

FS1Ch1 XMQs and MS

(Total: 74 marks)

1. FS1_2019 Q6 . 12 marks - FS1ch1 Discrete random variables
2. FS1_2019 Q7 . 12 marks - FS1ch3 Geometric and negative binomial distributions
3. FS1_2020 Q4 . 8 marks - FS1ch1 Discrete random variables
4. FS1_2021 Q4 . 10 marks - FS1ch1 Discrete random variables
5. FS1_2022 Q2 . 9 marks - FS1ch1 Discrete random variables
6. FS1_2023 Q1 . 9 marks - FS1ch1 Discrete random variables
7. FS1_2024 Q1 . 6 marks - FS1ch1 Discrete random variables
8. FS1_Specimen Q2 . 8 marks - FS1ch1 Discrete random variables

Qu	Scheme	Marks	AO
6 (a)	$G(1) = 1 \Rightarrow k \ln 2 = 1$ so $k = \frac{1}{\ln 2}$	B1	2.1
(b)	$\left\{ G(t) = \frac{1}{\ln 2} [\ln 2 - \ln(2-t)] \right\} \Rightarrow G'(t) = \frac{1}{\ln 2} \left[\frac{1}{2-t} \right]$ or $\frac{1}{\ln 2} (2-t)^{-1}$	M1 A1	2.1 1.1b
	$[E(X) =] G'(1) = \frac{1}{\ln 2}$	A1	1.1b
	$G''(t) = \frac{1}{\ln 2} \times \left[\frac{1}{(2-t)^2} \right]$	M1 A1	2.1 1.1b
	$\text{Var}(X) = G''(1) + G'(1) - [G'(1)]^2 = \frac{1}{\ln 2} + \frac{1}{\ln 2} - \left(\frac{1}{\ln 2} \right)^2$	M1	2.1
	$= \frac{1}{\ln 2} \left(2 - \frac{1}{\ln 2} \right)$	A1	1.1b
(c)	$P(X = 3) = \text{coefficient of } t^3 \text{ by Maclaurin need } G'''(0)$	M1	3.1a
	$G'''(t) = \frac{1}{\ln 2} \frac{2}{(2-t)^3}$	A1ft	1.1b
	$P(X = 3) = \frac{G'''(0)}{3!}$	M1	3.2a
	$= \frac{\frac{1}{4 \ln 2}}{6} = \frac{1}{24 \ln 2} = 0.0601122... \text{ awrt } \underline{\underline{0.0601}}$	A1	1.1b
		(4)	
		(12 marks)	
Notes			
(a)	B1 for finding k (must be exact)		
(b)	1 st M1 for an attempt to differentiate $G(t)$ e.g. $A(2-t)^{-1}$ (o.e.)		
	1 st A1 for a correct first derivative (condone k or use of $\frac{1}{\ln 2} = \text{awrt } 1.44$)		
	2 nd A1 for correct $E(X)$ or $G'(1)$ (allow awrt 1.44 calc: 1.442695...but not k) seen anywhere		
	2 nd M1 for attempting second derivative (ft their $G'(t)$)		
	3 rd A1 for a correct 2 nd derivative (condone k or use of $\frac{1}{\ln 2} = \text{awrt } 1.44$)		
	3 rd M1 for a correct method for $\text{Var}(X)$ (some substitution into the correct formula)		
	4 th A1 for $\frac{1}{\ln 2} \left(2 - \frac{1}{\ln 2} \right)$ o.e. but must simplify i.e. collect like terms		
	[Mark final answer – penalise incorrect log work etc]		
	NB 0.8040211.. is A0 unless exact answer seen		
(c)	1 st M1 for a suitable strategy to solve the problem (finding link with Maclaurin) Need mention of coefficient of t^3 and $[G'''(t)$ or $G'''(0)]$ (condone $G'''(1)$)		
	1 st A1ft for 3 rd derivative, ft their 2 nd derivative in (b) (provided $G''(t)$ not const) Correct $G'''(t)$ or $G'''(0)$ scores 1 st M1 1 st A1ft		
	2 nd M1 for translating Maclaurin to probability (a correct expression)		
	2 nd A1 for $\frac{1}{24 \ln 2}$ or awrt 0.0601		

