitable model for the random variable X

(1)

(b) Show that $P(X \leqslant 3) = \frac{91}{216}$

(2)

After 64 mornings Geethaka will calculate the mean number of times he rolled the die.

(c) Estimate the probability that the mean number of rolls is between 5.6 and 7.2

(5)

Nira wants to check Geethaka's die to decide whether or not the probability of rolling a 3 with his die is less than $\frac{1}{6}$

Nira rolls the die repeatedly until she rolls a 3
She obtains $x = 16$

(d) By carrying out a suitable test, determine what Nira's conclusion should be. You should state your hypotheses clearly and use a 5% level of significance.

(4)

Qu	Scheme	Mark	AO
4 (a)	$X \sim \text{Geo}(\frac{1}{6})$ accept in words: geometric distribution with $p = \frac{1}{6}$	B1	3.3
		(1)	
(b)	$P(X \leqslant 3) = 1 - P(X > 3)$ or $P(X = 1) + P(X = 2) + P(X = 3)$	M1	1.1b
	$= 1 - \left(\frac{5}{6}\right)^3$ or $\frac{1}{6} + \frac{5}{6} \times \frac{1}{6} + \left(\frac{5}{6}\right)^2 \times \frac{1}{6} = \frac{91}{216}$ (*)	A1cso	1.1b
		(2)	
(c)	$E(X) = 6$ $\quad \text{Var}(X) = \dfrac{\frac{5}{6}}{\left(\frac{1}{6}\right)^2}$ $[=30]$	M1 A1	3.4 1.1b
	$\bar{X} \approx \sim N\left(6, \left(\sqrt{\frac{30}{64}}\right)^2\right)$	M1 A1	3.3 1.1b
	$P(5.6 < \bar{X} < 7.2) = 0.68064\ldots$	A1	1.1b
		(5)	
(d)	$H_0: p = \frac{1}{6}$ $\quad H_1: p < \frac{1}{6}$	B1	2.5
	Probability approach: $P(X \geqslant 16)$ | CR approach: $\left(\frac{5}{6}\right)^{c-1} < 0.05$ or $\left(\frac{5}{6}\right)^{d} < 0.05$ with use of logs	M1	3.4
	$= P(X > 15) = \left(\frac{5}{6}\right)^{15} = 0.0649\ldots$ | CR: $X \geqslant 18$ or $X > 17$	A1	1.1b
	(Not significant) insufficient evidence that probability is $< \frac{1}{6}$ or (Not significant) insufficient evidence that dice is biased	A1	2.2b
		(4)	
		(12 marks)	

	Notes
(a)	B1 for both "geometric" or "Geo" **and** correct parameter of $\frac{1}{6}$. Probability must be seen in (a)
(b)	M1 for a correct method, may be implied by a correct expression A1cso* for a correct solution leading to printed answer with no incorrect working seen
(c)	1st M1 for correct use of formula for $E(X)$ or $\text{Var}(X)$ 1st A1 for both correct must see value of 6 for $E(X)$ and at least correct formula used for $\text{Var}(X)$ the value of 30 may be implied by 2nd A1 2nd M1 for use of CLT to get a normal distribution with correct mean and variance ≠ their 30 Condone use of X (or their letter) instead of $\bar{X}$ or seeing $N(6, \sqrt{\frac{30}{64}})$ 2nd A1 for a correct normal distribution stated or used. May be implied by correct answer. 3rd A1 for awrt 0.681 but allow 0.68 or 0.680 if correct distribution is seen
(d)	B1 for both hypotheses correct in terms of p M1 for sight or use of $P(X \geqslant 16)$ (may be implied) **or** correct expression with logs (use of logs may be implied by 17.43…) 1st A1 for 0.065 or better but accept 0.06 if a correct expression is seen, **or** correct CR 2nd A1 for a correct conclusion mentioning probability or biased Condone 'evidence suggests probability is equal to $\frac{1}{6}$' **NB**: Must have correct probability or correct CR to gain final A1. **NB:** Send responses comparing 0.935 with 0.95 to review

4. A random sample of 100 observations is taken from a Poisson distribution with mean 2.3

Estimate the probability that the mean of the sample is greater than 2.5

(4)

