Fp1Ch1 XMQs and MS

(Total: 114 marks)

1. FP1_Sample Q6 . 11 marks - FP1ch1 Vectors
2. FP1_Specimen Q2 . 4 marks - FP1ch1 Vectors
3. FP1_Specimen Q5 . 4 marks - FP1ch1 Vectors
4. FP1_2019 Q7 . 10 marks - FP1ch1 Vectors
5. FP1_2020 Q3 . 9 marks - FP1ch1 Vectors
6. FP1_2021 Q4 . 7 marks - FP1ch1 Vectors
7. FP1_2021 Q7 . 7 marks - FP1ch1 Vectors
8. FP1_2022 Q3 . 9 marks - FP1ch1 Vectors
9. FP1_2022 Q6 . 6 marks - FP1ch1 Vectors
10. FP1(AS)_2018 Q4 . 9 marks - FP1ch1 Vectors
11. FP1(AS)_2019 Q4 . 8 marks - FP1ch1 Vectors
12. FP1(AS)_2020 Q5 . 10 marks - FP1ch1 Vectors
13. FP1(AS)_2021 Q4 . 9 marks - FP1ch1 Vectors
14. FP1(AS)_2022 Q5 . 11 marks - FP1ch1 Vectors

6. A tetrahedron has vertices $A(1, 2, 1)$, $B(0, 1, 0)$, $C(2, 1, 3)$ and $D(10, 5, 5)$.

Find

(a) a Cartesian equation of the plane ABC.

(3)

(b) the volume of the tetrahedron $ABCD$.

(3)

The plane Π has equation $2x - 3y + 3 = 0$

The point E lies on the line AC and the point F lies on the line AD.

Given that Π contains the point B, the point E and the point F,

(c) find the value of k such that $\overrightarrow{AE} = k\overrightarrow{AC}$.

(3)

Given that $\overrightarrow{AF} = \frac{1}{9}\overrightarrow{AD}$

(d) show that the volume of the tetrahedron $ABCD$ is 45 times the volume of the tetrahedron $ABEF$.

(2)

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Question	Scheme	Marks	AOs
6(a)	$\mathbf{AB}\times\mathbf{AC}=\begin{pmatrix}-1\\-1\\-1\end{pmatrix}\times\begin{pmatrix}1\\-1\\2\end{pmatrix}=\begin{pmatrix}-2-1\\-1+2\\1+1\end{pmatrix}=\begin{pmatrix}-3\\1\\2\end{pmatrix}$	M1	1.1b
	$\mathbf{r}.\begin{pmatrix}-3\\1\\2\end{pmatrix}=\begin{pmatrix}0\\1\\0\end{pmatrix}.\begin{pmatrix}-3\\1\\2\end{pmatrix}=1$	M1	1.1b
	Hence $-3x+y+2z=1$	A1	1.1b
		(3)	
(b)	Volume of Tetrahedron = $\frac{1}{6}\left\|\mathbf{n}.(\mathbf{AD})\right\|$	M1	3.1a
	$=\frac{1}{6}\left\|\begin{pmatrix}-3\\1\\2\end{pmatrix}.\left(\begin{pmatrix}10\\5\\5\end{pmatrix}-\begin{pmatrix}1\\2\\1\end{pmatrix}\right)\right\|$	M1	1.1b
	$=\frac{1}{6}\left\|(-27+3+8)\right\|=\frac{8}{3}$	A1	1.1b
		(3)	
(c)	$\mathbf{AE}=k\mathbf{AC}$ so E is $(1+k,\ 2-k,\ 1+2k)$	M1	3.1a
	E lies on plane so $2(1+k)-3(2-k)+3=0$, leading to $k=...$	M1	3.1a
	Hence $k=\frac{1}{5}$	A1	1.1b
		(3)	
(d)	Volume $ABEF$ = $\frac{1}{6}(\mathbf{AB}\times\mathbf{AE}).\mathbf{AF}=\frac{1}{6}\left(\mathbf{AB}\times\frac{1}{5}\mathbf{AC}\right).\frac{1}{9}\mathbf{AD}$	M1	3.1a
	$=\frac{1}{45}\left(\frac{1}{6}(\mathbf{AB}\times\mathbf{AC}).\mathbf{AD}\right)$ and hence result *	A1*	2.2a
		(2)	
	(11 marks)		

Question 6 notes:	
(a)	
M1:	Attempting a suitable cross product. Accept use of unit vectors
M1:	Complete method that would lead to finding the Cartesian equation of plane
A1:	Accept any equivalent form
(b)	
M1:	Identifies suitable vectors and attempts to substitute into a correct formula. Accept use of unit vectors
M1:	Correct form of scalar triple product using their **n** from part (a)
A1:	$\frac{8}{3}$ or exact equivalent form
(c)	
M1:	Uses that E is on AC in order to find an expression for E
M1:	Uses that E is in the plane Π to form and solve an expression in k
A1:	$\frac{1}{5}$ o.e. only
(d)	
M1:	Uses formula for volume of tetrahedron and substitutes for $\mathbf{AE}$ and $\mathbf{AF}$
A1*:	Deduces result: Use of $\frac{1}{6}(\mathbf{AB}\times\mathbf{AC}).\mathbf{AD}$ is required and no errors seen in solution

2. Relative to a fixed origin O, the points A, B and C have position vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ respectively, where

$$\mathbf{a} = \mathbf{i} + \mathbf{j} \qquad \mathbf{b} = 3\mathbf{i} - \mathbf{j} + \mathbf{k} \qquad \mathbf{c} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$$

Find

(a) the area of triangle OBC

(2)

(b) the volume of the tetrahedron $OABC$

(2)

Question	Scheme	Marks	AOs
2(a)	$\mathbf{b} \times \mathbf{c} = 0\mathbf{i} + 5\mathbf{j} + 5\mathbf{k}$ or $\begin{pmatrix} 0 \\ 5 \\ 5 \end{pmatrix}$	M1	1.1b
	area of triangle $OBC = \frac{1}{2}\lvert 0\mathbf{i} + 5\mathbf{j} + 5\mathbf{k}\rvert = \frac{1}{2}\sqrt{50} = \frac{5}{2}\sqrt{2}$ o.e.	A1	2.2a
		(2)	
(b)	$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 5 \\ 5 \end{pmatrix} = 0 + 5 + 0 = 5$	M1	1.1b
	volume of tetrahedron $OABC = \frac{1}{6} \times \lvert 5 \rvert = \frac{5}{6}$	A1	2.2a
		(2)	
			(4 marks)

Notes:

(a)

M1: Attempts the vector product $\mathbf{b} \times \mathbf{c}$, with at least two correct terms.

A1: Deduces area of triangle $OBC = \frac{1}{2}\lvert \mathbf{b} \times \mathbf{c}\rvert$

(b)

M1: Attempt at the triple scaler product $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$

A1: Deduces volume of tetrahedron $OABC = \frac{1}{6}\lvert \mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})\rvert$

5. (i) Given that $\mathbf{a}$ and $\mathbf{b}$ are non-zero vectors, find the value of

$$\mathbf{a}.(\mathbf{b} \times \mathbf{a})$$

giving a reason for your answer.

(2)

(ii) Given vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ such that $\mathbf{a} \times \mathbf{b} = \mathbf{a} \times \mathbf{c}$, where $\mathbf{a} \neq \mathbf{0}$ and $\mathbf{b} \neq \mathbf{c}$, show that

$$\mathbf{b} - \mathbf{c} = \lambda\mathbf{a}$$

where λ is a scalar, giving a reason for your answer.

(2)

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Question	Scheme	Marks	AOs
5(i)	$\mathbf{b} \times \mathbf{a}$ is perpendicular to $\mathbf{a}$ (and/or $\mathbf{b}$)	M1	2.4
	Therefore $\mathbf{a}.(\mathbf{b} \times \mathbf{a}) = 0$	A1	1.1b
		(2)	
(ii)	$\mathbf{a} \times \mathbf{b} = \mathbf{a} \times \mathbf{c} \Rightarrow \mathbf{a} \times (\mathbf{b} - \mathbf{c}) = \mathbf{0}$	M1	3.1a
	As $\mathbf{a} \neq \mathbf{0}$ and $\mathbf{b} \neq \mathbf{c}$ then $\mathbf{a}$ is parallel to $(\mathbf{b} - \mathbf{c})$ therefore $\mathbf{b} - \mathbf{c} = \lambda\mathbf{a}$	A1	2.4
		(2)	
			(4 marks)

Notes:

(i)
M1: Reasoning that $\mathbf{b} \times \mathbf{a}$ is perpendicular to $\mathbf{a}$
A1: $\mathbf{a}.(\mathbf{b} \times \mathbf{a}) = 0$

(ii)
M1: Collecting on to one side and factorising
A1: Reasoning: as $\mathbf{a} \neq \mathbf{0}$ and $\mathbf{b} \neq \mathbf{c}$ then $\mathbf{a}$ is parallel to $(\mathbf{b} - \mathbf{c})$ therefore $\mathbf{b} - \mathbf{c} = \lambda\mathbf{a}$

7. With respect to a fixed origin O, the points A, B and C have coordinates (3, 4, 5), (10, –1, 5) and (4, 7, –9) respectively.

The plane Π has equation $4x - 8y + z = 2$

The line segment AB meets the plane Π at the point P and the line segment BC meets the plane Π at the point Q.

(a) Show that, to 3 significant figures, the area of quadrilateral $APQC$ is 38.5

(6)

The point D has coordinates $(k, 4, -1)$, where k is a constant.

Given that the vectors $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{AD}$ form three edges of a parallelepiped of volume 226

(b) find the possible values of the constant k.