ALT	Log series 1 st M1 attempt to write $G(t)$ in suitable form as far as: $k[\ln 2 - \ln(2[1 - \frac{t}{2}])]$
	1 st A1 reaching $-k \ln(1 - \frac{t}{2})$ 2 nd M1 use of $-\ln(1 - x)$ series (<u>some</u> correct substitution) NB $G(t) = \frac{1}{\ln 2} \left(\frac{t}{2} + \frac{t^2}{8} + \frac{t^3}{24} + \dots \right)$

Qu	Scheme	Marks	AO
7(a)(i)	$[B \sim \text{Geo}(\frac{1}{3})] P(B = 4) = (\frac{2}{3})^3 \times \frac{1}{3}$	M1	3.3
	$= \frac{8}{81}$	A1	1.1b
	(ii) $P(B \leq 5) = 1 - P(B > 5)$ <u>or</u> $1 - (\frac{2}{3})^5$	M1	2.1
	$= \frac{211}{243}$	A1	1.1b
		(4)	
	(b) $E(B^2) = \text{Var}(B) + [E(B)]^2$	M1	2.1
	From formula booklet: $E(B) = \frac{1}{\frac{1}{3}} = 3$ and $\text{Var}(B) = \frac{1 - \frac{1}{3}}{(\frac{1}{3})^2} = 6$	B1	1.1b
	So $E(B^2) = 6 + 9 = \underline{15}$	A1	1.1b
		(3)	
	(c) [Let $R =$ no. of the spin when it first lands on red] $X = R \sim \text{Geo}(\frac{2}{3})$	M1	3.3
	Require $E(e^X) = \sum_{x=1}^{\infty} e^x \left(\frac{1}{3}\right)^{x-1} \frac{2}{3}$	M1	3.1a
	$= \frac{2e}{3} \sum_{x=1}^{\infty} \left(\frac{e}{3}\right)^{x-1}$	M1	2.1
$= \frac{2e}{3} \times \frac{1}{1 - \frac{e}{3}}$ <u>or</u> $\frac{2e}{3 - e}$	A1	1.1b	
$E(e^X) = 19.297\dots \{ > 15 = E(B^2) \}$ so Tamara should choose red since it has the greater expected score	A1	2.2a	
	(5)		
	(12 marks)		

Notes			
(a)(i)	M1 for selecting the correct model i.e. $\text{Geo}(p)$ (May be implied by a correct expression)		
	A1 for $\frac{8}{81}$ (= 0.098765... accept awrt 0.0988)		
(ii)	M1 for a suitable strategy to use the geometric model to find a correct expression		
	A1 for $\frac{211}{243}$ (= 0.868312... accept awrt 0.868)		
(b)	M1 for a suitable strategy to find $E(B^2)$ [allow $G''(1) + G'(1)$]		
	B1 for use of the correct formulae to find $E(B) = 3$ <u>and</u> $\text{Var}(B) = 6$ <u>or</u> $G''(1) = 12$		
	A1 for 15		
SC	Formula for $E(B^2)$ Allow M1B1A0 for $E(B^2) = \frac{2-p}{p^2}$ (o.e.)		

Qu7	Notes
(c)	<p>1st M1 for choosing a suitable geometric model (sight of $\text{Geo}(\frac{2}{3})$ or at least 3 correct probabilities)</p> <p>2nd M1 for realising the need for appropriate expected value and using $E(g(X))$ [Need sum and $f(x)$]</p> <p style="padding-left: 40px;">NB simply finding $e^{E(X)} = e^{1.5} = \text{awrt } 4.48$ is M0 and probably no more marks.</p> <p>3rd M1 for a suitable strategy to turn the expression into a sum that can be found</p> <p>1st A1 for correct use of sum to infinity of geometric series</p> <p>2nd A1 for interpreting the outcome of the calculations in terms of a solution to the problem must</p> <p style="padding-left: 40px;">choose red and see the awrt 19.3 (and allow ft of their $E(B^2) < 19$)</p>

