FS1Ch4 XMQs and MS

(Total: 98 marks)

1. FS1_2019 Q5 . 12 marks - FS1ch8 Quality of tests
2. FS1_2020 Q1 . 13 marks - FS1ch2 Poisson distributions
3. FS1_2021 Q2 . 14 marks - FS1ch2 Poisson distributions
4. FS1_2021 Q5 . 18 marks - FS1ch4 Hypothesis testing
5. FS1_2022 Q3 . 14 marks - FS1ch2 Poisson distributions
6. FS1_2023 Q2 . 11 marks - FS1ch2 Poisson distributions
7. FS1_2024 Q2 . 7 marks - FS1ch2 Poisson distributions
8. FS1_Specimen Q5 . 9 marks - FS1ch4 Hypothesis testing

5. Information was collected about accidents on the *Seapron* bypass. It was found that the number of accidents per month could be modelled by a Poisson distribution with mean 2.5

Following some work on the bypass, the numbers of accidents during a series of 3-month periods were recorded. The data were used to test whether or not there was a change in the mean number of accidents per month.

(a) Stating your hypotheses clearly and using a 5% level of significance, find the critical region for this test. You should state the probability in each tail.

(5)

(b) State P(Type I error) using this test.

(1)

Data from the series of 3-month periods are recorded for 2 years.

(c) Find the probability that at least 2 of these 3-month periods give a significant result.

(3)

Given that the number of accidents per month on the bypass, after the work is completed, is actually 2.1 per month,

(d) find P(Type II error) for the test in part (a)

(3)

Qu	Scheme	Marks	AO
5(a)	$H_0 : \lambda = 2.5$ (or $\mu = 7.5$) $H_1 : \lambda \neq 2.5$ (or $\mu \neq 7.5$)	B1	2.5
	[X = no. of accidents in a 3-month period] $X \sim \text{Po}(7.5)$	M1	3.3
	$P(X \leqslant 2) = 0.0203$ (calc: 0.020256…) { or $P(X \leqslant 3) = 0.0591$}		
	$P(X \leqslant 13) = 0.9784$ so $P(X \geqslant 14) = 0.0216$ (calc: 0.0215646…)	M1	3.4
	{or $P(X \geqslant 15) = 0.0103$}		
	Giving Critical region of: $\boldsymbol{X \leqslant 2}$	A1	1.1b
	$\boldsymbol{X \geqslant 14}$	A1	1.1b
		(5)	
(b)	[0.0203 +0.0216] =awrt <u>**0.0419**</u> <u>or</u> (calc: 0.041821366… awrt <u>**0.0418**</u>)	B1ft	1.2
		(1)	
(c)	[Let M = no of 3-month periods with a significant result]		
	$M \sim B(8, \text{“}0.0419\text{”})$	M1	3.3
	$[P(M \geqslant 2)] = 1 - P(M \leqslant 1)$	M1	1.1b
	[= 1 – 0.9584…]		
	=0.04153…(calc: 0.041394…) <u>**[0.04139~ 0.04154]**</u>	A1cso	1.1b
		(3)	
(d)	$Y \sim \text{Po}(6.3)$	M1	3.3
	P(Type II error) = $P(3 \leqslant Y \leqslant 13)$ or $P(Y \leqslant 13) - P(Y \leqslant 2)$	M1	3.4
	[= 0.9945147… – 0.049846…]		
	= 0.9446… awrt <u>**0.945**</u>	A1	1.1b
		(3)	
		(12 marks)	

Notes

(a)
B1 for both hypotheses in terms of λ or μ (either way around)
1st M1 for selecting the correct Po model. Sight or use of Po(7.5) may be implied by 2nd M1
2nd M1 for using the correct model to find one of these probs with correct label (2sf or better)

1st A1 for one end correct | Allow any letter, even CR $\leqslant 2$ or set notation but **not** $P(X \leqslant 2)$
2nd A1 for a fully correct CR | Can have $X < 3$ and $X > 13$ etc

(b)
B1ft for awrt 0.0419 <u>or</u> awrt 0.0418
<u>or</u> ft addition of their **two** probs provided both are 0 < prob < 0.025 (awrt 3sf)