(4)

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Question	Scheme	Marks	AOs
7(a)	Examples: Area *APQC* = Area *ABC* – Area *PBQ* Area *APQC* = Area *APC* + Area *CPQ* Area *APQC* = Area *APQ* + Area *AQC* $\text{Area } APQC = \frac{1}{2}\lvert \mathbf{AQ}\times\mathbf{PC}\rvert$	M1	3.1a
	Line *AB*: $r=\begin{pmatrix}3\\4\\5\end{pmatrix}+\lambda\begin{pmatrix}10-3\\-1-4\\5-5\end{pmatrix}=\begin{pmatrix}3\\4\\5\end{pmatrix}+\lambda\begin{pmatrix}7\\-5\\0\end{pmatrix}$ **or** Line *BC*: $r=\begin{pmatrix}10\\-1\\5\end{pmatrix}+\mu\begin{pmatrix}10-4\\-1-7\\5+9\end{pmatrix}=\begin{pmatrix}10\\-1\\5\end{pmatrix}+\mu\begin{pmatrix}6\\-8\\14\end{pmatrix}$	M1	3.1a
	$4(3+7\lambda)-8(4-5\lambda)+5=2\Rightarrow\lambda=...\Rightarrow P$ is … **or** $4(10+6\mu)-8(-1-8\mu)+5+14\mu=2\Rightarrow\mu=...\Rightarrow Q$ is … $\left(\text{NB }\lambda=\frac{1}{4},\ \mu=-\frac{1}{2}\right)$	M1	2.1
	$P(4.75,\ 2.75,\ 5)$ **and** $Q(7,\ 3,\ -2)$	A1	1.1b
	$\text{Area } ABC=\frac{1}{2}\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\7&-5&0\\6&-8&14\end{vmatrix}=\frac{1}{2}\sqrt{70^2+98^2+26^2}$ $\text{Area } PBQ=\frac{1}{2}\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\5.25&-3.75&0\\3&-4&7\end{vmatrix}=\frac{1}{2}\sqrt{26.25^2+36.75^2+9.75^2}$ $\text{Area } APQC=\frac{1}{2}\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\-4&1&7\\0.75&-4.25&14\end{vmatrix}=\frac{1}{2}\sqrt{43.75^2+61.25^2+16.25^2}$ NB: Area *APQ* = 7.7004, Area *AQC* = 30.8018, Area *CPQ* = 23.101, Area *APC* = 15.4008	M1	2.1
	Area *ABC* – Area *PBQ* = 38.5*	A1*	1.1b
		(6)	
(b)	$\overrightarrow{AB}=\begin{pmatrix}7\\-5\\0\end{pmatrix},\ \overrightarrow{AC}=\begin{pmatrix}1\\3\\-14\end{pmatrix},\overrightarrow{AD}=\begin{pmatrix}k-3\\0\\-6\end{pmatrix}$	M1	3.1a

	$\overrightarrow{AB}\times\overrightarrow{AC}\bullet\overrightarrow{AD}=\begin{vmatrix}7 & -5 & 0\\ 1 & 3 & -14\\ k-3 & 0 & -6\end{vmatrix}=...$		
	$\overrightarrow{AB}\times\overrightarrow{AC}\bullet\overrightarrow{AD}=7\times-18+5(-6+14k-42)$	A1	1.1b
	$7\times-18+5(-6+14k-42)=\pm226\Rightarrow k=...$	**d**M1	3.1a
	$k=2$ or $\frac{296}{35}$	A1	1.1b
		(4)	
			(10 marks)

Notes

(a)

M1: Identifies a correct strategy to determine the area of the required quadrilateral. The attempt does **not need to be complete** for this mark so one of the statements (or intentions) in the markscheme would be sufficient.

M1: Correct attempt to find the equation of the line *AB* **or** the line *BC*

M1: Uses **at least one** of their lines and the equation of the given plane to determine the value of **at least one** of the parameters and hence the coordinates of *P* **or** *Q*

A1: **Both** coordinates correct – allow as vectors and may be implied if for example the candidate calculates the vectors e.g. AP, AQ, CP, CQ without stating the coordinates explicitly

M1: Uses all the required information to **calculate appropriate areas correctly** leading to the area of the quadrilateral. Needs to be a complete method here.

A1*: Reaches 38.5 with no errors

(b)

M1: Adopts a correct strategy by finding suitable vectors and forming the scalar triple product. This is often done in 2 steps e.g.

$$\overrightarrow{AB}\times\overrightarrow{AC}=\begin{pmatrix}70\\98\\26\end{pmatrix} \text{ or } \overrightarrow{AB}\times\overrightarrow{AD}=\begin{pmatrix}30\\42\\5k-15\end{pmatrix} \text{ or } \overrightarrow{AC}\times\overrightarrow{AD}=\begin{pmatrix}-18\\48-14k\\-3k+9\end{pmatrix}$$

$$\overrightarrow{AB}\times\overrightarrow{AC}.\overrightarrow{AD}=\begin{pmatrix}70\\98\\26\end{pmatrix}.\begin{pmatrix}k-3\\0\\-6\end{pmatrix}=70k-210-156$$

$$\text{or } \overrightarrow{AB}\times\overrightarrow{AD}.\overrightarrow{AC}=\begin{pmatrix}30\\42\\5k-15\end{pmatrix}.\begin{pmatrix}1\\3\\-14\end{pmatrix}=30+126-70k+210$$

$$\text{or } \overrightarrow{AC}\times\overrightarrow{AD}.\overrightarrow{AB}=\begin{pmatrix}-18\\48-14k\\-3k+9\end{pmatrix}.\begin{pmatrix}7\\-5\\0\end{pmatrix}=-126+70k-240$$

If it is not clear that the vector product is being used, at least 2 of the components should be correct.

A1: Correct expression for the triple product in terms of *k* (should be $\pm(70k-366)$)

Ignore the presence or absence of "1/6" for the first 2 marks

dM1: Realises that ±226 is possible for the value of the triple product and attempts to solve to obtain 2 values for *k*. **Dependent on the previous method mark.**

A1: Correct values (must be exact)

3. The points A, B and C, with position vectors $\mathbf{a} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$, $\mathbf{b} = \mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$ and $\mathbf{c} = -2\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$ respectively, lie on the plane Π

(a) Find $\overrightarrow{AB} \times \overrightarrow{AC}$

(3)

(b) Find an equation for Π in the form $\mathbf{r}.\mathbf{n} = p$

(2)

The point D has position vector $8\mathbf{i} + 7\mathbf{j} + 5\mathbf{k}$

(c) Determine the volume of the tetrahedron $ABCD$

(4)

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Question	Scheme	Marks	AOs
3(a)	$\overrightarrow{AB} = -2\mathbf{i} + 6\mathbf{j} + 4\mathbf{k}$ and $\overrightarrow{AC} = -5\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$	B1	1.1b
	$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} -2 & 6 & 4 \\ -5 & 5 & 2 \end{vmatrix}$ $= (6\times2-4\times5)\mathbf{i} - (-2\times2-4\times-5)\mathbf{j} + (-2\times5-6\times-5)\mathbf{k}$	M1	1.1b
	$= -8\mathbf{i} - 16\mathbf{j} + 20\mathbf{k}$	A1	1.1b
		(3)	
(b)	E.g. $\mathbf{n} = -8\mathbf{i} - 16\mathbf{j} + 20\mathbf{k}$ gives $p = (3\mathbf{i} - 2\mathbf{j} + \mathbf{k}).(-8\mathbf{i} - 16\mathbf{j} + 20\mathbf{k}) = ...$ E.g. $\mathbf{n} = 2\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}$ gives $p = (3\mathbf{i} - 2\mathbf{j} + \mathbf{k}).(2\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}) = ...$	M1	1.1b
	Equation is $\mathbf{r}.(-8\mathbf{i} - 16\mathbf{j} + 20\mathbf{k}) = 28$ or $\mathbf{r}.(2\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}) = -7$ (oe)	A1	2.5
		(2)	
(c)	$\overrightarrow{AD}.\left(\overrightarrow{AB} \times \overrightarrow{AC}\right) = "\overrightarrow{AD}".(-8\mathbf{i} - 16\mathbf{j} + 20\mathbf{k}) = ...$	M1	1.1b
	$\overrightarrow{AD} = 5\mathbf{i} + 9\mathbf{j} + 4\mathbf{k}$	B1	1.1b
	Volume = $\frac{1}{6}\left\|\overrightarrow{AD}.(\overrightarrow{AB} \times \overrightarrow{AC})\right\| = \frac{1}{6}\left\|(5\mathbf{i} + 9\mathbf{j} + 4\mathbf{k}).(-8\mathbf{i} - 16\mathbf{j} + 20\mathbf{k})\right\| = ...$	M1	3.1a
	$= \frac{52}{3}$ o.e. $17\frac{1}{3}$	A1	1.1b
		(4)	
			(9 marks)

Notes:

(a)

B1: Both $\overrightarrow{AB}$ and $\overrightarrow{AC}$ correct.

M1: Applies the cross product to their $\overrightarrow{AB}$ and their $\overrightarrow{AC}$. There must be at least two correct components if no method seen. Method can be implied by $\mathbf{i}\begin{vmatrix} 6 & 4 \\ 5 & 2 \end{vmatrix} - \mathbf{j}\begin{vmatrix} -2 & 4 \\ -5 & 2 \end{vmatrix} + \mathbf{k}\begin{vmatrix} -2 & 6 \\ -5 & 5 \end{vmatrix} = ...$ with at least one correct component.

A1: Correct vector

(b)

M1: Uses their $\mathbf{n}$ (which may be any multiple of their $\overrightarrow{AB} \times \overrightarrow{AC}$) and any point on the plane in an attempt to find p. (Use of $\overrightarrow{AB}$ or $\overrightarrow{AC}$ is M0.)

A1: Correct equation in form stated. Accept any multiples, e.g. $\mathbf{r}.(-8\mathbf{i} - 16\mathbf{j} + 20\mathbf{k}) = 28$

(c)

M1: Attempts a suitable scalar triple product, e.g. $\overrightarrow{AD}.\left(\overrightarrow{AB} \times \overrightarrow{AC}\right)$. Must include a complete method to use all necessary vectors.

B1: Correct '$\overrightarrow{AD}$' if using $\overrightarrow{AD}.\left(\overrightarrow{AB} \times \overrightarrow{AC}\right)$, or all vectors correct if using a different product.

M1: Use of volume = $\frac{1}{6}\left|\text{their } \overrightarrow{AD}.\left(\overrightarrow{AB}\times\overrightarrow{AC}\right)\right|$ (oe full method to find the volume).

A1: Correct exact answer.

4.

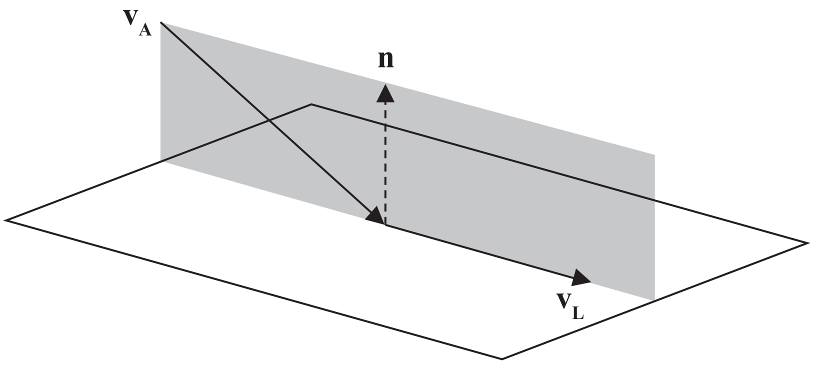

Figure 2

A small aircraft is landing in a field.

In a model for the landing the aircraft travels in different straight lines before and after it lands, as shown in Figure 2.

The vector $\mathbf{v}_A$ is in the direction of travel of the aircraft as it approaches the field.

The vector $\mathbf{v}_L$ is in the direction of travel of the aircraft after it lands.

With respect to a fixed origin, the field is modelled as the plane with equation

$$x - 2y + 25z = 0$$

and

$$\mathbf{v}_A = \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix}$$

(a) Write down a vector **n** that is a normal vector to the field.

(1)

(b) Show that $\mathbf{n} \times \mathbf{v}_A = \lambda \begin{pmatrix} 13 \\ 19 \\ 1 \end{pmatrix}$, where λ is a constant to be determined.