Question	Scheme	Marks	AOs															
4(a)	$[E(X) =](-5) \times \frac{1}{12} + (-2) \times \frac{1}{6} + (3) \times \frac{1}{4} + (4) \times \frac{1}{2} [= 2]$	M1	1.1b															
	$[E(X^2) =](-5)^2 \times \frac{1}{12} + (-2)^2 \times \frac{1}{6} + (3)^2 \times \frac{1}{4} + (4)^2 \times \frac{1}{2} [= 13] \text{ (oe)}$	M1	1.1b															
	$\text{Var}(X) = E(X^2) - [E(X)]^2 = 13 - 2^2 = \underline{9}$	A1	1.1b															
		(3)																
(b)	<table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>x</td> <td>(-5)</td> <td>-2</td> <td>3</td> <td>(4)</td> </tr> <tr> <td>y</td> <td>(25)</td> <td>4</td> <td>7</td> <td>(10)</td> </tr> <tr> <td>p</td> <td>($\frac{1}{12}$)</td> <td>$\frac{1}{6}$</td> <td>$\frac{1}{4}$</td> <td>($\frac{1}{2}$)</td> </tr> </table>	x	(-5)	-2	3	(4)	y	(25)	4	7	(10)	p	($\frac{1}{12}$)	$\frac{1}{6}$	$\frac{1}{4}$	($\frac{1}{2}$)	M1	3.1a
	x	(-5)	-2	3	(4)													
	y	(25)	4	7	(10)													
	p	($\frac{1}{12}$)	$\frac{1}{6}$	$\frac{1}{4}$	($\frac{1}{2}$)													
$P(Y < 9) = P(X = -2) + P(X = 3) [= \frac{1}{6} + \frac{1}{4}]$	M1	1.1b																
	$= \underline{\frac{5}{12}}$	A1	1.1b															
		(3)																
(c)	$E(XY) = (-5)(25) \frac{1}{12} + (-2)(4) \times \frac{1}{6} + (3)(7) \times \frac{1}{4} + (4)(10) \times \frac{1}{2}$	M1	3.1a															
		$= \underline{13.5}$	A1	1.1b														
		(2)																
(8 marks)																		
Notes																		
(a)	M1: Attempt at $E(X)$ with at least 3 correct products seen M1: Attempt at $E(X^2)$ with at least 3 correct products seen A1: 9 cao																	
	Alternative M1: Attempt at $E(X)$ with at least 3 correct products seen M1: Attempt at expression for $E((X - \mu)^2) = (-5 - 2)^2 \times \frac{1}{12} + (-2 - 2)^2 \times \frac{1}{6} + (3 - 2)^2 \times \frac{1}{4} + (4 - 2)^2 \times \frac{1}{2}$ with at least 3 correct terms A1: 9 cao																	
(b)	M1: Finding distribution of Y M1: $P(X = -2) + P(X = 3)$ or $P(Y = 4) + P(Y = 7)$ A1: $\frac{5}{12}$ (condone awrt 0.417)																	
(c)	M1: Attempt at $E(XY)$ with at least 2 correct terms A1: 13.5																	

