FS1Ch3 XMQs and MS

(Total: 72 marks)

1. FS1_2019 Q1 . 6 marks - FS1ch3 Geometric and negative binomial distributions
2. FS1_2019 Q7 . 12 marks - FS1ch3 Geometric and negative binomial distributions
3. FS1_2020 Q3 . 9 marks - FS1ch3 Geometric and negative binomial distributions
4. FS1_2022 Q4 . 13 marks - FS1ch3 Geometric and negative binomial distributions
5. FS1_2023 Q7 . 13 marks - FS1ch3 Geometric and negative binomial distributions
6. FS1_Sample Q5 . 8 marks - FS1ch3 Geometric and negative binomial distributions
7. FS1_Specimen Q7 . 11 marks - FS1ch3 Geometric and negative binomial distributions

Answer ALL questions. Write your answers in the spaces provided.

1. A chocolate manufacturer places special tokens in 2% of the bars it produces so that each bar contains at most one token. Anyone who collects 3 of these tokens can claim a prize.

Andreia buys a box of 40 bars of the chocolate.

(a) Find the probability that Andreia can claim a prize.

(2)

Barney intends to buy bars of the chocolate, one at a time, until he can claim a prize.

(b) Find the probability that Barney can claim a prize when he buys his 40th bar of chocolate.

(3)

(c) Find the expected number of bars that Barney must buy to claim a prize.

(1)

Qu	Scheme	Marks	AO
1(a)	[Let X = no. of prizes Andreia wins] $X \sim B(40, 0.02)$	M1	3.3
	[Require P(X …3) = 1 – P(X ,, 2)] = 0.04567… awrt **0.0457**	A1	1.1b
		(2)	
(b)	[Let Y = no. of the bar when Barney wins] Y ~NegBin(3, 0.02)	M1	3.3
	[P(Y = 40) =] $\binom{39}{2} \times 0.02^2 \times 0.98^{37} \times 0.02$	M1	3.4
	= 0.0028071… awrt **0.00281**	A1	1.1b
		(3)	
(c)	$\text{E}(Y) = \dfrac{3}{0.02}$ = **150**	B1	1.1b
		(1)	
		(6 marks)	

	Notes
(a)	M1 for selecting a suitable model i.e. B(40, p) where p is any probability Written or used, may be implied by a correct ans or 0.037429… from P(X = 3) A1 for awrt 0.0457 (correct answer only 2/2)
(b)	1st M1 for selecting a suitable model (NB(3, 0.02)) May be implied by a correct expression 2nd M1 for use of model to form a correct expression
SC	$\boldsymbol{p \neq 0.02}$ Allow prob of the form $\binom{39}{2} p^3 (1-p)^{37}$ where $0 < p < 1$ scores M0M1
	A1 for awrt 0.00281 (accept awrt 2.81×10^{-3}) [correct answer with no working scores 3/3]
(c)	B1 for 150

7. A spinner can land on red or blue. When the spinner is spun, there is a probability of $\frac{1}{3}$ that it lands on blue. The spinner is spun repeatedly.

The random variable B represents the number of the spin when the spinner first lands on blue.

(a) Find (i) $P(B = 4)$

(ii) $P(B \leqslant 5)$

(4)

(b) Find $E(B^2)$

(3)

Steve invites Tamara to play a game with this spinner.

Tamara must choose a colour, either red or blue.

Steve will spin the spinner repeatedly until the spinner first lands on the colour Tamara has chosen. The random variable X represents the number of the spin when this occurs.

If Tamara chooses red, her score is e^X

If Tamara chooses blue, her score is X^2

(c) State, giving your reasons and showing any calculations you have made, which colour you would recommend that Tamara chooses.