(2)

When the aircraft lands it remains in contact with the field and travels in the direction $\mathbf{v}_L$

The vector $\mathbf{v}_L$ is in the same plane as both $\mathbf{v}_A$ and **n** as shown in Figure 2.

(c) Determine a vector which has the same direction as $\mathbf{v}_L$

(3)

(d) State a limitation of the model.

(1)

Question	Scheme	Marks	AOs
4(a)	$\mathbf{n}=\begin{pmatrix}1\\-2\\25\end{pmatrix}$ or any non-zero scalar multiple thereof	**B1**	1.2
		(1)	
(b)	$\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\1 & -2 & 25\\3 & -2 & -1\end{vmatrix}=\begin{pmatrix}(-2)(-1)-(-2)(25)\\-((1)(-1)-(3)(25))\\(1)(-2)-(3)(-2)\end{pmatrix}=...$	**M1**	1.1b
	$=\begin{pmatrix}52\\76\\4\end{pmatrix}=4\begin{pmatrix}13\\19\\1\end{pmatrix}$ (or correct multiple for their normal vector used)	**A1**	2.1
		(2)	
(c)	Landing direction is perpendicular to $\mathbf{n}\times\mathbf{v_A}$ and $\mathbf{n}$ so required direction is given by $\begin{pmatrix}13\\19\\1\end{pmatrix}\times\begin{pmatrix}1\\-2\\25\end{pmatrix}=...$ **Alternatively** recognises the recognises the landing direction is the line of intersection of the plane containing $\mathbf{n}$ and $\mathbf{v_A}$ and the plane representing the field. Finds the equation of the plane containing $\mathbf{n}$ and $\mathbf{v_A}$ $\begin{pmatrix}x\\y\\z\end{pmatrix}.\begin{pmatrix}13\\19\\1\end{pmatrix}=\begin{pmatrix}3\\2\\-1\end{pmatrix}.\begin{pmatrix}13\\19\\1\end{pmatrix}$ $x-2y+25z=0$ and $13x+19y+z=0$	**M1**	3.1b
	$=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\13 & 19 & 1\\1 & -2 & 25\end{vmatrix}=\begin{pmatrix}(19)(25)-(-2)(1)\\-((13)(25)-(1)(1))\\(13)(-2)-(1)(19)\end{pmatrix}=...$ **Alternative** Selects a value for either *x*, *y* or *z* and solves simultaneously e.g $z = -5$ leading to $x-2y=125$ and $13x+19y=5\Rightarrow x=...,\ y=..$	**M1**	3.4
	$=\begin{pmatrix}477\\-324\\-45\end{pmatrix}$ or any positive multiple thereof, e.g $\begin{pmatrix}53\\-36\\-5\end{pmatrix}$ or $\begin{pmatrix}1908\\-1296\\-180\end{pmatrix}$	**A1**	1.1b
		(3)	
(d)	Any acceptable reason e.g - Paths would not be linear - May have some lateral movement	**B1**	3.5b

	- Could be affected by cross winds - Field might not be flat		
		(1)	
			(7 marks)

Notes:

(a)
B1: Correct normal vector (any non-zero scalar multiple thereof is fine)

(b)
M1: Uses their **n** and $\mathbf{v}_A$ in cross product formula. Allow slips in coordinates as long as the intent is clear.
A1: Correct work leading to a multiple of the required vector – may be a different multiple to the one shown if their **n** was different.

(c)
M1: Uses a correct strategy to find the direction, ie realises $\mathbf{v}_A$ must be perpendicular to both the vector from (b) and **n.** Allow for vectors used either way round.
Other methods may be possible – e.g finds the plane containing **n** and $\mathbf{v}_A$ and solves with the plane representing the field.
M1: Uses their answer to (b) with the normal vector to find a vector in the direction required. Allow for vectors either way round.
Alternative find the line of intersection of the plane containing **n** and $\mathbf{v}_A$ and the field.
A1: A correct direction vector, as shown or any positive multiple. For this mark the direction should be correct – so order of vectors must have been correct, or adapted to correct direction if initially incorrect.

(d)
B1: Any correct limitation given. See scheme for examples.

7. With respect to a fixed origin O, the line l has equation

$$(\mathbf{r} - (12\mathbf{i} + 16\mathbf{j} - 8\mathbf{k})) \times (9\mathbf{i} + 6\mathbf{j} + 2\mathbf{k}) = \mathbf{0}$$

The point A lies on l such that the direction cosines of $\overrightarrow{OA}$ with respect to the **i**, **j** and **k** axes are $\frac{3}{7}$, β and γ.

Determine the coordinates of the point A.

(7)

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Question	Scheme	Marks	AOs
7 **Way 1**	Position of A is given by $\overrightarrow{OA}=\begin{pmatrix}12+9\lambda\\16+6\lambda\\-8+2\lambda\end{pmatrix}$	**B1**	3.1a
	So have $\dfrac{12+9\lambda}{\sqrt{(12+9\lambda)^2+(16+6\lambda)^2+(-8+2\lambda)^2}}=\dfrac{3}{7}$	**M1**	1.1b
	$\Rightarrow 49\left(3\left(4+3\lambda\right)\right)^2=9\left((12+9\lambda)^2+(16+6\lambda)^2+(-8+2\lambda)^2\right)$ $\Rightarrow 2880\lambda^2+7200\lambda+2880=0$ or $2\lambda^2+5\lambda+2=0$	**M1** **A1**	3.1a 2.1
	$\Rightarrow (2\lambda+1)(\lambda+2)=0\Rightarrow \lambda=...$	**M1**	1.1b
	Substitutes a value of λ to find a position for A e.g. $\overrightarrow{OA}=\begin{pmatrix}12+9\left(-\frac{1}{2}\right)\\16+6\left(-\frac{1}{2}\right)\\-8+2\left(-\frac{1}{2}\right)\end{pmatrix}=...$	**M1**	1.1b
	Coordinates of A are $\left(\dfrac{15}{2},13,-9\right)$ only	**A1**	2.3
		(7)	
7 **Way 2**	Direction of $\overrightarrow{OA}$ is given by $\mathbf{d}=\begin{pmatrix}\frac{3}{7}k\\\beta k\\\gamma k\end{pmatrix}$ or use of $\left(\dfrac{3}{7}\right)^2+\beta^2+\gamma^2=1$	**B1**	3.1a
	$\begin{pmatrix}\frac{3}{7}k-12\\\beta k-16\\\gamma k+8\end{pmatrix}\times\begin{pmatrix}9\\6\\2\end{pmatrix}=\mathbf{0}\Rightarrow\begin{cases}6\left(\frac{3}{7}k-12\right)-9\left(\beta k-16\right)=0\\2\left(\frac{3}{7}k-12\right)-9\left(\gamma k+8\right)=0\\2\left(\beta k-16\right)-6\left(\gamma k+8\right)=0\end{cases}$	**M1**	2.1
	$\Rightarrow \beta k=\dfrac{2}{7}k+8$ and $\gamma k=\dfrac{1}{3}\left(\dfrac{2}{7}k-32\right)$ $\Rightarrow \dfrac{9k^2}{49}+\left(\dfrac{2}{7}k+8\right)^2+\dfrac{1}{9}\left(\dfrac{2}{7}k-32\right)^2=k^2\Rightarrow 2k^2-7k-490=0$	**M1** **A1**	3.1a 1.1b
	$\Rightarrow (2k-35)(k+14)=0\Rightarrow k=...$	**M1**	1.1b

	$k>0$ as direction cosine for first ordinate is positive, so need $k=\frac{35}{2}$ hence $\overrightarrow{OA}=\begin{pmatrix}\frac{3}{7}\times\frac{35}{2}\\ \frac{2}{7}\times\frac{35}{2}+8\\ \frac{1}{3}\left(\frac{2}{7}\times\frac{35}{2}-32\right)\end{pmatrix}=\ldots$	**M1**	2.3
	Coordinates of *A* are $\left(\frac{15}{2},13,-9\right)$ only	**A1**	1.1b
		(7)	
			(7 marks)

Way 1

B1: Starts a correct procedure by parametrising the line correctly.

M1: Uses the direction cosine of $\frac{3}{7}$ to form an equation in λ

M1: Realises need to square, to form quadratic in λ and gathers terms.

A1: Correct quadratic – three terms only or rearranged to complete square and solve, but need not have common factors all cancelled.

M1: Solves their three term quadratic, any valid method.

M1: Substitutes aa value for λ into the equation of the line to find a position for *A*.

A1: Correct coordinates only

Way 2

B1: Starts correct procedure by using the direction cosines to parametrise $\overrightarrow{OA}$ or attempting to use the Pythagorean property of the direction cosines.

M1: Uses their $\overrightarrow{OA}$ as multiple of direction cosines in the line equation to produce simultaneous equations.

M1: Solves the system (no need to see check for consistency of third equation) to find βk and γk, or just β and γ in terms of *k* and proceeds to form a quadratic in *k* using the Pythagorean property of the direction cosines.

A1: A correct quadratic in *k* reduced to three terms etc.

M1: Solves their three term quadratic, any valid method.

M1: Substitutes aa value for λ into the equation of the line to find a position for *A*.

A1: Correct coordinates only

3. With respect to a fixed origin O, the points A and B have coordinates (2, 2, –1) and (4, $2p$, 1) respectively, where p is a constant.

For each of the following, determine the possible values of p for which,

(a) OB makes an angle of 45° with the positive x-axis
(3)

(b) $\overrightarrow{OA} \times \overrightarrow{OB}$ is parallel to $\begin{pmatrix} 4 \\ -p \\ 2 \end{pmatrix}$
(3)

(c) the area of triangle OAB is $3\sqrt{2}$
(3)

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Question	Scheme	Marks	AOs
3 (a)	$(\|OB\| =)\sqrt{4^2+(2p)^2+1^2}\left(=\sqrt{17+4p^2}\right)$	B1	1.1b
	$\cos 45 = \dfrac{4}{\sqrt{17+4p^2}} \Rightarrow p = \ldots$	M1	3.1a
	$p = \pm\dfrac{\sqrt{15}}{2}$	A1	1.1b
		(3)	
(b)	$\overrightarrow{OA}\times\overrightarrow{OB} = \begin{pmatrix}2+2p\\-6\\4p-8\end{pmatrix}$	B1	1.1b
	E.g. Sets $\begin{pmatrix}2+2p\\-6\\4p-8\end{pmatrix} = \begin{pmatrix}4\lambda\\-p\lambda\\2\lambda\end{pmatrix}$ and solves to find a value for *p*	M1	3.1a
	p = 3 only	A1	2.2a
		(3)	
(c)	$\dfrac{1}{2}\left\|\overrightarrow{OA}\times\overrightarrow{OB}\right\| = 3\sqrt{2} \Rightarrow (2+2p)^2+(-6)^2+(4p-8)^2 = \left(6\sqrt{2}\right)^2$	M1	3.1a
	Solves a 3TQ to find a value for *p* $20p^2-56p+32=0 \Rightarrow p = \ldots$	dM1	1.1b
	$p = 2,\ \dfrac{4}{5}$	A1	1.1b
		(3)	
			(9 marks)

Notes:

(a)

B1: Correct expression for the magnitude for $\overrightarrow{OB}$ (may be seen in formula)

M1: A complete method to find a value for *p*. E,g, Sets cos 45 = 4/their magnitude of $\overrightarrow{OB}$ and solves to find a value for *p*. Note this may arise from attempts using dot products, but the same equation is reached and a full method to find *p* is required.