(c)
1st M1 for selecting a correct binomial model, ft their answer to part (b)
2nd M1 for a correct probability statement of $1 - P(M \leqslant 1)$ **dep on** <u>a</u> binomial selected
A1cso for answer in range [0.04139, 0.04154] **dep on** use of B(8, “0.0419”) or better

(d)
1st M1 for selecting a Po(6.3) model
2nd M1 for a correct probability statement using their Poisson model and their CR in (a) which may have just one tail.
A1 for awrt 0.945

1. The number of customers entering Jeff's supermarket each morning follows a Poisson distribution.

Past information shows that customers enter at an average rate of 2 every 5 minutes.

Using this information,

(a) (i) find the probability that exactly 26 customers enter Jeff's supermarket during a randomly selected 1-hour period one morning,

(2)

(ii) find the probability that at least 21 customers enter Jeff's supermarket during a randomly selected 1-hour period one morning.

(2)

A rival supermarket is opened nearby. Following its opening, the number of customers entering Jeff's supermarket over a randomly selected 40-minute period is found to be 10

(b) Test, at the 5% significance level, whether or not there is evidence of a decrease in the rate of customers entering Jeff's supermarket. State your hypotheses clearly.

(4)

A further randomly selected 20-minute period is observed and the hypothesis test is repeated. Given that the true rate of customers entering Jeff's supermarket is now 1 every 5 minutes,

(c) calculate the probability of a Type II error.

(5)

Question	Scheme	Marks	AOs
1(a)(i)	$X \sim \text{Po}(24)$	B1	3.4
	$\text{P}(X = 26) = 0.071912\ldots$ awrt **0.0719**	B1	1.1b
		(2)	
(ii)	$\text{P}(X \geq 21) = 1 - \text{P}(X \leq 20)$ [$= 1 - 0.24263\ldots$]	M1	3.4
	$= 0.75736\ldots$ awrt **0.757**	A1	1.1b
		(2)	
(b)	$\text{H}_0: \lambda = 2$ [$\mu = 16$] $\text{H}_1: \lambda < 2$ [$\mu < 16$]	B1	2.5
	$\text{P}(Y \leq 10 \mid Y \sim \text{Po}(16)) = 0.077396\ldots$ awrt **0.0774**	B1	1.1b
	Not significant / Do not reject H_0 / 10 is not in the CR	M1	1.1b
	There is not sufficient evidence to suggest a decrease/change in the rate of customers entering Jeff's supermarket.	A1	2.2b
		(4)	
(c)	Use of Po(8) to attempt critical region	M1	2.1
	Critical region is $Y \leq 3$/ H_0 is not rejected when $Y \geq 4$	A1	1.1b
	True distribution is $W \sim \text{Po}(4)$	B1	2.1
	$\text{P}(W \geq 4 \mid W \sim \text{Po}(4)) = 1 - \text{P}(W \leq 3)$ [$= 1 - 0.43347\ldots$]	M1	1.1b
	$=0.56652\ldots$ awrt **0.567**	A1	1.1b
		(5)	
			(13 marks)

Notes

(a)(i) **(ii)**	**B1:** For realising the distribution is Po(24) (May be seen or implied in part (ii)) **B1:** awrt 0.0719 **M1:** Writing or using $1 - \text{P}(X \leq 20)$ **A1:** awrt 0.757
(b)	**B1:** Both hypotheses correct (must use μ or λ) **B1:** awrt 0.0774 Allow awrt 0.08 from a correct probability statement. allow CR: $X \leq 9$ **M1:** Correct non-contextual conclusion (may be implied by correct contextual conclusion). Allow a f.t. comparison of 'their p' with 0.05 (Ignore any contradictory contextual comments for this mark) **A1:** A fully correct solution drawing a correct inference in context **with all previous marks in (b) scored.**
(c)	**M1:** Use of Po(8) to attempt critical region [$\text{P}(Y \leq 3)=0.0423..$ $\text{P}(Y \leq 4)=0.0996..$] **A1:** Finding critical region for the test $Y \leq 3$ which must come from Po(8). **B1:** Identifying the need to use Po(4) as the true distribution. Allow Po(4) seen or used for this mark. **M1:** Writing or using $\text{P}(W \geq \text{'4'})$ or $1 - \text{P}(W \leq \text{'3'})$ from Po(4). Allow f.t. on their **identified** CR but must be using Po(4) **A1:** awrt 0.567

2. On a weekday, a garage receives telephone calls randomly, at a mean rate of 1.25 per 10 minutes.

(a) Show that the probability that on a weekday at least 2 calls are received by the garage in a 30-minute period is 0.888 to 3 decimal places.