(5)

ALT	**Log series** 1st M1 attempt to write G(t) in suitable form as far as: $k[\ln 2 - \ln(2[1-\frac{t}{2}])]$
	1st A1 reaching $-k\ln(1-\frac{t}{2})$ 2nd M1 use of $-\ln(1-x)$ series (<u>some</u> correct substitution) NB $\text{G}(t) = \frac{1}{\ln 2}\left(\frac{t}{2}+\frac{t^2}{8}+\frac{t^3}{24}+\ldots\right)$

Qu	Scheme	Marks	AO
7(a)(i)	[B~Geo($\frac{1}{3}$)] P(B = 4) = $\left(\frac{2}{3}\right)^3 \times \frac{1}{3}$	M1	3.3
	= $\underline{\frac{8}{81}}$	A1	1.1b
(ii)	P($B \leqslant 5$) = 1 – P(B > 5) <u>or</u> $1-\left(\frac{2}{3}\right)^5$	M1	2.1
	= $\underline{\frac{211}{243}}$	A1	1.1b
		(4)	
(b)	$\text{E}(B^2) = \text{Var}(B) + [\text{E}(B)]^2$	M1	2.1
	From formula booklet: E(B) = $\frac{1}{\frac{1}{3}}$ = 3 and Var(B) = $\frac{1-\frac{1}{3}}{\left(\frac{1}{3}\right)^2}$ = 6	B1	1.1b
	So E(B^2) = 6 + 9 = **<u>15</u>**	A1	1.1b
		(3)	
(c)	[Let R = no. of the spin when it first lands on red] $X = R$~Geo($\frac{2}{3}$)	M1	3.3
	Require E(e^X) = $\sum_{x=1}^{\infty} e^x \left(\frac{1}{3}\right)^{x-1} \frac{2}{3}$	M1	3.1a
	= $\frac{2e}{3}\sum_{x=1}^{\infty}\left(\frac{e}{3}\right)^{x-1}$	M1	2.1
	= $\frac{2e}{3} \times \frac{1}{1-\frac{e}{3}}$ <u>or</u> $\frac{2e}{3-e}$	A1	1.1b
	E(e^X) = 19.297… {> 15 = E(B^2)} so Tamara should **choose red** since it has the greater expected score	A1	2.2a
		(5)	
		(12 marks)	

	Notes
(a)(i)	M1 for selecting the correct model i.e. Geo(p) (May be implied by a correct expression) A1 for $\frac{8}{81}$ (= 0.098765… accept awrt 0.0988)
(ii)	M1 for a suitable strategy to use the geometric model to find a correct expression A1 for $\frac{211}{243}$ (= 0.868312…accept awrt 0.868)
(b)	M1 for a suitable strategy to find E(B^2) [allow $\text{G}''(1)+\text{G}'(1)$] B1 for use of the correct formulae to find E(B) = 3 <u>and</u> Var(B) = 6 <u>or</u> $\text{G}''(1) = 12$ A1 for 15
SC	**Formula for E(B^2)** Allow M1B1A0 for $\text{E}(B^2) = \frac{2-p}{p^2}$ (o.e.)

Qu7	Notes
(c)	1st M1 for choosing a suitable geometric model (sight of Geo($\frac{2}{3}$) or at least 3 correct probabilities) 2nd M1 for realising the need for appropriate expected value and using E(g(X)) [Need sum and f(x)] NB simply finding $e^{E(X)} = e^{1.5}$ = awrt 4.48 is M0 and probably no more marks. 3rd M1 for a suitable strategy to turn the expression into a sum that can be found 1st A1 for correct use of sum to infinity of geometric series 2nd A1 for interpreting the outcome of the calculations in terms of a solution to the problem must choose red and see the awrt 19.3 (and allow ft of their E(B^2) < 19)

3. Suzanne and Jon are playing a game.

They put 4 red counters and 1 blue counter in a bag.

Suzanne reaches into the bag and selects one of the counters at random. If the counter she selects is blue, she wins the game. Otherwise she puts it back in the bag and Jon selects one at random. If the counter he selects is blue, he wins the game. Otherwise he puts it back in the bag and they repeat this process until one of them selects the blue counter.