Question	Scheme		Marks	AOs														
4(a)	$4E(N) + 2 = 14.8$ or $E(N) = 3.2$		M1	3.1a														
	$0.2 + 0.1 + 0.75 + 4b + 5c = 3.2$		M1	1.1b														
	$\frac{c}{0.25 + b + c} = 0.5$ or $0.25 = c - b$		M1	3.1a														
	$b = 0.1$ and $c = 0.35$																	
	$E(N^2) = 1 \times 0.2 + 4 \times 0.05 + 9 \times 0.25 + 16 \times "0.1" + 25 \times "0.35" [= 13]$		M1	1.1b														
	$\text{Var}(N) = "13" - "3.2" ^2$		dM1	1.1b														
	$= 2.76$ *		A1*	2.1														
			(6)															
(b)	<table border="1"> <tr> <td>fee</td> <td>0</td> <td>50</td> <td>70</td> <td>90</td> <td>100</td> <td>100</td> </tr> <tr> <td>$P(N = n)$</td> <td>a</td> <td>0.2</td> <td>0.05</td> <td>0.25</td> <td>b</td> <td>c</td> </tr> </table>		fee	0	50	70	90	100	100	$P(N = n)$	a	0.2	0.05	0.25	b	c	M1	3.3
	fee	0	50	70	90	100	100											
	$P(N = n)$	a	0.2	0.05	0.25	b	c											
	$50 \times 0.2 + 70 \times 0.05 + 90 \times 0.25 + 100 \times "0.1" + 100 \times "0.35"$		M1	1.1b														
$= 81\text{p}$		A1	1.1b															
		(3)																
(c)	Poisson distribution will assign substantial probability to $N > 5$		B1	3.5b														
			(1)															
(10 marks)																		
Notes																		
(a)	M1:	For using the given information to find $E(N)$																
		ALT $a + b + c = 0.5$ oe																
	M1:	For use of $\sum nP(N = n) = "3.2"$ At least 3 terms correct																
		ALT $\sum (4n + 2)P(N = n) = 14.8 \Rightarrow 2a + 1.2 + 0.5 + 3.5 + 18b + 22c = 14.8$ At least 3 terms correct																
	M1:	Forming an equation in b and c using conditional probability																
	M1:	For using $\sum n^2P(N = n)$ Allow with the letters b and c																
	dM1:	Dependent on previous method mark. Correct method to find $\text{Var}(N)$																
	A1*:	All previous marks must be awarded and 2.76 stated																
(b)	M1:	Setting up a new model with the correct fees. At least 3 terms correct. Allow 0.5, 0.7, 0.9, 1																
	M1:	Correct method for calculating $E(\text{fee})$ Allow with the letters b and c																
	A1:	81[p] No units needed. Allow 0.81 if fees are in pounds																
(c)	B1:	A correct limitation.																

Qu 1	Scheme	Marks	AO
(a)	$[E(X) =] -2 \times 0.25 + -1 \times a + 0 \times b + 1 \times a + 3 \times 0.3$ $= \underline{0.4}$	M1 A1	1.1b 1.1b
(b)	$E(X^2) = (-2)^2 \times 0.25 + (-1)^2 \times a + 0 + 1^2 \times a + 3^2 \times 0.3 (= 2a + 3.7)$ $[Var(X) =] 3.9 = 2a + 3.7 - "0.4^2"$ $a = \underline{0.18}$ $[Use of sum of probs = 1 implies 2a + b = 0.45] \quad b = \underline{0.09}$	M1 dM1 A1 A1ft	2.1 1.1b 1.1b 1.1b
(c)	$X_1 + X_2 > 3$ when $X_1 = 3, X_2 = 1 \quad X_1 = 1, X_2 = 3 \quad X_1 = 3, X_2 = 3$ $[P (X_1 + X_2 > 3) =]$ $"0.18" \times 0.3 + 0.3 \times "0.18" + 0.3 \times 0.3 \quad \text{or} \quad 2 \times 0.3 \times (0.3 + "0.18") - 0.3^2$ $= \underline{0.198}$	M1 M1 A1	3.4 1.1b 1.1b
(9 marks)			
Notes			
(a)	M1 for a correct attempt (at least 3 correct non-zero terms or products and addition) division by k ($k \neq 1$) is M0 A1 for 0.4 o.e. (correct answer only scores 2 out of 2)		
(b)	1 st M1 for a correct attempt at $E(X^2)$ (at least 3 correct non-zero products and addition) Missing brackets around -2 and -1 is M0 unless recovered 2 nd dM1 (dep on 1 st M1) for use of $3.9 = \text{their } E(X^2) - [E(X)]^2$ ft their $E(X) = 0.4$ 1 st A1 for $a = 0.18$ o.e. 2 nd A1 (dep on 1 st M1 only) for $b = 0.09$ o.e. <u>or</u> their $b = 0.45 - 2 \times "a"$ (provided both a and b are probabilities)		
(c)	1 st M1 for identifying at least 2 cases e.g. $X_1 = 3, X_2 \geq 1$ counts as 2 cases (ignore extras including any incorrect pairs identified) implied by at least two correct products of probs. or correct ft products of probs. 2 nd M1 for a correct numerical expression for the probability ft their "0.18" A1 for 0.198 o.e.		