(2)

(b) Calculate the probability that at least 2 calls are received by the garage in fewer than 4 out of 6 randomly selected, non-overlapping 30-minute periods on a weekday.

(2)

The manager of the garage randomly selects 150 non-overlapping 30-minute periods on weekdays.
She records the number of calls received in each of these 30-minute periods.

(c) Using a Poisson approximation show that the probability of the manager finding at least 3 of these 30-minute periods when exactly 8 calls are received by the garage is 0.664 to 3 significant figures.

(4)

(d) Explain why the Poisson approximation may be reasonable in this case.

(1)

The manager of the garage decides to test whether the number of calls received on a Saturday is different from the number of calls received on a weekday. She selects a Saturday at random and records the number of telephone calls received by the garage in the first 4 hours.

(e) Write down the hypotheses for this test.

(1)

The manager found that there had been 40 telephone calls received by the garage in the first 4 hours.

(f) Carry out the test using a 5% level of significance.

(4)

Question	Scheme	Marks	AOs
2(a)	$C \sim$ Poisson (3.75)	M1	3.3
	$\mathrm{P}(C \geqslant 2) =$ 0.88829…*. awrt 0.8883*	A1*cso	1.1b
		(2)	
(b)	$D \sim$ B(6, "0.888")	M1	3.3
	$\mathrm{P}(D \leqslant 3) = 0.02163...$ awrt 0.0216 / 0.0215	A1	1.1b
		(2)	
(c)	$\mathrm{P}(C = 8) = 0.02281...$	B1	1.1b
	$E \sim$ B(150, "0.02281..") $\Rightarrow$ mean $= 150 \times$"0.02281..."[= 3.4215…]	M1	3.3
	$E \sim$ Po("3.4215…") $\Rightarrow$ $\mathrm{P}(E \geq 3) = [1 - \mathrm{P}(E \leqslant 2)]$	M1	3.4
	= 0.664 *	A1*cso	2.1
		(4)	
(d)	The number of periods is large and the probability of receiving 8 calls in 30-minutes is small.	B1 (1)	2.4
(e)	H_0: $\lambda = 30$ H_1: $\lambda \neq 30$	B1	2.5
		(1)	
(f)	$X \sim$ Po(30)	B1	3.3
	$\mathrm{P}(X \geqslant 40) = 1 - \mathrm{P}(X \leqslant 39)$	M1	1.1b
	= 0.04625…	A1	1.1b
	0.046… > 0.025 or no evidence to reject H_0 There is insufficient evidence at the 5% level of significance that the number of **calls** received is different on a Saturday	A1 (4)	2.2b
			(14 marks)

Notes:

(a)	**M1:**	For calculating the mean and setting up the correct model. Poisson may be implied by 0.8883 or better or 1 – awrt 0.1117 but must see 3.75 or 1.25×3
	A1*cso:	$\mathrm{P}(C \geqslant 2) =$ awrt 0.8883 or 1 – awrt 0.1117 = 0.888 Must see $\mathrm{P}(C \geqslant 2)$ oe
(b)	**M1:**	Setting up a new model using their answer to (a) Implied by correct answer
	A1:	awrt 0.0216 or awrt 0.0215
(c)	**B1:**	awrt 0.0228
	M1:	Setting up a new model B(150, "0.0228") and using *np* (working seen if incorrect)
	M1:	Using the model Po(their *np*) Must be clearly stated and $\mathrm{P}(E \geqslant 3)$ oe seen
	A1*cso:	Only award if the previous 3 marks have been awarded and 0.664 is stated. NB Use of B(150 0.02281) gives 0.668
(d)	**B1:**	Idea that n = 150 (number of periods selected) is large and p is 0.022… (exactly 8 calls in the time period) is small.
(e)	**B1:**	Both hypotheses correct using λ or μ allow 1.25 or 3.75
(f)	**B1:**	Realising Po(30) needs to be used. NB Implied by correct answer or $\mathrm{P}(X = 40) = 0.0139…$
	M1:	Writing or using $1 - \mathrm{P}(X \leqslant 39)$ or if CR method for $\mathrm{P}(X \geqslant 42) = 0.0221…$
	A1:	0.04… or awrt 0.05 or CR $X \geqslant 42$ oe must be CR and not probability
	A1:	A fully correct solution and correct inference in context. Calls required If put this prob but then give Cr X >= 40 M1A1A0