(a) Find the probability that Suzanne selects the blue counter on her 4th selection.
(2)

(b) Find the probability that the blue counter is first selected on or after Jon's third selection.
(2)

(c) Find the mean and standard deviation of the number of selections made until the blue counter is selected.
(2)

(d) Find the probability that Suzanne wins the game.
(3)

Question	Scheme	Marks	AOs
3(a)	[$X \sim \text{Geo}(0.2)$ Suzanne's 4th selection is the 7th selection overall] $P(X = 7) = (0.8)^6(0.2)$ or $(0.64)^3(0.2)$	M1	3.3
	$= 0.05242\ldots$ awrt **0.0524**	A1	1.1b
		(2)	
(b)	$P(X \geq 6)$ [= $(1-0.2)^5$]	M1	1.1b
	= 0.32768 awrt **0.328**	A1	1.1b
		(2)	
(c)	Mean = 5	B1	1.1b
	Standard deviation $\left[= \sqrt{\frac{1-0.2}{0.2^2}}\right] = \sqrt{20}$ awrt **4.47**	B1	1.1b
		(2)	
(d)	P(Suzanne wins) = $0.2 + (0.8)^2(0.2) + (0.8)^4(0.2) + \ldots$	M1	3.1b
	Infinite geometric series = $\frac{0.2}{1-0.8^2}$ (oe)	M1	2.1
	$= \frac{5}{9}$	A1	1.1b
		(3)	
			(9 marks)

Notes

(a)	**M1:** Selecting geometric distribution with p = 0.2 and attempting required probability. Allow $(0.8)^n(0.2)$ to imply M1 with n = 6 or n = 3 **A1:** awrt 0.0524 Allow exact fraction $\frac{4096}{78125}$
(b)	**M1:** $P(X \geq 6)$ may be implied by $(1 - p)^5$ or $1 - (p + pq + pq^2 + pq^3 + pq^4)$ **A1:** awrt 0.328 Allow exact fraction $\frac{1024}{3125}$
(c)	**B1:** Mean = 5 **B1:** Standard deviation = $\sqrt{20}$ o.e. or awrt 4.47
(d)	**M1:** Determining the probability that Suzanne wins with at least three terms seen (may be implied by 2nd M1) **M1:** Recognising need to sum terms of an infinite geometric series with correct r = 0.8^2 (with numerator less than denominator) **A1:** $\frac{5}{9}$ (allow awrt 0.556)

4. In a game a spinner is spun repeatedly. When the spinner is spun, the probability of it landing on blue is 0.11

(a) Find the probability that the spinner lands on blue

(i) for the first time on the 6th spin,

(2)

(ii) for the first time before the 6th spin,

(2)

(iii) exactly 4 times during the first 6 spins,

(2)

(iv) for the 4th time on or before the 6th spin.

(4)

Zac and Izana play the game. They take turns to spin the spinner. The winner is the first one to have the spinner land on blue. Izana spins the spinner first.

(b) Show that the probability of Zac winning is 0.471 to 3 significant figures.

(3)

Question	Scheme	Marks	AOs
4(a) (i)	[$W \sim \text{Geo}(0.11)$] $\quad P(W=6)=(0.89)^5(0.11)$	M1	3.3
	$= 0.06142\ldots$ awrt **<u>0.0614</u>**	A1	1.1b
		(2)	
(ii)	$P(W \leqslant 5) = 1-(0.89)^5$	M1	3.1b
	$= 0.44159\ldots$ awrt **<u>0.442</u>**	A1	1.1b
		(2)	
(iii)	$X \sim B(6, 0.11)$	M1	3.3
	$P(X=4)=0.001739\ldots$ awrt **<u>0.00174</u>**	A1	1.1b
		(2)	
(iv)	[$Y \sim NB(4, 0.11)$] using a neg bin **or** $V \sim B(6, 0.11)$	M1	3.3
	$P(Y \leqslant 6)=P(Y=4)+P(Y=5)+P(Y=6)$ **and** $P(V \geqslant 4)$ for **M2**	M1	3.1b
	$= (0.11)^4+\binom{4}{3}(0.11)^3(0.89)^1\times 0.11+\binom{5}{3}(0.11)^3(0.89)^2\times 0.11$	M1	3.4
	$= 0.001827$ awrt **<u>0.00183</u>**	A1	1.1b
		(4)	
(b)	P(Zac wins) = $0.89\times 0.11+(0.89)^3\times 0.11+(0.89)^5\times 0.11+\ldots$	M1	3.1b
	$=\dfrac{0.89\times 0.11}{1-(0.89)^2}$ oe	M1	1.1b
	$= 0.47089\ldots = 0.471$*	A1cso*	2.1
		(3)	
			Total 13