A1: $p = \pm\frac{\sqrt{15}}{2}$

(b)

B1: Correct vector product, allow if seen anywhere in the question. Alternatively, if method 2 below is used, the cross product is not necessary, and this mark may be awarded for a correct equation in *p* from either dot product.

M1: A complete method to find a value for *p*. E.g.

- Set the vector product equal to a multiple of the parallel vector and solves to find a value for *p*,

- Attempts dot products of both $\overrightarrow{OA}$ **and** $\overrightarrow{OB}$ with $\begin{pmatrix} 4 \\ -p \\ 2 \end{pmatrix}$, solves and finds an answer from both,
- Finds cross product of $\overrightarrow{OA} \times \overrightarrow{OB}$ with $\begin{pmatrix} 4 \\ -p \\ 2 \end{pmatrix}$ and sets at least one coefficient to zero to find *p*.

A1cso: A complete argument leading to *p* = 3 only, which must be consistent with their work. Where a method leads to more than one value for *p* the candidate will need to check which values hold and give the answer *p* = 3 only. No method needs be seen for this but other values must be rejected.

(c)

M1: A complete method to set up a polynomial in *p*. E.g. sets half magnitude of their vector product $= 3\sqrt{2}$ and reaches a quadratic expression in *p*. An alternative approach is:

$$cos \quad \angle AOB = \frac{2 \times 4 + 2 \times 2p - 1 \times 1}{\sqrt{4+4+1}\sqrt{16+4p^2+1}} = \frac{7+4p}{3\sqrt{17+4p^2}}$$

$$\Rightarrow 3\sqrt{2} = \frac{1}{2} OA.OB \, sin \angle AOB = \frac{1}{2} 3\sqrt{17+4p^2} \sqrt{1 - \frac{(7+4p)^2}{9(17+4p^2)}}$$

$$\Rightarrow 72 = 9(17+4p^2) - (49+56p+16p^2)$$

dM1: Dependent on the previous method mark. Solve a 3TQ to find a value for *p*.

A1: $p = 2, \frac{4}{5}$ only. Note: allow this mark if their $\overrightarrow{OA} \times \overrightarrow{OB}$ was correct apart from the **j** component sign.

6. The points P, Q and R have position vectors $\begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix}$, $\begin{pmatrix} 3 \\ 1 \\ -5 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix}$ respectively.

(a) Determine a vector equation of the plane that passes through the points P, Q and R, giving your answer in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$, where λ and μ are scalar parameters.

(2)

(b) Determine the coordinates of the point of intersection of the plane with the x-axis.

(4)

Question	Scheme	Marks	AOs
6(a)	Finds any two vectors $\pm\overrightarrow{PQ},\ \pm\overrightarrow{PR}$ or $\pm\overrightarrow{QR}$ $$\pm\begin{pmatrix}2\\3\\-9\end{pmatrix} \text{ or } \pm\begin{pmatrix}1\\2\\-1\end{pmatrix} \text{ or } \pm\begin{pmatrix}-1\\-1\\8\end{pmatrix}$$	M1	1.1b
	A correct equation for the plane $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$ $$\mathbf{a} = \begin{pmatrix}1\\-2\\4\end{pmatrix} \text{ or } \begin{pmatrix}3\\1\\-5\end{pmatrix} \text{ or } \begin{pmatrix}2\\0\\3\end{pmatrix}$$ **b** and **c** are any two vectors from $\pm\begin{pmatrix}2\\3\\-9\end{pmatrix}$ or $\pm\begin{pmatrix}1\\2\\-1\end{pmatrix}$ or $\pm\begin{pmatrix}-1\\-1\\8\end{pmatrix}$	A1	1.1b
		(2)	
(b)	Forms two simultaneous equations by setting $y = 0$ and $z = 0$ e.g. $-2 + 3\lambda + 2\mu = 0$ $$4 - 9\lambda - \mu = 0$$	M1	3.1a
	Solves their simultaneous equations to find a value for μ and a value for λ e.g. $\left.\begin{matrix}-2 + 3\lambda + 2\mu = 0\\ 4 - 9\lambda - \mu = 0\end{matrix}\right\} \Rightarrow \lambda = 0.4,\ \mu = 0.4$	dM1	1.1b
	Uses their values of μ and λ to find the *x* coordinate $$x = 1 + 2\lambda + \mu = 1 + 2(0.4) + (0.4) = \ldots$$	ddM1	1.1b
	$(2.2,\ 0,\ 0)$	A1	1.1b
		(4)	
	Alternative $$\begin{vmatrix}2 & 3 & -9\\1 & 2 & -1\end{vmatrix} = (-3 + 18)\mathbf{i} - (-2 + 9)\mathbf{j} + (4 - 3)\mathbf{k}$$	M1	3.1a
	$\begin{pmatrix}15\\-7\\1\end{pmatrix}.\begin{pmatrix}1\\-2\\4\end{pmatrix} = 15 + 14 + 4 = 33$ leading to $15x - 7y + z = 33$	dM1	1.1b
	$15x - 7(0) + (0) = 33 \Rightarrow x = \ldots$	ddM1	1.1b
	$(2.2,\ 0,\ 0)$	A1	1.1b
		(4)	
			(6 marks)

Notes:

Accept alternative vector forms throughout.

(a)

M1: Finds any two vectors $\pm\overrightarrow{PQ}$, $\pm\overrightarrow{PR}$or $\pm\overrightarrow{QR}$ by subtracting relevant vectors. Two out of three values correct is sufficient to imply the correct method

A1: Any correct equation for the plane. Must start with **r** =…

(b)

M1: Uses their equation for the plane to form two simultaneous equations by setting $y = 0$ and $z = 0$

dM1: Dependent on the previous method mark. Solves their simultaneous equations from the *y* and *z* coordinates to find a value for $\mu$and a value for λ

ddM1: Depends on both method marks. Uses their value for $\mu$and their value for λ to find the *x* coordinate

A1: Correct coordinates. Accept as a column vector or listed separately (*y* = 0 and *z* = 0 may be implied). Accept equivalent fractions, e.g. $\left(\frac{11}{5}, 0,0\right)$ or $\left(\frac{33}{15}, 0,0\right)$

Alternative

M1: Finds the cross product of the vectors **b** and **c** for their plane. Allow one slip in expansion.

dM1: Finds the Cartesian equation of the plane

ddM1: Depends on both previous method marks. Sets$y = 0$and $z = 0$ to find the *x* coordinate.

A1: Correct coordinates. Accept as a column vector or listed separately (*y* = 0 and *z* = 0 may be implied). Accept equivalent fractions, e.g. $\left(\frac{11}{5}, 0,0\right)$ or $\left(\frac{33}{15}, 0,0\right)$

4. A scientist is investigating the properties of a crystal. The crystal is modelled as a tetrahedron whose vertices are $A(12, 4, -1)$, $B(10, 15, -3)$, $C(5, 8, 5)$ and $D(2, 2, -6)$, where the length of unit is the millimetre. The mass of the crystal is 0.5 grams.

(a) Show that, to one decimal place, the area of the triangular face ABC is $52.2\,\text{mm}^2$

(3)

(b) Find the density of the crystal, giving your answer in g cm^{-3}

(6)

Question	Scheme	Marks	AOs
4	$A(12, 4, -1),\ B(10, 15, -3),\ C(10, 15, -3),\ D(2, 2, -6)$		
(a)	$\overrightarrow{AB} = \begin{pmatrix} -2 \\ 11 \\ -2 \end{pmatrix},\ \overrightarrow{AC} = \begin{pmatrix} -7 \\ 4 \\ 6 \end{pmatrix},\ \left\{ \overrightarrow{BC} = \begin{pmatrix} -5 \\ -7 \\ 8 \end{pmatrix} \right\}$	M1	1.1b
	$\text{Area } = \frac{1}{2}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 11 & -2 \\ -7 & 4 & 6 \end{vmatrix} = \frac{1}{2}\left\| \begin{pmatrix} 74 \\ 26 \\ 69 \end{pmatrix} \right\| = \frac{1}{2}\sqrt{(74)^2 + (26)^2 + (69)^2}$	M1	1.1b
	$\{= 52.23265...\} = 52.2\ (\text{mm}^2)\ (1\text{ dp})\ *$	A1*	2.2a
		(3)	
(b)	Finds appropriate vectors to find the volume of *ABCD* and makes a complete attempt to find the volume of the tetrahedron	M1	3.1a
	e.g. $\left\| \begin{pmatrix} -10 \\ -2 \\ -5 \end{pmatrix} \bullet \begin{pmatrix} 74 \\ 26 \\ 69 \end{pmatrix} \right\| = ...$ or $\begin{vmatrix} -2 & 11 & -2 \\ -7 & 4 & 6 \\ -10 & -2 & -5 \end{vmatrix} = ...$	M1	1.1b
	$= \lvert -740 - 52 - 345 \rvert$ or $\lvert -2(-8) - 11(95) - 2(54) \rvert\ \ \{= 1137\}$	A1	1.1b
	$V = \frac{1137}{6}\ (\text{mm}^3)\ \left\{ \text{or } \frac{379}{2} \text{ or } 189.5 \right\}$	A1	1.1b
	$\text{Density } = \frac{0.5}{189.5} \times 1000\ \ (\text{g cm}^{-3})$	M1	2.1
	$\{= 2.638522427...\} =$ awrt $2.6\ (\text{g cm}^{-3})$	A1	1.1b
		(6)	
		(9 marks)	

Notes

(a)	
M1:	Uses a correct method to find any 2 edges of triangle *ABC*
M1:	Complete process of taking the vector product between 2 edges of triangle *ABC*, applying Pythagoras and multiplying the result by 0.5
A1*:	Deduces the correct area of $52.2\ (\text{mm}^2)$. Condone awrt 52.2
Note:	Condone $\frac{1}{2}\lvert 74\mathbf{i} - 26\mathbf{j} + 69\mathbf{k} \rvert = \frac{1}{2}\sqrt{(74)^2 + (-26)^2 + (69)^2} = 52.2$, o.e. for M1M1A1
Note:	As an alternative, $\frac{1}{2}\sqrt{129}\sqrt{101}\sin(66.2343...) = 52.2\ (1\text{ dp})$, where the angle has been found by applying the scalar product between $\overrightarrow{AB}$ and $\overrightarrow{AC}$
(b)	
M1:	See scheme
M1:	Uses appropriate vectors to in an attempt at the scalar triple product
A1:	Correct numerical expression for the scalar triple product (allow $\pm$)
A1:	Correct volume (in mm^3) (allow $\pm$)
M1:	A correct method for changing their units for their volume and for finding density
A1:	Obtains the correct density in g cm^{-3}. Allow awrt 2.6