5. Asha, Davinda and Jerry each have a bag containing a large number of counters, some of which are white and the rest are red.
Each person draws counters from their bag one at a time, notes the colour of the counter and returns it to their bag.

The probability of Asha getting a red counter on any one draw is 0.07

(a) Find the probability that Asha will draw at least 3 white counters before a red counter is drawn.
(2)

(b) Find the probability that Asha gets a red counter for the second time on her 9th draw.
(2)

The probability of Davinda getting a red counter on any one draw is p.
Davinda draws counters until she gets n red counters. The random variable D is the number of counters Davinda draws.

Given that the mean and the standard deviation of D are 4400 and 660 respectively,

(c) find the value of p.
(4)

Jerry believes that his bag contains a smaller proportion of red counters than Asha's bag. To test his belief, Jerry draws counters from his bag until he gets a red counter. Jerry defines the random variable J to be the number of counters drawn up to and including the first red counter.

(d) Stating your hypotheses clearly and using a 10% level of significance, find the critical region for this test.
(5)

Jerry gets a red counter for the first time on his 34th draw.

(e) Giving a reason for your answer, state whether or not there is evidence that Jerry's bag contains a smaller proportion of red counters than Asha's bag.
(2)

Given that the probability of Jerry getting a red counter on any one draw is 0.011

(f) show that the power of the test is 0.702 to 3 significant figures.
(3)

Question	Scheme	Marks	AOs
5(a)	P(at least 3 whites) = $(1-0.07)^3$ **or** $1-0.07-0.93\times0.07-0.93^2\times0.07$	M1	1.1b
	= 0.8043… awrt 0.804	A1	1.1b
		(2)	
(b)	P(2nd red on 9th draw) = $\binom{8}{1}0.93^7\times0.07^2$	M1	3.3
	= 0.02358… awrt 0.0236	A1	1.1b
		(2)	
(c)	$\frac{n}{p}=4400$ and $\frac{n(1-p)}{p^2}=660^2$	M1 A1	3.1b 1.1b
	$1-p=99p$ oe	M1	1.1b
	$p=0.01$	A1	1.1b
		(4)	
(d)	H_0: p = 0.07 H_1: p < 0.07	B1	2.5
	J ~ Geo(0.07)	M1	3.3
	$P(J\geq c)<0.1\Rightarrow(1-0.07)^{c-1}<0.1$	M1	3.4
	$c-1>\frac{\log 0.1}{\log 0.93}$	M1	1.1b
	$c>32.72...$ $\therefore$ CR $J\geq33$	A1	1.1b
		(5)	
(e)	34 is in the Critical region	M1	1.1b
	There is evidence to suggest that Jerry's **bag** contains a smaller **proportion** of **red counters** than Asha's bag.	A1	2.2b
		(2)	
(f)	Power of test = $P(J\geq33 \mid p=0.011)$	M1	2.1
	= $(1-0.011)^{32}$ oe	M1	1.1b
	= 0.7019…*	A1*	1.1b
		(3)	
			(18 marks)