(a)(i)	**M1**	Correct method to find $P(W=6)$ eg $(p)^5(1-p)$ for p = 0.11 or 0.89
	A1	awrt 0.0614 (Correct ans with no incorrect working 2/2)
(ii)	**M1**	Correct method to find $P(W \leqslant 5)$
	A1	awrt 0.442 (Correct ans with no incorrect working 2/2)
(iii)	**M1**	For using the model B(6, 0.11) allow B(6, 0.89) [Implied by 0.0017 or awrt 0.114]
	A1	awrt 0.00174 (Correct ans with no incorrect working 2/2)
(iv)		In part (iv) we can accept correct expressions or values for probabilities
	1st M1	For using a negative binomial model implied by correct $P(Y=5)$ or $P(Y=6)$
	2nd M1	Correct method to find $P(Y \leqslant 6)$
	3rd M1	At least two correct terms <u>or</u> 1 – 0.99817.. from $1-P(V \leqslant 3)$
	A1	awrt 0.00183
(b)	**1st M1**	Forming the correct probability of Zac winning <u>or</u> identify a and r of GP. Allow for $p=(0.11)\times 0+(1-0.11)(1-p)$
	2nd M1	Using sum to infinity of a GP. Allow for $p=\dfrac{0.89}{1+0.89}$
	A1*	Previous method marks must be seen leading to an answer 0.471 (NOT awrt 0.471)

a	4	5	6
$P(Y=a)$	1.46×10^{-4}	5.21×10^{-4}	1.16×10^{-3}
$P(V=a)$	1.74×10^{-3}	8.60×10^{-5}	1.77×10^{-6}

7. Each time a spinner is spun, the probability that it lands on red is 0.2

(a) Find the probability that the spinner lands on red

(i) for the 1st time on the 4th spin

(2)

(ii) for the 3rd time on the 8th spin

(2)

(iii) exactly 4 times during 10 spins

(2)

Each time the spinner is spun, the probability that it lands on yellow is 0.4

In a game with this spinner, a player must choose one of two events

R is the event that the spinner lands on **red** for the **1st** time in at most 4 spins

Y is the event that the spinner lands on **yellow** for the **3rd** time in at most 7 spins

(b) Showing your calculations clearly, determine which of these events has the greater probability.

(7)