Notes Continued	
(b)	
Note:	Using any of $\overrightarrow{OA}$, $\overrightarrow{OB}$, $\overrightarrow{OC}$ or $\overrightarrow{OD}$ in their scalar triple product is M0M0A0A0
Note:	Allow M1M1A0A0 for $V = \frac{1}{6}\left\lvert\begin{pmatrix}-10\\-2\\-5\end{pmatrix}\bullet\begin{pmatrix}74\\26\\69\end{pmatrix}\right\rvert = \frac{1}{6}\lvert -740\mathbf{i} - 52\mathbf{j} - 345\mathbf{k}\rvert = \frac{1}{6}\sqrt{(-740)^2+(-52)^2+(-345)^2}$ $= \frac{1}{6}(818.125296...) = 135.354216...$
Note:	Some vector product calculations for reference:
	$\lvert\overrightarrow{AD}.(\overrightarrow{AB}\times\overrightarrow{AC})\rvert = \begin{vmatrix}-10 & -2 & -5\\-2 & 11 & -2\\-7 & 4 & 6\end{vmatrix} = \left\lvert\begin{pmatrix}-10\\-2\\-5\end{pmatrix}\bullet\begin{pmatrix}74\\26\\69\end{pmatrix}\right\rvert = \lvert -740-52-345\rvert = 1137$
	$\lvert\overrightarrow{AB}.(\overrightarrow{AC}\times\overrightarrow{AD})\rvert = \begin{vmatrix}-2 & 11 & -2\\-7 & 4 & 6\\-10 & -2 & -5\end{vmatrix} = \left\lvert\begin{pmatrix}-2\\11\\-2\end{pmatrix}\bullet\begin{pmatrix}-8\\-95\\54\end{pmatrix}\right\rvert = \lvert 16-1045-108\rvert = 1137$
	$\lvert\overrightarrow{AC}.(\overrightarrow{AB}\times\overrightarrow{AD})\rvert = \begin{vmatrix}-7 & 4 & 6\\-2 & 11 & -2\\-10 & -2 & -5\end{vmatrix} = \left\lvert\begin{pmatrix}-7\\4\\6\end{pmatrix}\bullet\begin{pmatrix}-59\\10\\114\end{pmatrix}\right\rvert = \lvert 413+40+684\rvert = 1137$
Note:	Some candidates apply $\overrightarrow{AB}\times\overrightarrow{AC}$ incorrectly to give $\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\-2 & 11 & -2\\-7 & 4 & 6\end{vmatrix} = 74\mathbf{i} - 26\mathbf{j} + 69\mathbf{k}$ This leads to an incorrect $\lvert\overrightarrow{AD}.(\overrightarrow{AB}\times\overrightarrow{AC})\rvert = \left\lvert\begin{pmatrix}-10\\-2\\-5\end{pmatrix}\bullet\begin{pmatrix}74\\-26\\69\end{pmatrix}\right\rvert = \lvert -740+52-345\rvert = 1033$

4.

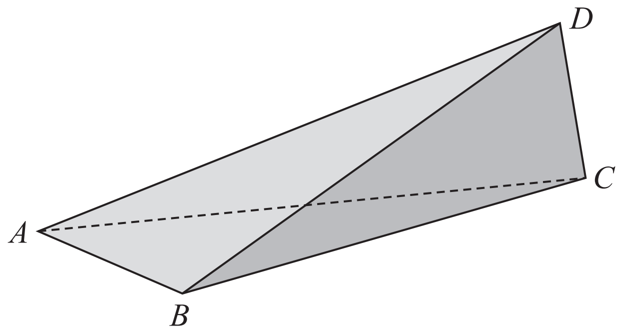

Figure 1

Figure 1 shows a sketch of a solid doorstop made of wood. The doorstop is modelled as a tetrahedron.

Relative to a fixed origin O, the vertices of the tetrahedron are A (2, 1, 4), B (6, 1, 2), C (4, 10, 3) and D (5, 8, d), where d is a positive constant and the units are in centimetres.

(a) Find the area of the triangle ABC.

(4)

Given that the volume of the doorstop is 21 cm^3

(b) find the value of the constant d.

(4)

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P 6 1 8 6 2 A 0 8 1 6

Question	Scheme	Marks	AOs
4	$A(2,1,4)$, $B(6,1,2)$, $C(4,10,3)$, $D(5,8,d)$		
(a) Way 1	Uses appropriate vectors in a correct method to make a complete attempt to find the area of triangle *ABC*.	M1	3.1b
	$\overrightarrow{AB} = \begin{pmatrix} 4 \\ 0 \\ -2 \end{pmatrix}$, $\overrightarrow{AC} = \begin{pmatrix} 2 \\ 9 \\ -1 \end{pmatrix}$, $\left\{\overrightarrow{BC} = \begin{pmatrix} -2 \\ 9 \\ 1 \end{pmatrix}\right\}$	M1	1.1b
	e.g. $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 0 & -2 \\ 2 & 9 & -1 \end{vmatrix} = \ldots$ or $\begin{pmatrix} 4 \\ 0 \\ -2 \end{pmatrix} \times \begin{pmatrix} 2 \\ 9 \\ -1 \end{pmatrix} = \ldots$	M1	1.1b
	$= 18\mathbf{i} + 0\mathbf{j} + 36\mathbf{k}$		
	Area $ABC = \frac{1}{2}\sqrt{(18)^2 + (0)^2 + (36)^2}$		
	$\{= 20.1246...\} = 9\sqrt{5}\ (\text{cm}^2)$ or awrt $20.1\ (\text{cm}^2)$	A1	2.2a
		(4)	
(a) Way 2	Uses appropriate vectors to find an angle or perpendicular height in triangle *ABC* and uses a correct method to make a complete attempt to find the area of triangle *ABC*.	M1	3.1b
	$\overrightarrow{AB} = \begin{pmatrix} 4 \\ 0 \\ -2 \end{pmatrix}$, $\overrightarrow{AC} = \begin{pmatrix} 2 \\ 9 \\ -1 \end{pmatrix}$, $\left\{\overrightarrow{BC} = \begin{pmatrix} -2 \\ 9 \\ 1 \end{pmatrix}\right\}$	M1	1.1b
	Uses a correct method to find an angle or perpendicular height in triangle *ABC*	M1	1.1b
	Note: $B\hat{A}C = 27.905...$, $A\hat{B}C = 76.047...$, $B\hat{C}A = 76.047...$ or perpendicular height $= 9$		
	Area $ABC = \frac{1}{2}\sqrt{86}\sqrt{20}\sin 76.047...$ or $\frac{1}{2}\sqrt{86}\sqrt{86}\sin 27.905...$ or $\frac{1}{2}\sqrt{20}(9)$		
	$\{= 20.1246...\} = 9\sqrt{5}\ (\text{cm}^2)$ or awrt $20.1\ (\text{cm}^2)$	A1	2.2a
		(4)	
(b)	Finds appropriate vectors to form the equation volume tetrahedron $ABCD = 21$ to give a linear equation in d **Note:** The volume must include $\frac{1}{6}$	M1	3.1a
	e.g. $\left\lvert \begin{pmatrix} 3 \\ 7 \\ d-4 \end{pmatrix} \bullet \begin{pmatrix} 18 \\ 0 \\ 36 \end{pmatrix} \right\rvert = \ldots$ or $\begin{vmatrix} 4 & 0 & -2 \\ 2 & 9 & -1 \\ 3 & 7 & d-4 \end{vmatrix} = \ldots$	M1	1.1b
	$= \lvert 54 + 36d - 144 \rvert$ or $\lvert 4(9d - 36 + 7) - 2(14 - 27) \rvert$ $\{= \lvert 36d - 90 \rvert\}$	A1	1.1b
	$\left\{\frac{1}{6}\lvert 36d - 90 \rvert = 21 \Rightarrow \lvert 36d - 90 \rvert = 126 \Rightarrow\right\}$ $d = 6$	A1	1.1b
		(4)	
			(8 marks)

Question	Scheme	Marks	AOs
4	$A(2, 1, 4)$, $B(6, 1, 2)$, $C(4, 10, 3)$, $D(5, 8, d)$		
(a) **Way 3**	Complete attempt to find the area of triangle ABC by applying $\frac{1}{2}\left\|\overrightarrow{OA}\times\overrightarrow{OB} + \overrightarrow{OB}\times\overrightarrow{OC} + \overrightarrow{OC}\times\overrightarrow{OA}\right\|$ or equivalent	M1	3.1b
	$\overrightarrow{OA}\times\overrightarrow{OB} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 1 & 4 \\ 6 & 1 & 2 \end{vmatrix} = ...$ **and** $\overrightarrow{OB}\times\overrightarrow{OC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & 1 & 2 \\ 4 & 10 & 3 \end{vmatrix} = ...$, **and** $\overrightarrow{OC}\times\overrightarrow{OA} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 10 & 3 \\ 2 & 1 & 4 \end{vmatrix} = ...$	M1	1.1b
	$\{\overrightarrow{OA}\times\overrightarrow{OB} + \overrightarrow{OB}\times\overrightarrow{OC} + \overrightarrow{OC}\times\overrightarrow{OA}\} = \begin{pmatrix} -2 \\ 20 \\ -4 \end{pmatrix} + \begin{pmatrix} -17 \\ -10 \\ 56 \end{pmatrix} + \begin{pmatrix} 37 \\ -10 \\ -16 \end{pmatrix}$	M1	1.1b
	Area $ABC = \frac{1}{2}\sqrt{(18)^2 + (0)^2 + (36)^2}$		
	$\{= 20.1246...\} = 9\sqrt{5}\ (\text{cm}^2)$ or awrt $20.1\ (\text{cm}^2)$	A1	2.2a
		(4)	

Notes for Question 4

(a)	**Way 1**
M1:	Complete correct process of taking the vector product between 2 edges of triangle ABC, applying Pythagoras and multiplying the result by 0.5
M1:	Uses a correct method to find any 2 edges of triangle ABC
M1:	Attempts to take the vector cross product between 2 edges of triangle ABC
A1:	Deduces the correct area of either $9\sqrt{5}\ (\text{cm}^2)$ or awrt $20.1\ (\text{cm}^2)$
(a)	**Way 2**
M1:	See scheme
M1:	Uses a correct method to find any 2 edges of triangle ABC
M1:	Either • finds an angle in ABC by using a correct scalar product method • finds an angle in ABC by using the cosine rule in the correct direction • realises triangle ABC is isosceles and applies Pythagoras in the correct direction to find the perpendicular height
A1:	Deduces the correct area as either $9\sqrt{5}\ (\text{cm}^2)$ or awrt $20.1\ (\text{cm}^2)$
Note:	For Way 1 and Way 2, using any of $\overrightarrow{OA}$, $\overrightarrow{OB}$ or $\overrightarrow{OC}$ in their vector product is M0 M0 A0 A0
(a)	**Way 3**
M1:	See scheme
M1:	Attempts to apply $\overrightarrow{OA}\times\overrightarrow{OB}$, $\overrightarrow{OB}\times\overrightarrow{OC}$ and $\overrightarrow{OC}\times\overrightarrow{OA}$
A1:	Attempts to add (as vectors) the results of applying $\overrightarrow{OA}\times\overrightarrow{OB}$, $\overrightarrow{OB}\times\overrightarrow{OC}$ and $\overrightarrow{OC}\times\overrightarrow{OA}$
A1:	Deduces the correct area as either $9\sqrt{5}\ (\text{cm}^2)$ or awrt $20.1\ (\text{cm}^2)$