Question	Scheme	Marks	AOs
Q4.	Po(2.3) $n = 100$ $\mu = 2.3$ $\sigma^2 = 2.3$		
	CLT $\Rightarrow \bar{X} \approx \mathrm{N}\left(2.3, \dfrac{2.3}{100}\right)$	M1 A1	3.1a 1.1b
	$\mathrm{P}(\bar{X} > 2.5) = \mathrm{P}\left(Z > \dfrac{2.5-2.3}{\sqrt{0.023}}\right)$	M1	3.4
	$= \mathrm{P}(Z > 1.318..)$		
	$= 0.09632\ldots$	A1	1.1b
		(4)	
		(4 marks)	

M1: For realising the need to use the CLT to set $\bar{X} \approx$ normal with correct mean
May be implied by using the correct normal distribution

A1: For fully correct normal stated or used

M1: Use of the normal model to find $\mathrm{P}(\bar{X} > 2.5)$. Can be awarded for $\dfrac{2.5-2.3}{\sqrt{0.023}}$ **or** awrt 1.32

A1: awrt 0.0963

7. A radio station is running a contest each day for 30 days.

Each day it awards a prize to each of the first 12 callers who phone in and answer a question correctly. Once 12 prizes are awarded, no more calls are taken that day.

It can be assumed that each caller has a $\frac{3}{4}$ chance of answering a question correctly, independently of all other callers.

(a) Find the probability that there are exactly 15 calls taken on the first day.
(2)

(b) Find the probability that there are more than 13 calls taken on the first day.
(3)

(c) Estimate the probability that the mean number of calls taken over the 30 days is more than 15.5
(6)

Question	Scheme	Marks	AOs
7(a)	$[X \sim \text{NB}(12, \tfrac{3}{4})]$ $\binom{14}{11} \times \left(\frac{3}{4}\right)^{12} \times \left(\frac{1}{4}\right)^{3}$	M1	3.3
	= awrt **<u>0.180</u>**	A1	1.1b
		(2)	
(b)	$\text{P}(X > 13) = 1 - [\text{P}(X = 12) + \text{P}(X = 13)]$	B1	3.1b
	$1 - \left(\left(\frac{3}{4}\right)^{12} + \binom{12}{11}\left(\frac{3}{4}\right)^{12} \times \left(\frac{1}{4}\right)\right)$	M1	1.1b
	= awrt **<u>0.873</u>**	A1	1.1b
		(3)	
(c)	$\text{E}(X) = \frac{12}{\frac{3}{4}} = 16$	M1	3.1b
	$\text{Var}(X) = \frac{12(\frac{1}{4})}{(\frac{3}{4})^2} = \frac{16}{3}$	A1	1.1b
	$\bar{X} \sim \text{N}\left(16, \frac{\frac{16}{3}}{30} (= 0.1\dot{7})\right)$	M1 A1ft	3.1b 1.1b
	$\text{P}(\bar{X} > 15.5) = \text{P}\left(Z > \frac{15.5 - 16}{\sqrt{0.1\dot{7}}}\right)$	M1	3.4
	$= \text{P}(Z > -1.1858\ldots)$		
	= awrt **<u>0.882/0.883</u>**	A1	1.1b
		(6)	
			(11 marks)

Notes

(a)	M1 Selecting correct model: negative binomial **or** $\text{B}(14, \tfrac{3}{4})$ with extra success A1 0.18 or awrt 0.180
(b)	B1 Realising that $\text{P}(X > 13) = 1 - [\text{P}(X = 12) + \text{P}(X = 13)]$ M1 Correct form using negative binomial A1 awrt 0.873
(c)	1st M1 Realising that both the mean and variance of NB are required 1st A1 Both mean and variance correct (may be implied by correct standardisation) 2nd M1 Using CLT to model $\bar{X} \sim \text{N}(\text{'}16\text{'}, \ldots)$ 2nd A1ft Fully correct (or correct ft) normal distribution model for $\bar{X}$ 3rd M1 Using the normal model to find $\text{P}(\bar{X} > 15.5)$. Can be awarded for correct (ft) standardisation 3rd A1 awrt 0.882 or 0.883