Notes:		
(a)	M1:	A correct method to find $P(X \geqslant 3)$
	A1:	awrt 0.804
(b)	M1:	For selecting the appropriate model negative binomial or binomial with an extra trial
	A1:	awrt 0.0236
(c)	M1:	Forming an equation for the mean and variance. At least one correct.
	A1:	Both equations correct
		Allow M1 A1 if both equations correct with the same number subst for n
	M1:	Solving the 2 equations leading to $1-p=99p$ oe Allow $p-p^2=99p^2$ ft their 4400 and 660 Allow $1-p=0.15p$
	A1:	0.01
(d)	M1:	Both hypotheses correct using correct notation allow eg $p > 0.93$
	M1:	Realising the need to use Geo(0.07) ft their Hypotheses
	M1:	Using the model to find $P(J \geqslant c)$ Condone $(1-0.07)^c < 0.1$ ft their 0.07 $\neq 0.93$ **ALT** $P(J \geqslant 32) = 0.1[054...\}]$ or $P(J \geqslant 33) = 0.09[8...]$ Implied by correct CR
	M1:	For a valid method to solve the inequality or $P(J \geqslant 32) = 0.1[054]$ and $P(J \geqslant 33) = 0.09[81]$ Implied by correct CR
	A1:	Correct CR(any letter) A0 if given as a probability statement. Must be integer
(e)	M1:	Comparing 34 with their CR eg 34 > 33 $34 \geqslant 33$ or $P(J \geqslant 34) = 0.09[12]$
	A1:	Fully correct conclusion in context. Allow Jerry's belief is true. Allow probability for proportion
(f)	M1:	Realising they need to find P(their CR in (d)) Allow 1– $P(J \leqslant 32)$
	M1:	For a Correct method. Allow 1 – 0.2981… May be implied by 0.7019… If the CR is incorrect $(1-0.011)^{\text{"CR"}-1}$ or 1 – $\{1-(1-0.011)^{\text{"CR"}-1}\}$ must be seen
	A1*:	Only award if both method marks awarded.

3. During the summer, mountain rescue team A receives calls for help randomly with a rate of 0.4 per day.

(a) Find the probability that during the summer, mountain rescue team A receives at least 19 calls for help in 28 randomly selected days.

(2)

The leader of mountain rescue team A randomly selects 250 summer days from the last few years.
She records the number of calls for help received on each of these days.

(b) Using a Poisson approximation, estimate the probability of the leader finding at least 20 of these days when more than 1 call for help was received by mountain rescue team A.

(4)

Mountain rescue team A believes that the number of calls for help per day is lower in the winter than in the summer. The number of calls for help received in 42 randomly selected winter days is 8

(c) Use a suitable test, at the 5% level of significance, to assess whether or not there is evidence that the number of calls for help per day is lower in the winter than in the summer. State your hypotheses clearly.

(4)

During the summer, mountain rescue team B receives calls for help randomly with a rate of 0.2 per day, independently of calls to mountain rescue team A.

The random variable C is the total number of calls for help received by mountain rescue teams A and B during a period of n days in the summer.
On a Monday in the summer, mountain rescue teams A and B each receive a call for help.

Given that over the next n days $\mathrm{P}(C = 0) < 0.001$

(d) calculate the minimum value of n

(3)

(e) Write down an assumption that needs to be made for the model to be appropriate.

(1)