ALT	M1 for identifying connection $Y = 3X + 4$ 1st A1 for a correct numerical probability expression for $P(X = 5)$		
Qu7	**Scheme**	**Marks**	**AO**
(a)(i)	X~Geo(0.2) or $P(X = 4) = 0.8^3 \times 0.2$ $= \underline{\mathbf{0.1024}}$	M1 A1 **(2)**	3.3 1.1b
(ii)	T~NegBin(3,0.2) or $P(T = 8) = \binom{7}{2} 0.2^2 \times 0.8^5 \times 0.2$ $= 0.05505\ldots$ awrt $\underline{\mathbf{0.0551}}$	M1 A1 **(2)**	3.3 1.1b
(iii)	F~B(10, 0.2) or $P(F = 4) = \binom{10}{4} 0.2^4 \times 0.8^6$ $P(F = 4) = 0.088080\ldots$ awrt $\underline{\mathbf{0.0881}}$	M1 A1 **(2)**	3.3 1.1b
(b)	$P(R) = P(X \leqslant 4)$ and X~Geo(0.2) | $P(X \geqslant 1)$, $X \sim B(4, 0.2)$ $= 1 - P(X > 4) = 1 - 0.8^4$ | $= 1 - P(Y = 0) = 1 - 0.8^4$ $= \underline{0.59(04)}$	M1 M1 A1	3.1b 3.4 1.1b
	$P(Y) = P(N \leqslant 7)$ and N~NegBin(3, 0.4)	M1	3.1b
	$0.4^3 + \binom{3}{2}0.4^3 0.6^1 + \binom{4}{2}0.4^3 0.6^2 + \binom{5}{2}0.4^3 0.6^3 + \binom{6}{2}0.4^3 0.6^4$ | $1 - \left(\binom{7}{2}0.4^2 0.6^5 + \binom{7}{1}0.4^1 0.6^6 + \binom{7}{0}0.6^7\right)$	M1	3.4
ALT	$P(Y) = P(W > 2)$ where W~B(7, 0.4) $= 1 - P(W \leqslant 2)$ [$= 1 - 0.419904$]	M1 M1	
	$= \underline{0.58(0096\ )}$ R (has the greater probability)	A1 A1 **(7)**	1.1b 3.2b
		(13 marks)	

	Notes
(a)(i)	M1 for selecting the correct model. Stated or used which may be implied by ans. A1 for 0.1024 or $\frac{64}{625}$ (accept 0.102) (correct answer scores 2 out of 2)
(ii)	M1 for selecting the correct model. Stated or used which may be implied by ans. Allow $0.2 \times P(V = 2)$ from V~B(7, 0.2) A1 for awrt 0.0551 (correct answer scores 2 out of 2)
(iii)	M1 for selecting the correct model. Stated or used may be implied by ans of 0.967(2) A1 for awrt 0.0881 (correct answer scores 2 out of 2)
(b)	1st M1 for a correct distribution and prob. expression for $P(R)$ (may be implied by 2nd M1) 2nd M1 for a correct numerical expression for $P(R)$ (allow any equivalent expression) 1st A1 for awrt 0.590 or $\frac{369}{625}$ (accept 0.59 or better) awrt 0.590 implies M1M1A1 3rd M1 for a correct distribution and prob. expression for $P(Y)$ (may be implied by 4th M1) 4th M1 for a correct numerical expression for $P(Y)$ (allow any equivalent expression) 2nd A1 for awrt 0.580 or (accept 0.58 or better) awrt 0.580 implies M1M1A1

	3rd A1 dep on all other marks for R or correct description in words Condone P(R) > P(Y)

5. The probability of Richard winning a prize in a game at the fair is 0.15

Richard plays a number of games.

(a) Find the probability of Richard winning his second prize on his 8th game,

(2)

(b) State two assumptions that have to be made, for the model used in part (a) to be valid.

(2)

Mary plays the same game, but has a different probability of winning a prize. She plays until she has won r prizes. The random variable G represents the total number of games Mary plays.

(c) Given that the mean and standard deviation of G are 18 and 6 respectively, determine whether Richard or Mary has the greater probability of winning a prize in a game.

(4)

Question	Scheme	Marks	AOs
Q5(a)	$\binom{7}{1}\times 0.15^2 \times (0.85)^6$	M1	3.3
	= 0.05940… = awrt **<u>0.0594</u>**	A1	1.1b
		(2)	
(b)	The model is only valid if:		
	the games (trials) are **independent**	B1	3.5b
	the probability of winning a prize, 0.15, is **constant** for each game	B1	3.5b
		(2)	
(c)	$18=\frac{r}{p}$ and $6^2=\frac{r(1-p)}{p^2}$	M1 A1	3.1b 1.1b
	Solving: $2p = 1 - p$	M1	1.1b
	$p=\frac{1}{3}$ (> 0.15) so Mary has the greater chance of winning a prize	A1	3.2a
		(4)	
	(8 marks)		

Notes:

5(a)

M1: For selecting an appropriate model negative binomial or B(7, 0.15) with an extra success in 8th trial e.g.