Notes for Question 4 Continued	
(b)	
M1:	See scheme
M1:	Uses appropriate vectors in an attempt at the scalar triple product
A1:	Correct applied expression for the scalar triple product (allow $\pm$ and ignore modulus sign)
A1:	Correct solution leading to $d=6$
Note:	Using any of $\overrightarrow{OA}, \overrightarrow{OB}, \overrightarrow{OC}$ or $\overrightarrow{OD}$ in their scalar triple product is M0 M0 A0 A0
Note:	Some vector product calculations for reference:
	$\left\lvert\overrightarrow{AD}.\left(\overrightarrow{AB}\times\overrightarrow{AC}\right)\right\rvert = \begin{vmatrix} 3 & 7 & d-4 \\ 4 & 0 & -2 \\ 2 & 9 & -1 \end{vmatrix} = \left\lvert \begin{pmatrix} 3 \\ 7 \\ d-4 \end{pmatrix} \bullet \begin{pmatrix} 18 \\ 0 \\ 36 \end{pmatrix} \right\rvert = \lvert 54+36d-144 \rvert = \lvert 36d-90 \rvert$
	$\left\lvert\overrightarrow{AB}.\left(\overrightarrow{AC}\times\overrightarrow{AD}\right)\right\rvert = \begin{vmatrix} 4 & 0 & -2 \\ 2 & 9 & -1 \\ 3 & 7 & d-4 \end{vmatrix} = \left\lvert \begin{pmatrix} 4 \\ 0 \\ -2 \end{pmatrix} \bullet \begin{pmatrix} 9d-29 \\ 5-2d \\ -13 \end{pmatrix} \right\rvert = \lvert 36d-116+26 \rvert = \lvert 36d-90 \rvert$
	$\left\lvert\overrightarrow{AC}.\left(\overrightarrow{AB}\times\overrightarrow{AD}\right)\right\rvert = \begin{vmatrix} 2 & 9 & -1 \\ 4 & 0 & -2 \\ 3 & 7 & d-4 \end{vmatrix} = \left\lvert \begin{pmatrix} 2 \\ 9 \\ -1 \end{pmatrix} \bullet \begin{pmatrix} 14 \\ 10-4d \\ 28 \end{pmatrix} \right\rvert = \lvert 28+90-36d-28 \rvert = \lvert 90-36d \rvert$

5.

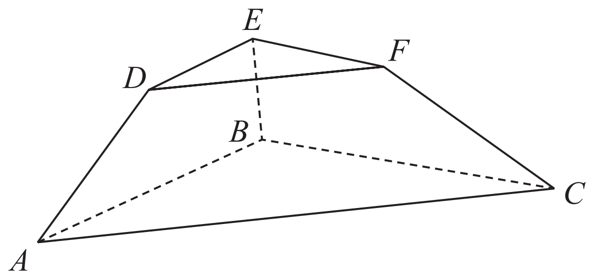

Figure 3

Figure 3 shows a solid display stand with parallel triangular faces *ABC* and *DEF*.
Triangle *DEF* is similar to triangle *ABC*.

With respect to a fixed origin *O*,
the points *A*, *B* and *C* have coordinates (3, –3, 1), (–5, 3, 3) and (1, 7, 5) respectively and
the points *D*, *E* and *F* have coordinates (2, –1, 8), (–2, 2, 9) and (1, 4, 10) respectively.
The units are in centimetres.

(a) Show that the area of the triangular face *DEF* is $\frac{1}{2}\sqrt{339}$ cm^2

(3)

(b) Find, in cm^3, the exact volume of the display stand.

(7)

Question	Scheme	Marks	AOs
5(a)	$\pm\overrightarrow{DE}=\pm\begin{pmatrix}-4\\3\\1\end{pmatrix},\ \pm\overrightarrow{DF}=\pm\begin{pmatrix}-1\\5\\2\end{pmatrix},\ \pm\overrightarrow{EF}=\pm\begin{pmatrix}3\\2\\1\end{pmatrix}$	M1	1.1b
	Area = $\frac{1}{2}\left\|\overrightarrow{DE}\times\overrightarrow{DF}\right\|=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\-4&3&1\\-1&5&2\end{vmatrix}=\frac{1}{2}\begin{vmatrix}1\\7\\-17\end{vmatrix}=\frac{1}{2}\sqrt{1^2+7^2+17^2}$	M1	1.1b
	$=\frac{1}{2}\sqrt{339}(\text{cm}^2)*$	A1*	2.2a
		(3)	
	Alternative for (a) using trigonometry:		
	$\pm\overrightarrow{DE}=\pm\begin{pmatrix}-4\\3\\1\end{pmatrix},\ \pm\overrightarrow{DF}=\pm\begin{pmatrix}-1\\5\\2\end{pmatrix},\ \pm\overrightarrow{EF}=\pm\begin{pmatrix}3\\2\\1\end{pmatrix}$	M1	1.1b
	$DE=\sqrt{26},\ DF=\sqrt{30},\ EF=\sqrt{14}$ $\cos DEF=\frac{26+14-30}{2\sqrt{26}\sqrt{14}}\Rightarrow DEF=\cos^{-1}\frac{5}{\sqrt{26}\sqrt{14}}$ $\Rightarrow \text{Area } DEF=\frac{1}{2}\times\sqrt{26}\sqrt{14}\sqrt{1-\frac{25}{364}}=...$	M1	1.1b
	$=\frac{1}{2}\times\sqrt{26}\sqrt{14}\frac{\sqrt{339}}{\sqrt{26}\sqrt{14}}=\frac{1}{2}\sqrt{339}*$	A1	2.2a
(b)	Attempt to find "T", the 4th vertex of the tetrahedron e.g. $\begin{pmatrix}2\\-1\\8\end{pmatrix}+\lambda\overrightarrow{AD}=\begin{pmatrix}1\\4\\10\end{pmatrix}+\mu\overrightarrow{CF}\Rightarrow\lambda=...\text{ or }\mu=...$ or e.g. $DF=\sqrt{30},\ AC=\sqrt{120}\Rightarrow$ Linear SF $=2$ $AT=2AD\Rightarrow T$ is ...	M1	3.1b
	$T(1, 1, 15)$	A1	1.1b
	e.g $\overrightarrow{AT}=\begin{pmatrix}-2\\4\\14\end{pmatrix},\ \overrightarrow{BT}=\begin{pmatrix}6\\-2\\12\end{pmatrix},\ \overrightarrow{CT}=\begin{pmatrix}0\\-6\\10\end{pmatrix}$ $\overrightarrow{AT}\times\overrightarrow{BT}\bullet\overrightarrow{CT}=\begin{vmatrix}-2&4&14\\6&-2&12\\0&-6&10\end{vmatrix}=...$	M1	1.1b
	e.g $V=\frac{1}{6}\left\|-2(-20+72)-4(60)+14(-36)\right\|\left(=\frac{424}{3}\right)$	A1	1.1b
	$\frac{1}{6}\overrightarrow{DT}\times\overrightarrow{ET}\bullet\overrightarrow{FT}=\begin{vmatrix}-1&2&7\\3&-1&6\\0&-3&5\end{vmatrix}=\frac{1}{6}\left\|-1(13)-2(15)+7(-9)\right\|\left(=\frac{53}{3}\right)$	M1	3.1b

	or length scale factor = 2 $\Rightarrow$ volume scale factor = 8		
	e.g Volume $= \frac{424}{3} - \frac{53}{3} = ...$ or Volume $= \frac{7}{8} \times \frac{424}{3} = ...$ or Volume $= 7 \times \frac{53}{3} = ...$	dM1	3.1a
	$= \frac{371}{3}$ cm^3	A1	1.1b
		(7)	
	See below for alternative for part (b)		
			(10 marks)

Notes

(a)
M1: Attempts to find 2 edges of triangle *DEF*. Must be subtracting components.
M1: Uses the correct process of the vector product to attempt the area including use of Pythagoras
A1*: Deduces the correct area with no errors
Alternative:
M1: Attempts to find 3 edges of triangle *DEF*. Must be subtracting components.
M1: A complete method for the area. Allow work in decimals for this mark but must work in exact terms to obtain the A mark
A1*: Deduces the correct area with no errors and no decimal work
(b)
M1: Adopts a correct strategy to find the fourth vertex of the tetrahedron e.g. finding where two edges intersect or uses the linear scale factor
A1: Correct coordinates for the other vertex
M1: Uses the information from the design to attempt a scalar triple product between appropriate vectors to find the volume of the smaller or larger tetrahedron.
A1: Correct volume for either tetrahedron
M1: Makes further progress with the solution by finding the volume of the other tetrahedron or calculates the volume scale factor using an appropriate method. E.g. using ratios or by finding the area of triangle *DEF* and comparing with triangle *ABC*
dM1: Completes the problem by finding the required volume of the frustum. Depends on all previous method marks
A1: Correct answer

Alternative for part (b) – splits into 4 tetrahedra:
This example takes *M* as the midpoint of *AC* and finds the volume of *ABMD*, *MBFC*, *DMFB*, *EDFB*

(b)	$Vol_{ABMD} = \frac{1}{6}\overrightarrow{AB} \times \overrightarrow{AD} \bullet \overrightarrow{AM} = ...$	M1	3.1b
	$= \frac{106}{3}$	A1	1.1b
	$Vol_{MBFC} = \frac{1}{6}\overrightarrow{BF} \times \overrightarrow{BC} \bullet \overrightarrow{BM} = ...\left(\frac{106}{3}\right)$ $Vol_{DMFB} = \frac{1}{6}\overrightarrow{DF} \times \overrightarrow{DM} \bullet \overrightarrow{DB} = ...\left(\frac{106}{3}\right)$	M1 A1	1.1b 1.1b

	$Vol_{EDFB} = \frac{1}{6}\overrightarrow{ED} \times \overrightarrow{EF} \bullet \overrightarrow{EB} = ...\left(\frac{53}{3}\right)$	M1	3.1b
	ABMD + *MBFC* + *DMFB* + *EDFB* = $3 \times \frac{106}{3} + \frac{53}{3}$	dM1	3.1a
	$= \frac{371}{3}$ cm^3	A1	1.1b

Notes:

M1: Adopts a correct strategy to find the volume of one tetrahedron
A1: Any correct volume
M1: Adopts a correct strategy to find the volumes of at least 2 other tetrahedra
A1: Correct volumes
M1: Makes further progress with the solution by finding the volume of all relevant tetrahedra
dM1: Completes the problem by finding the required volume of the frustum. Depends on all previous method marks
A1: Correct answer

4. With respect to a fixed origin O, the points A, B and C have position vectors given by

$$\overrightarrow{OA} = 18\mathbf{i} - 14\mathbf{j} - 2\mathbf{k} \quad \overrightarrow{OB} = -7\mathbf{i} - 5\mathbf{j} + 3\mathbf{k} \quad \overrightarrow{OC} = -2\mathbf{i} - 9\mathbf{j} - 6\mathbf{k}$$

The points O, A, B and C form the vertices of a tetrahedron.

