Cp1Ch1 XMQs and MS

(Total: 7 marks)

1. CP1_2022   Q7 . 7 marks - CP1ch1 Complex numbers

7. Given that $z = a + b\mathrm{i}$ is a complex number where a and b are real constants,

(a) show that zz^* is a real number.

(2)

Given that

- $zz^* = 18$
- $\dfrac{z}{z^*} = \dfrac{7}{9} + \dfrac{4\sqrt{2}}{9}\mathrm{i}$

(b) determine the possible complex numbers z

(5)

Question	Scheme	Marks	AOs
7(a)	$z* = a - bi$ then $zz* = (a+bi)(a-bi) = ..$	M1	1.1b
	$zz* = a^2 + b^2$ therefore, a real number	A1	2.4
		(2)	
(b)	$\frac{z}{z*} = \frac{a+bi}{a-bi} = \frac{(a+bi)(a+bi)}{(a-bi)(a+bi)} = \frac{(a^2-b^2)+2abi}{a^2+b^2} = \frac{7}{9} + \frac{4\sqrt{2}i}{9}$ or $\frac{z}{z^*} = \frac{z^2}{zz^*} = \frac{z^2}{18} \Rightarrow$ $z^2 = 14 + 8\sqrt{2}i$ or $a + bi = \left(\frac{7}{9} + \frac{4\sqrt{2}i}{9}\right)(a - bi) = .. + .. i$	M1	1.1b
	Forms two equations from $a^2 + b^2 = 18$ or $\frac{a^2-b^2}{18} = \frac{7}{9}$ or $\frac{a^2-b^2}{a^2+b^2} = \frac{7}{9}$ or $\frac{2ab}{18} = \frac{4\sqrt{2}}{9}$ or $\frac{2ab}{a^2+b^2} = \frac{4\sqrt{2}}{9}$ or $a = \frac{7}{9}a + \frac{4\sqrt{2}}{9}b$ oe	M1 A1	3.1a 1.1b
	Solves the equations simultaneously e.g. $a^2 + b^2 = 18$ and $a^2 - b^2 = 14$ leading to a value for *a* or *b*	dM1	1.1b
	$z = \pm(4 + \sqrt{2}i)$	A1	2.2a
		(5)	
			(7 marks)

Notes:

(a)(i)

M1: States or implies $z* = a - bi$ and finds an expression for $zz*$

A1: Achieves $zz* = a^2 + b^2$ and draws the conclusion that $zz*$ is a real number. Accept $\in \mathbb{R}$ as conclusion, but not just "no imaginary part".

(b)

M1: Starts the process of solving by using the conjugate to form an equation with real denominators, and without z^* or i^2 in the equation. Accept as shown in scheme, or may multiply through by $a - bi$ and expand and gather terms. May be implied by correct extraction of equation(s).

M1: Uses the given information to form two equations involving *a* and *b* at least one of which includes both. It must involve equating real or imaginary parts of $\frac{z}{z*} = \frac{7}{9} + \frac{4\sqrt{2}i}{9}$

A1: Any two correct equations arising from use of both given facts. (Note: if multiplying through by $a - bi$ then equating real and imaginary terms gives the same equation.)

dM1: Dependent on previous method mark, solves the equations to find a value for either *a* or *b*.

A1: Deduces the correct complex numbers and no extras. Do not accept $\pm 4 \pm \sqrt{2}i$

Note: it is possible to solve via polar coordinates, but unlikely to succeed. If you see responses you think are worthy of credit but are unsure how to mark, use review. Example solutions shown below.

(b) **Alt**	$\frac{z}{z^*}=\frac{z^2}{zz^*}=\frac{z^2}{18}\Rightarrow z^2=14+8\sqrt{2}i$ or let arg $z=\theta$. then $\frac{z}{z*}=\frac{re^{i\theta}}{re^{-i\theta}}=e^{2i\theta}=cos\,2\,\theta+\text{i}\sin 2\theta$	M1	1.1b
	$z^2=18(cos\,\alpha+\text{i}\sin\alpha)$where $\tan\alpha=\frac{4\sqrt{2}}{7}\Rightarrow z=\pm\sqrt{18}\left(cos\frac{1}{2}\alpha+\text{i}\sin\frac{1}{2}\alpha\right)$Or $cos\,2\,\theta+\text{i}\sin 2\theta=\frac{7}{9}+\frac{4\sqrt{2}i}{9}\Rightarrow 2\,cos^2\theta-1=\frac{7}{9}, 2\,sin\,\theta\,cos\,\theta=\frac{4\sqrt{2}}{9}$	M1 A1	1.1b 1.1b
	$cos\frac{1}{2}\alpha=\sqrt{\frac{1}{2}(1+cos\,\alpha)}=\sqrt{\frac{1}{2}\left(1+\frac{7}{9}\right)}=\ldots$ and $\sin\frac{1}{2}\alpha=\sqrt{\frac{1}{2}(1-cos\,\alpha)}=\sqrt{\frac{1}{2}\left(1-\frac{7}{9}\right)}=\ldots$or $\Rightarrow cos\,\theta=\frac{2\sqrt{2}}{3}, sin\,\theta=\frac{1}{3}, r=\|z\|=\sqrt{zz^*}=\sqrt{18}$	dM1	3.1a
	$z=\pm\left(4+\sqrt{2}i\right)$	A1	2.2a
		(5)	