$\binom{7}{1}0.15\times(0.85)^6\times 0.15$ Allow $\binom{7}{1}0.85\times(0.15)^6\times 0.85$ may be implied by awrt 0.0594

A1: awrt 0.0594

(b)

B1: Stating the first assumption that games are independent

B1: Stating the second assumption that the probability remains constant

(c)

M1: Forming an equation for the mean or for the standard deviation

A1: Both equations correct

M1: Solving the 2 equations leading to $2p = 1 - p$

A1: For $p = \frac{1}{3}$ followed by a correct deduction

7. A radio station is running a contest each day for 30 days.

Each day it awards a prize to each of the first 12 callers who phone in and answer a question correctly. Once 12 prizes are awarded, no more calls are taken that day.

It can be assumed that each caller has a $\frac{3}{4}$ chance of answering a question correctly, independently of all other callers.

(a) Find the probability that there are exactly 15 calls taken on the first day.

(2)

(b) Find the probability that there are more than 13 calls taken on the first day.

(3)

(c) Estimate the probability that the mean number of calls taken over the 30 days is more than 15.5

(6)

Question	Scheme	Marks	AOs
7(a)	$[X \sim \text{NB}(12, \tfrac{3}{4})]$ $\binom{14}{11} \times \left(\frac{3}{4}\right)^{12} \times \left(\frac{1}{4}\right)^{3}$	M1	3.3
	= awrt **0.180**	A1	1.1b
		(2)	
(b)	$\text{P}(X > 13) = 1 - [\text{P}(X = 12) + \text{P}(X = 13)]$	B1	3.1b
	$1 - \left(\left(\frac{3}{4}\right)^{12} + \binom{12}{11}\left(\frac{3}{4}\right)^{12} \times \left(\frac{1}{4}\right)\right)$	M1	1.1b
	= awrt **0.873**	A1	1.1b
		(3)	
(c)	$\text{E}(X) = \frac{12}{\frac{3}{4}} = 16$	M1	3.1b
	$\text{Var}(X) = \frac{12(\frac{1}{4})}{(\frac{3}{4})^2} = \frac{16}{3}$	A1	1.1b
	$\bar{X} \sim \text{N}\left(16, \frac{\frac{16}{3}}{30} (= 0.1\dot{7})\right)$	M1 A1ft	3.1b 1.1b
	$\text{P}(\bar{X} > 15.5) = \text{P}\left(Z > \frac{15.5 - 16}{\sqrt{0.1\dot{7}}}\right)$	M1	3.4
	$= \text{P}(Z > -1.1858\ldots)$		
	= awrt **0.882/0.883**	A1	1.1b
		(6)	
			(11 marks)

Notes	
(a)	M1 Selecting correct model: negative binomial **or** $\text{B}(14, \tfrac{3}{4})$ with extra success A1 0.18 or awrt 0.180
(b)	B1 Realising that $\text{P}(X > 13) = 1 - [\text{P}(X = 12) + \text{P}(X = 13)]$ M1 Correct form using negative binomial A1 awrt 0.873
(c)	1st M1 Realising that both the mean and variance of NB are required 1st A1 Both mean and variance correct (may be implied by correct standardisation) 2nd M1 Using CLT to model $\bar{X} \sim \text{N}(\text{'16'}, \ldots)$ 2nd A1ft Fully correct (or correct ft) normal distribution model for $\bar{X}$ 3rd M1 Using the normal model to find $\text{P}(\bar{X} > 15.5)$. Can be awarded for correct (ft) standardisation 3rd A1 awrt 0.882 or 0.883