Qu	Scheme	Marks	AOs
3(a)	$W \sim \text{Po}(11.2)$ and $\text{P}(W \geqslant 19) = 1 - \text{P}(W \leqslant 18)$ <u>or</u> suitable 3sf probs	M1	3.4
	$\text{P}(W \geqslant 19) = 0.020776...$ awrt **<u>0.021</u>**	A1	1.1b
		(2)	
(b)	[S = # calls per day, $S \sim \text{Po}(0.4)$] $\text{P}(S > 1) = 0.061551...$ awrt 0.0616	B1	1.1b
	$X \sim$ B(250, "0.061551..")	M1	3.3
	$Y \sim$ Po("15.3879…") [Accept Po(15.4) or better] <u>or</u> suitable 3sf probs	M1	3.4
	= 0.14751… awrt **<u>0.148</u>**	A1	1.1b
		(4)	
(c)	$\text{H}_0\text{: } \lambda = 16.8$ $\text{H}_1\text{: } \lambda < 16.8$	B1	2.5
	$U \sim$ Po(16.8)	B1	3.3
	$\text{P}(U \leqslant 8) = 0.014$	M1	1.1b
	[0.014 < 0.05 or there is sufficient evidence to reject H_0] There is sufficient evidence at the 5% level of significance that the <u>number of calls</u> received <u>per day</u> is <u>lower</u> in winter **or** <u>rate of calls</u> is <u>lower</u> in winter **or** <u>less calls</u> <u>per day</u> in <u>winter</u> (o.e.)	A1	2.2b
		(4)	
(d)	$C \sim \text{Po}(0.4 \times n + 0.2 \times n)$ [= Po(0.6n)] <u>or</u> $D \sim \text{B}(n,\ \text{e}^{-0.6}$ or awrt 0.549)	M1	3.1b
	$\text{e}^{-0.6n} < 0.001$ <u>or</u> $-0.6n < \ln(0.001)$ <u>or</u> $n > 11.5…$	M1	1.1b
	n = **<u>12</u>**	A1	1.1b
		(3)	
(e)	The <u>rate of calls</u> per day is <u>constant</u> **or** the <u>number of calls</u> occurring in non-overlapping time <u>interval</u>s is <u>independent</u>. **or** <u>number of calls per day is independent</u> (o.e.)	B1	2.4
		(1)	
	Total 14		

(a)	**M1**	For using the model Po(11.2) implied by sight of: 0.02077… or 0.9889.. or 0.9792..
	A1	awrt 0.021
(b)	**B1**	awrt 0.0616
	1st M1	Setting up a new model B(250, "0.0616") [condone B("0.0616", 250)]
	2nd M1	Seeing the model Po(their np) implied by sight of: 0.1475.. or 0.89975 or 0.8524…
	A1	awrt 0.148
SC		if **no approximation** used(and 1st M1 not seen) an answer of awrt 0.140 could get B1M1M0A0
(c)	**1st B1**	Both hypotheses correct using λ or μ and 16.8 or 0.4 [Accept their ans to 0.4×42]
	2nd B1	Realising Po(16.8) needs to be used. Sight or use of, implied by correct prob or CR
	M1	For 0.014 or better (0.0141..) or CR $X \leqslant 9$ oe must be CR and not probability. [Allow CR $X \leqslant 10$ with probability P($X \leqslant 10$) = 0.054 or better]
	A1	**Indep of 1st B1** (must see 2nd B1 and M1 scored) for a correct inference in context
(d)	**1st M1**	Selecting a suitable model. Sight of Po(0.6n) <u>or</u> B(n, $\text{e}^{-0.6}$) or implied by 2nd M1
	2nd M1	For a correct inequality or equality involving n [Condone slips in solving] Allow **MR** i.e. misread of 0.01 for 0.001 (or similar) to score M1M1A0
	A1	n = 12 cao [Correct answer with no incorrect working seen scores 3/3]
(e)	**B1**	Allow equivalent statements. Underlined words required.

2. Telephone calls arrive at a call centre randomly, at an average rate of 1.7 per minute. After the call centre was closed for a week, in a random sample of 10 minutes there were 25 calls to the call centre.

(a) Carry out a suitable test to determine whether or not there is evidence that the rate of calls arriving at the call centre has changed.
Use a 5% level of significance and state your hypotheses clearly.

(4)

Only 1.2% of the calls to the call centre last longer than 8 minutes.

One day Tiang has 70 calls.

(b) Find the probability that out of these 70 calls Tiang has more than 2 calls lasting longer than 8 minutes.

(3)

The call centre records show that 95% of days have at least one call lasting longer than 30 minutes.
On Wednesday 900 calls arrived at the call centre and none of them lasted longer than 30 minutes.

(c) Use a Poisson approximation to estimate the proportion of calls arriving at the call centre that last longer than 30 minutes.