(a) Show that the area of the triangular face ABC of the tetrahedron is 108 to 3 significant figures.

(3)

(b) Find the volume of the tetrahedron.

(4)

An oak wood block is made in the shape of the tetrahedron, with centimetres taken for the units.

The density of oak is $0.85\,\mathrm{g\,cm^{-3}}$

(c) Determine the mass of the block, giving your answer in kg.

(2)

Question	Scheme	Marks	AOs
4(a)	E.g. $\overrightarrow{AB}=\begin{pmatrix}-25\\9\\5\end{pmatrix},\ \overrightarrow{AC}=\begin{pmatrix}-20\\5\\-4\end{pmatrix}$	M1	1.1b
	$\left\|\overrightarrow{AB}\times\overrightarrow{AC}\right\|=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\-25&9&5\\-20&5&-4\end{vmatrix}=...$	M1	1.1b
	$\text{Area}=\frac{1}{2}\begin{vmatrix}-61\\-200\\55\end{vmatrix}=\frac{1}{2}\sqrt{61^2+200^2+55^2}=108*$	A1*	2.2a
		(3)	
	(a) Alternative:		
	$AB=\sqrt{25^2+9^2+5^2},\ AC=\sqrt{20^2+5^2+4^2},\ BC=\sqrt{5^2+4^2+9^2}$	M1	1.1b
	$AC^2=AB^2+BC^2-2AB\times BC\cos ABC$ $\Rightarrow 441=731+122-2\times\sqrt{731}\times\sqrt{122}\cos ABC$ $\Rightarrow \cos ABC=\frac{731+122-441}{2\times\sqrt{731}\times\sqrt{122}}$	M1	1.1b
	$\text{Area}=\frac{1}{2}\sqrt{731}\sqrt{122}\sin ABC=108*$	A1	2.2a
(b)	A complete attempt to find the volume of the tetrahedron	M1	3.1a
	E.g. $\begin{vmatrix}18&-14&-2\\-7&-5&3\\-2&-9&-6\end{vmatrix}=...$	M1	1.1b
	$=1592$	A1	1.1b
	$V=\frac{1592}{6}$ or e.g. $V=\frac{796}{3}$	A1	1.1b
		(4)	
(c)	$\text{Mass}=\frac{1592}{6}\times 0.85\div 1000\ \ (\text{kg})$	M1	2.1
	$\{=0.2255333...\}=$ awrt 0.226 (kg)	A1	1.1b
		(2)	
			(9 marks)

Notes

(a)
M1: Attempts to find 2 edges of the required triangle
M1: Uses the correct process of the vector product for 2 appropriate vectors

A1*: Deduces the correct area with no errors but condone sign slips on the components provided the work is otherwise correct e.g. allow $\text{Area} = \frac{1}{2}\begin{vmatrix} -61 \\ -200 \\ -55 \end{vmatrix} = \frac{1}{2}\sqrt{61^2 + 200^2 + 55^2} = 108*$

Alternative

M1: Attempts lengths of all 3 sides

M1: Applies the cosine rule to find one of the angles of the triangle

A1*: Deduces the correct area with no errors

(b)

M1: See scheme

M1: Uses appropriate vectors in an attempt at the scalar triple product

A1: Correct numerical expression for the scalar triple product (allow $\pm$)

A1: Correct volume

(c)

M1: A correct method for changing their units for their volume and for finding the mass in kg

A1: Correct answer

5.

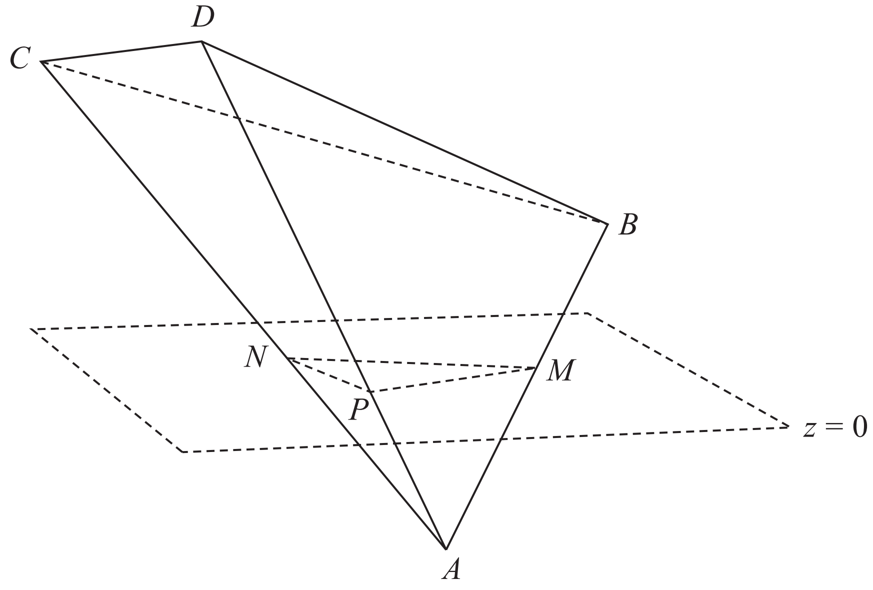

Figure 1

The points $A(3, 2, -4)$, $B(9, -4, 2)$, $C(-6, -10, 8)$ and $D(-4, -5, 10)$ are the vertices of a tetrahedron.

The plane with equation $z = 0$ cuts the tetrahedron into two pieces, one on each side of the plane.

The edges AB, AC and AD of the tetrahedron intersect the plane at the points M, N and P respectively, as shown in Figure 1.

Determine

(a) the coordinates of the points M, N and P,

(3)

(b) the area of triangle MNP,

(2)

(c) the exact volume of the solid $BCDPNM$.

(6)

Question	Scheme	Marks	AOs
5(a)	A correct method to find one coordinates of *M*, *N* or *P* For example $\overrightarrow{AB} = \begin{pmatrix} 6 \\ -6 \\ 6 \end{pmatrix}$ so $\overrightarrow{OM} = \begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix} + \frac{4}{6}\begin{pmatrix} 6 \\ -6 \\ 6 \end{pmatrix} = \ldots$ $\overrightarrow{AC} = \begin{pmatrix} -9 \\ -12 \\ 12 \end{pmatrix}$ so $\overrightarrow{ON} = \begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix} + \frac{4}{12}\begin{pmatrix} -9 \\ -12 \\ 12 \end{pmatrix} = \ldots$ $\overrightarrow{AD} = \begin{pmatrix} -7 \\ -7 \\ 14 \end{pmatrix}$ so $\overrightarrow{OP} = \begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix} + \frac{4}{14}\begin{pmatrix} -7 \\ -7 \\ 14 \end{pmatrix} = \ldots$	M1	3.1a
	One of $(M =)(7,-2,0)$, $(N =)(0,-2,0)$ or $(P =)(1,0,0)$	A1	1.1b
	All of $(M =)(7,-2,0)$, $(N =)(0,-2,0)$ and $(P =)(1,0,0)$	A1	1.1b
		(3)	
(b)	Correct method, e.g. realises *MN* is parallel to *x* axis, so base is 7 and height 2, hence area of intersection is $\frac{1}{2}\times 7\times 2 = \ldots$ **Alternatively** using $\frac{1}{2}\lvert a\times b\rvert$ $\overrightarrow{PM} = \pm\begin{pmatrix} 6 \\ -2 \\ 0 \end{pmatrix}$ $\overrightarrow{PN} = \pm\begin{pmatrix} -1 \\ -2 \\ 0 \end{pmatrix}$ $\overrightarrow{NM} = \pm\begin{pmatrix} 7 \\ 0 \\ 0 \end{pmatrix}$ For example $\frac{1}{2}\left\lvert\overrightarrow{MP}\times\overrightarrow{PN}\right\rvert = \frac{1}{2}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & -2 & 0 \\ -1 & -2 & 0 \end{vmatrix} = \frac{1}{2}\left\lvert\begin{pmatrix} 0 \\ 0 \\ -14 \end{pmatrix}\right\rvert = \ldots$	M1	1.1b
	$= 7$ cso	A1	1.1b
		(2)	
(c)	Vol $NMPA = \frac{1}{3}A_b h = \frac{1}{3}\times 7\times 4 = \frac{28}{3}$ Or using triple scalar product $NMPA = \frac{1}{6}\left\lvert\overrightarrow{AM}\cdot\left(\overrightarrow{AN}\times\overrightarrow{AP}\right)\right\rvert = \frac{1}{6}\left\lvert\begin{pmatrix} 4 \\ -4 \\ 4 \end{pmatrix}\cdot\left(\begin{pmatrix} -3 \\ -4 \\ 4 \end{pmatrix}\times\begin{pmatrix} -2 \\ -2 \\ 4 \end{pmatrix}\right)\right\rvert$ $= \frac{1}{6}\left\lvert\begin{pmatrix} 4 \\ -4 \\ 4 \end{pmatrix}\cdot\begin{pmatrix} -8 \\ 4 \\ -2 \end{pmatrix}\right\rvert = \frac{28}{3}$	M1 A1	3.1a 1.1b

	Vol $ABCD = \frac{1}{6}\left\|\overrightarrow{AB}\cdot\left(\overrightarrow{AC}\times\overrightarrow{AD}\right)\right\| = \ldots$ $= \frac{1}{6}\left\|\begin{pmatrix}6\\-6\\6\end{pmatrix}\cdot\left(\begin{pmatrix}-9\\-12\\12\end{pmatrix}\times\begin{pmatrix}-7\\-7\\14\end{pmatrix}\right)\right\| = \frac{1}{6}\left\|\begin{pmatrix}6\\-6\\6\end{pmatrix}\cdot\begin{pmatrix}-84\\42\\-21\end{pmatrix}\right\| = \ldots$	**M1**	1.1b
	$= 147$	**A1**	1.1b
	So volume required is $'147' - \frac{'28'}{3} = \ldots$	**M1**	3.1a
	$= \frac{413}{3}$	**A1**	1.1b
		(6)	
			(11 marks)

Notes:

(a)
M1: Correct method for finding at least one of the three points. Allow one slip in coordinates but should have correct fraction to make the value of *z* to be 0.
A1: Any one of the three points correct, ignoring the labelling.
A1: All three points correct, ignoring the labelling

(b)
M1: Correct method for finding the area of the triangle, e.g realises that *MN* is parallel to the *x*-axis so uses $\frac{1}{2}bh$ with $b = MN$ and *h* is distance of *MN* from axis.