Qu	Scheme	Mark	AO
1 (a)	$E(X) = [-1 \times 0.2 + 0 + 1 \times 0.2] + 3 \times 0.25 + 5 \times 0.25 [= 2]$	M1	1.1b
	$E(X^2) = (-1)^2 \times 0.2 + 0 + 1 \times 0.2 + 3^2 \times 0.25 + 5^2 \times 0.25 [= 8.9]$	M1	1.1b
	$[\text{Var}(X) = 8.9 - 4 =]$ <u>4.9</u>	A1	1.1b
		(3)	
	(b) $E(X^4) = (-1)^4 \times 0.2 + 0 + 1 \times 0.2 + 3^4 \times 0.25 + 5^4 \times 0.25 [= 176.9]$	M1	2.1
	$\text{Var}(X^2) = "176.9" - "8.9" ^2$ $= 97.69$ awrt <u>97.7</u>	M1 A1	1.1b 1.1b
	(3)		
(6 marks)			
Notes			
(a)	1 st M1 for an attempt at $E(X)$, at least the final 2 products seen <u>or</u> an answer of 2 2 nd M1 for attempting a correct expression for $E(X^2)$, at least 3 non-zero products seen A1 for 4.9 or exact equivalent		
(b)	1 st M1 for attempting a correct expression, at least 3 non-zero products seen (implied by 176.9) 2 nd M1 for a correct method, ft their 176.9 and their 8.9 but must be intending $E(X^4)$ and $E(X^2)$ A1 for 97.69 or awrt 97.7		

Question	Scheme	Marks	AOs
2(a)	$\text{Var}(Y) = E(Y^2) - [E(Y)]^2$	M1	3.1a
	$E(Y) = \frac{1}{2}a + 2 \times \frac{3}{10} + 7 \times \frac{1}{5} [= \frac{1}{2}a + 2]$	B1	1.1b
	$E(Y^2) = \frac{1}{2}a^2 + 4 \times \frac{3}{10} + 49 \times \frac{1}{5} [= \frac{1}{2}a^2 + 11]$	B1	2.1
	$28 = \frac{1}{2}a^2 + 11 - (\frac{1}{2}a + 2)^2$	M1	1.1b
	$\frac{1}{4}a^2 - 2a - 21 = 0 \rightarrow a = \dots$	M1	1.1b
	$a = -6$ since $E(Y) < 0$	A1	2.2a
		(6)	
(b)	$\left(\frac{1}{3 - (-6)}\right) \times \frac{1}{2} + \left(\frac{1}{3 - 2}\right) \times \frac{3}{10} + \left(\frac{1}{3 - 7}\right) \times \frac{1}{5}$	M1	2.1
	$= \frac{11}{36}$	A1ft	1.1b
		(2)	
(8 marks)			
Notes			
(a)	1 st M1 Realising that $\text{Var}(Y) = E(Y^2) - [E(Y)]^2$ is required 1 st B1 Correct expression for $E(Y)$ 2 nd B1 Correct expression for $E(Y^2)$ 2 nd M1 Equating their expression for $\text{Var}(Y) = 28$ 3 rd M1 Solving the equation to find at least 1 value of a A1 -6 only		
(b)	M1 Correct expression for $E\left(\frac{1}{3 - Y}\right)$ or for finding all values of $\frac{1}{3 - Y}$ A1ft $\frac{11}{36}$ or awrt 0.306 ft on $a < -4$		