(4)

Qu 2	Scheme	Marks	AO
(a)	$H_0 : \lambda = 1.7 \qquad H_1 : \lambda \neq 1.7$	B1	2.5
	[X = no. of calls in 10 mins] $X \sim \text{Po}(17)$	M1	3.3
	$[\text{P}(X \geqslant 25) = 1 - \text{P}(X \leqslant 24)]$ = 0.0406463… <u>or</u> CR: $X \geqslant 27$	A1	3.4
	[0.04..> 0.025/ 25 is not in CR so not significant] insufficient evidence of a change in <u>rate</u> of <u>calls</u>	A1	2.2b
		(4)	
(b)	[T = no. of calls longer than 8 minutes] $T \sim \text{B}(70, 0.012)$	M1	3.3
	[P(T > 2) =] P($T \geqslant 3$) = $1 - \text{P}(T \leqslant 2)$ = 1 – 0.947725…	M1	3.4
	= awrt <u>**0.0523**</u>	A1	1.1b
		(3)	
(c)	[C = no. of calls out of 900 longer than 30 mins] [$C \sim \text{B}(900, p)$] $C \approx\sim \text{Po}(900p)$	M1	3.3
	$\text{P}(C = 0) \approx e^{-900p} = 0.05$	M1	3.4
	$900p = -\ln(0.05)$ [= 2.9957…]	M1	1.1b
	p = 0.003328… awrt <u>**0.00333**</u>	A1	1.1b
		(4)	
		(11 marks)	

Notes

(a)	B1 for both hypotheses correct which must be attached to H_0 and H_1; must be in terms of λ or μ; allow either 1.7 or 17
	M1 for stating or using the correct Poisson model. may be implied by sight of awrt 0.0406/7 <u>or</u> awrt 0.959 <u>or</u> 0.9747… or better
	1st A1 for correct prob of awrt 0.04 or for correct CR found $X \geqslant 27$ ($X > 26$) (ignore lower tail CR if found) allow CV $X = 27$
	2nd A1 (dep on M1A1) for a correct conclusion in context mentioning "rate of calls" o.e. Allow e.g. 'The rate of calls is 1.7 per minute/17 per 10 minutes' Must be rate o.e. not "number" A0 if inconsistent comments are seen e.g. "reject H_0, no change in rate of calls"
(b)	1st M1 for sight or use of the correct binomial model. may be implied by sight of awrt: 0.0523 <u>or</u> 0.948 <u>or</u> 0.795 <u>or</u> 0.205
	2nd M1 for correct interpretation of more than 2 (allow 1 – 0.95 or better)
	A1 for awrt 0.0523 (correct answer only scores 3 out of 3)
SC:	Use of Po(70 ×0.012) leading to an answer of 0.0533(45…) and scores M1M1A0
(c)	1st M1 for sight or use of Po(900p) (as a suitable approx. to B(900, p)) (may be implied by correct answer awrt 0.00333)
	2nd M1 for a correct equation in p <u>or</u> correct use of P(C = 0) from Po e.g. $e^{-\lambda} = 0.05$
	3rd M1 for a correct method to solve for p (allow $p = \pm\ln(0.05)/900$) <u>or</u> to solve for λ, i.e. λ = awrt 3(.00)
	A1 for p = awrt 0.00333 **Must see Po used** condone $\frac{1}{300}$ o.e. Allow standard form (awrt 3.33×10^{-3}) or percentage (awrt 0.333%)
SC:	Use of Binomial gives 0.003323… awrt 0.00332 scores M0M0M0A1

2. The number of errors made by a secretary is modelled by a Poisson distribution with a mean of 2.4 per 100 words.

A 100-word piece of work completed by the secretary is selected at random.

(a) Find the probability that

(i) there are exactly 3 errors,

(ii) there are fewer than 2 errors.

(2)

After a long holiday, a randomly selected piece of work containing 250 words completed by the secretary is examined to see if the rate of errors has changed.

(b) Stating your hypotheses clearly, and using a 5% level of significance, find the critical region for a suitable test.