Alternative using $\frac{1}{2}|a \times b|$ with vectors $\overrightarrow{PM} = \pm\begin{pmatrix}6\\-2\\0\end{pmatrix}$ $\overrightarrow{PN} = \pm\begin{pmatrix}-1\\-2\\0\end{pmatrix}$ $\overrightarrow{NM} = \pm\begin{pmatrix}7\\0\\0\end{pmatrix}$ follow through on their answers in part (a). Condone sign slips except they must be using $-\mathbf{j}$ in the cross product

For example $\frac{1}{2}\left|\overrightarrow{MP}\times\overrightarrow{PN}\right| = \frac{1}{2}\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 6 & -2 & 0\\ -1 & -2 & 0\end{vmatrix} = \frac{1}{2}\left\|\begin{pmatrix}0\\0\\-14\end{pmatrix}\right\| = \ldots$

$$\frac{1}{2}\left|\overrightarrow{PN}\times\overrightarrow{NM}\right| = \frac{1}{2}\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ -1 & -2 & 0\\ 7 & 0 & 0\end{vmatrix} = \frac{1}{2}\left\|\begin{pmatrix}0\\0\\14\end{pmatrix}\right\| = \ldots$$

$$\frac{1}{2}\left|\overrightarrow{PM}\times\overrightarrow{NM}\right| = \frac{1}{2}\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 6 & -2 & 0\\ 7 & 0 & 0\end{vmatrix} = \frac{1}{2}\left\|\begin{pmatrix}0\\0\\14\end{pmatrix}\right\| = \ldots$$

or attempting to find an angle using dot product or cosine rule followed by $\frac{1}{2}ab\sin C$.

A1: Correct area of 7 from correct vectors

(c) On ePen this is M1 A1 M1 M1 M1 A1

M1: Formulates a correct method to find the volume of *NMPA*. May use method shown, or e.g. $\frac{1}{6}\left|\overrightarrow{AM}\cdot\left(\overrightarrow{AN}\times\overrightarrow{AP}\right)\right|$ or equivalent method.

A1: For $\frac{28}{3}$.

Note there are many ways to find the required volume of *AMNP* applying the triple scalar product to a combination of the following vectors

$$\overrightarrow{AM}=\begin{pmatrix}4\\-4\\4\end{pmatrix}\ \overrightarrow{AN}=\begin{pmatrix}-3\\-4\\4\end{pmatrix}\ \overrightarrow{AP}=\begin{pmatrix}-2\\-2\\4\end{pmatrix}\ \overrightarrow{NA}=\begin{pmatrix}3\\4\\-4\end{pmatrix}\ \overrightarrow{NM}=\begin{pmatrix}7\\0\\0\end{pmatrix}\ \overrightarrow{NP}=\begin{pmatrix}1\\2\\0\end{pmatrix}$$

$$\overrightarrow{MA}=\begin{pmatrix}-4\\4\\-4\end{pmatrix}\ \overrightarrow{MN}=\begin{pmatrix}-7\\0\\0\end{pmatrix}\ \overrightarrow{MP}=\begin{pmatrix}-6\\2\\0\end{pmatrix}\ \overrightarrow{PA}=\begin{pmatrix}2\\2\\-4\end{pmatrix}\ \overrightarrow{PM}=\begin{pmatrix}6\\-2\\0\end{pmatrix}\ \overrightarrow{PN}=\begin{pmatrix}-1\\-2\\0\end{pmatrix}$$

For example

$$\frac{1}{6}\left|\overrightarrow{AM}\cdot\left(\overrightarrow{AN}\times\overrightarrow{AP}\right)\right|=\frac{1}{6}\left|\begin{pmatrix}4\\-4\\4\end{pmatrix}\cdot\left(\begin{pmatrix}-3\\-4\\4\end{pmatrix}\times\begin{pmatrix}-2\\-2\\4\end{pmatrix}\right)\right|=\frac{1}{6}\left|\begin{pmatrix}4\\-4\\4\end{pmatrix}\cdot\begin{pmatrix}-8\\4\\-2\end{pmatrix}\right|=\frac{1}{6}\times56$$

$$\frac{1}{6}\left|\overrightarrow{NA}\cdot\left(\overrightarrow{NM}\times\overrightarrow{NP}\right)\right|=\frac{1}{6}\left|\begin{pmatrix}3\\4\\-4\end{pmatrix}\cdot\left(\begin{pmatrix}7\\0\\0\end{pmatrix}\times\begin{pmatrix}1\\2\\0\end{pmatrix}\right)\right|=\frac{1}{6}\left|\begin{pmatrix}3\\4\\-4\end{pmatrix}\cdot\begin{pmatrix}0\\0\\14\end{pmatrix}\right|=\frac{1}{6}\times56$$

$$\frac{1}{6}\left|\overrightarrow{MA}\cdot\left(\overrightarrow{MN}\times\overrightarrow{MP}\right)\right|=\frac{1}{6}\left|\begin{pmatrix}-4\\4\\-4\end{pmatrix}\cdot\left(\begin{pmatrix}-7\\0\\0\end{pmatrix}\times\begin{pmatrix}-6\\2\\0\end{pmatrix}\right)\right|=\frac{1}{6}\left|\begin{pmatrix}-4\\4\\-4\end{pmatrix}\cdot\begin{pmatrix}0\\0\\14\end{pmatrix}\right|=\frac{1}{6}\times56$$

$$\frac{1}{6}\left|\overrightarrow{PA}\cdot\left(\overrightarrow{PM}\times\overrightarrow{PN}\right)\right|=\frac{1}{6}\left|\begin{pmatrix}2\\2\\-4\end{pmatrix}\cdot\left(\begin{pmatrix}6\\2\\0\end{pmatrix}\times\begin{pmatrix}-1\\-2\\0\end{pmatrix}\right)\right|=\frac{1}{6}\left|\begin{pmatrix}2\\2\\-4\end{pmatrix}\cdot\begin{pmatrix}0\\0\\-14\end{pmatrix}\right|=\frac{1}{6}\times56$$

Note candidates may write as $\frac{1}{6}\begin{vmatrix}4&-4&4\\-3&-4&4\\-2&-2&4\end{vmatrix}=\frac{1}{6}\left|4(-16+8)+4(-12+8)+4(6-8)\right|=\frac{1}{6}\left|-56\right|=\frac{28}{3}$

M1: A complete attempt at the volume of *ABCD*, with correct method for cross product (oe in other methods). Condone sign slips except they must be using – **j** in the cross product

A1 (M1 on ePen): 147

M1: Finds difference of the two volumes must have used a correct method to find the volumes.

A1: $\frac{413}{3}$

Note there are many ways to find the required volume of *ABCD* applying the triple scalar product to a combination of the following vectors

$$\overrightarrow{AB}=\begin{pmatrix}6\\-6\\6\end{pmatrix}\ \overrightarrow{AC}=\begin{pmatrix}-9\\-12\\12\end{pmatrix}\ \overrightarrow{AD}=\begin{pmatrix}-7\\-7\\14\end{pmatrix}\ \overrightarrow{BA}=\begin{pmatrix}-6\\6\\-6\end{pmatrix}\ \overrightarrow{BC}=\begin{pmatrix}15\\-6\\6\end{pmatrix}\ \overrightarrow{BD}=\begin{pmatrix}-13\\-1\\8\end{pmatrix}$$

$$\overrightarrow{CA}=\begin{pmatrix}9\\12\\-12\end{pmatrix}\ \overrightarrow{CB}=\begin{pmatrix}-15\\6\\-6\end{pmatrix}\ \overrightarrow{CD}=\begin{pmatrix}2\\5\\2\end{pmatrix}\ \overrightarrow{DA}=\begin{pmatrix}7\\7\\-14\end{pmatrix}\ \overrightarrow{DB}=\begin{pmatrix}13\\1\\-8\end{pmatrix}\ \overrightarrow{DC}=\begin{pmatrix}-2\\-5\\2\end{pmatrix}$$

For example

$$\frac{1}{6}\left|\overrightarrow{AB}\cdot\left(\overrightarrow{AC}\times\overrightarrow{AD}\right)\right|=\frac{1}{6}\left|\begin{pmatrix}6\\-6\\6\end{pmatrix}\cdot\left(\begin{pmatrix}-9\\-12\\12\end{pmatrix}\times\begin{pmatrix}-7\\-7\\14\end{pmatrix}\right)\right|=\frac{1}{6}\left|\begin{pmatrix}6\\-6\\6\end{pmatrix}\cdot\begin{pmatrix}-84\\42\\-21\end{pmatrix}\right|=\frac{1}{6}\times 882=147$$

$$\frac{1}{6}\left|\overrightarrow{BA}\cdot\left(\overrightarrow{BC}\times\overrightarrow{BD}\right)\right|=\frac{1}{6}\left|\begin{pmatrix}-6\\6\\-6\end{pmatrix}\cdot\left(\begin{pmatrix}15\\-6\\6\end{pmatrix}\times\begin{pmatrix}-13\\-1\\8\end{pmatrix}\right)\right|=\frac{1}{6}\left|\begin{pmatrix}-6\\6\\-6\end{pmatrix}\cdot\begin{pmatrix}-42\\42\\-63\end{pmatrix}\right|=\frac{1}{6}\times 882=147$$

$$\frac{1}{6}\left|\overrightarrow{CA}\cdot\left(\overrightarrow{CD}\times\overrightarrow{CB}\right)\right|=\frac{1}{6}\left|\begin{pmatrix}9\\12\\-12\end{pmatrix}\cdot\left(\begin{pmatrix}2\\5\\2\end{pmatrix}\times\begin{pmatrix}15\\6\\-6\end{pmatrix}\right)\right|=\frac{1}{6}\left|\begin{pmatrix}9\\12\\-12\end{pmatrix}\cdot\begin{pmatrix}-42\\42\\-63\end{pmatrix}\right|=\frac{1}{6}\times 882=147$$

$$\frac{1}{6}\left|\overrightarrow{DA}\cdot\left(\overrightarrow{DB}\times\overrightarrow{DC}\right)\right|=\frac{1}{6}\left|\begin{pmatrix}7\\7\\-14\end{pmatrix}\cdot\left(\begin{pmatrix}13\\1\\-8\end{pmatrix}\times\begin{pmatrix}-2\\-5\\2\end{pmatrix}\right)\right|=\frac{1}{6}\left|\begin{pmatrix}7\\7\\-14\end{pmatrix}\cdot\begin{pmatrix}42\\-42\\63\end{pmatrix}\right|=\frac{1}{6}\times 882=147$$

Note candidates may write as

$$\frac{1}{6}\begin{vmatrix}6 & -6 & 6\\-9 & -12 & 12\\-7 & -7 & 14\end{vmatrix}=\frac{1}{6}\left|6(-168+84)+6(-126+84)+6(63-84)\right|=\frac{1}{6}\left|-882\right|=147$$