(4)

(c) Find P(Type I error) for the test in part (b)

(1)

Qu	Scheme	Mark	AO
2 (a)	**(i)** 0.20901… awrt **0.209**	B1	3.4
	(ii) 0.30844.. awrt **0.308**	B1	1.1b
		(2)	
(b)	H_0: $\lambda = 2.4$ (or $\mu = 6$) H_1: $\lambda \neq 2.4$ (or $\mu \neq 6$)	B1	2.5
	[E = no. of errors] E ~Po(6)	M1	3.3
	P($E \leqslant 1$) = 0.0174 or P($E \leqslant 2$) = 0.0620 **and** P($E \leqslant 11$) = 0.980 or P($E \geqslant 12$) = 0.0201	M1	3.4
	Critical region: $E \leqslant 1$ or $E \geqslant 12$	A1	1.1b
		(4)	
(c)	[P(Type I error) = 0.0174 + 0.0201 =] **0.0375** (Calc gives: 0.017351… + 0.0200919… = 0.037443…)	B1ft	1.2
		(1)	
		(7 marks)	

	Notes
(a)	1st B1 for awrt 0.209 2nd B1 for awrt 0.308
(b)	B1 for both hypotheses correct in terms of λ or μ (allow λ = 6 etc) 1st M1 for selecting the correct model. Sight or use of Po(6) 2nd M1 for use of the correct model with two probs correct to 2.s.f. (accept P($E \geqslant 12$) = 0.02) Must see attempt at lower **and** upper limit. Probabilities may be seen in (c). A1 for correct critical region (both parts). Allow $E \leqslant 1$ and $E \geqslant 12$ or $E \leqslant 1$, $E \geqslant 12$ etc Writing CR as probability statements is A0 **NB**: Completely correct CR implies M1M1A1 SC: 1-tailed test B0 as hypotheses are incorrect M1 for sight or use of Po(6) M1 (dep on H_1) for sight of P($E \leqslant 1$) = 0.0174 or P($E \geqslant 11$) = 0.0426, in line with their H_1 A1 for CR: $E \leqslant 1$ or CR: $E \geqslant 11$, in line with their hypotheses
(c)	B1ft for 0.0375 or 0.0374 or summing their two appropriate probs (ft their CR) **NB:** If candidate uses a 1-tailed test, this mark cannot be gained

5. A company claims that the proportion of visitors to its website who make a purchase is 0.03

Nina believes that the proportion of visitors to the website who make a purchase is less than 0.03 and asks the company for some data to test this.

The company tells Nina, for a particular day, the number of visits to its website until a purchase is made. Nina assumes that visits to the website are made independently and that each visitor has the same probability of making a purchase.

(a) Using a 5% level of significance, find the critical region for this test.
State your hypotheses clearly.

(5)

(b) Find the actual level of significance for this test.

(2)

The 94th visitor to the website was the first person to make a purchase.

(c) Test, at the 5% level of significance, whether or not there is evidence that the proportion of visitors who make a purchase is less than 0.03.

(2)

Question	Scheme	Marks	AOs
5(a)	$H_0 : p = 0.03$ $H_1 : p < 0.03$	B1	2.5
	$X \sim \text{Geo}(0.03)$	M1	3.3
	$P(X \geq c) < 0.05$ $(1 - 0.03)^{c-1} < 0.05$	M1	3.4
	$c - 1 > \dfrac{\log 0.05}{\log 0.97}$	M1	1.1b
	$c > 99.35\ldots$ critical region $X \geq 100$	A1	2.2a
		(5)	
(b)	$P(X \geq 100) = 0.97^{99}$	M1	3.4
	$=$ **<u>0.0490</u>**	A1	1.1b
		(2)	
(c)	Critical region $X \geq 100$ 94 is not in the critical region $[P(X \geq 94) = 0.0588\ldots > 0.05]$	M1	1.1b
	Do not reject H_0 There is insufficient evidence at the 5% level of significance that the proportion of visitors making a purchase is less than 0.03	A1	2.2b
		(2)	
			(9 marks)

Notes	
(a)	B1 Both hypotheses correct using correct notation 1st M1 Realising that the model Geo (0.03) is needed. May be implied by its use 2nd M1 Using the model to find an expression for $P(X \geq c)$ 3rd M1 Finding a valid method to solve the inequality A1 Correct critical region
(b)	M1 Using Geo(0.03) model with 100 A1 0.049 or awrt 0.0490
(c)	M1 Comparing 94 with their critical value A1 Fully correct solution and drawing a correct inference in context.