Trebuchet Projectiles

Warwick castle boasts the world's largest working trebuchet. Standing at $18m$ high, and weighing $22\ tonnes$, it is capable of firing a projectile of mass $150kg$ a horizontal distance of up to $300m$ at a top speed of $70m/s$. The projectile can be assumed to be launched from ground level.

1.
Assume the angle of launch is altered to $45°$.

a) If the top speed stayed the same, what would the range become?

b) If the range stayed the same, what would the top speed become?

2.
Given the top speed and the horizontal range, at what angle does the trebuchet actually launch its projectile? You may find the identity $\sin 2\theta = 2\sin\theta\cos\theta$ useful.

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Trebuchet Projectiles SOLUTIONS

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1.

Assume the angle of launch is altered to $45°$.

a) If the top speed stayed the same, what would the range become?

Vertical motion:

$s = 0$
$u = 70\sin 45$
$v = -$
$a = -9.8$
$t = t$

$s = ut + \frac{1}{2}at^2$
$\Rightarrow \quad 0 = 70t\sin 45 - 4.9t^2$
$\Rightarrow \quad t(70\sin 45 - 4.9t) = 0$
$\Rightarrow \quad t = 0 \ or \ t = \frac{70\sin 45}{4.9}$
$From\ problem\ context: t = \frac{70\sin 45}{4.9}$

Horizontal motion:

$v = 70\cos 45$
$x = x$
$t = \frac{70\sin 45}{4.9}$

$v = \frac{x}{t}$
$\Rightarrow \quad 70\cos 45 = \frac{x}{\left(\frac{70\sin 45}{4.9}\right)}$
$\Rightarrow \quad \boldsymbol{x = 500}$

b) If the range stayed the same, what would the top speed become?

Horizontal motion:

$v = V\cos 45$
$x = 300$
$t = t$

$v = \frac{x}{t}$
$\Rightarrow \quad V\cos 45 = \frac{300}{t}$
$\Rightarrow \quad t = \frac{300}{V\cos 45}$

Vertical motion:

$s = 0$
$u = V\sin 45$
$v = -$
$a = -9.8$
$t = \frac{300}{V\cos 45}$

$s = ut + \frac{1}{2}at^2$
$\Rightarrow \quad 0 = V\sin 45\left(\frac{300}{V\cos 45}\right) - 4.9\left(\frac{300}{V\cos 45}\right)^2$
$\Rightarrow \quad 300\tan 45 = \frac{441000}{V^2\cos^2 45}$
$\Rightarrow \quad V^2 = \frac{882000}{300} = 2940$
$\Rightarrow \quad \boldsymbol{V = 54.2 ms^{-1}\ to\ 3\ s.f.}$

2.

Given the top speed and the horizontal range, at what angle does the trebuchet actually launch its projectile? You may find the identity $\sin 2\theta = 2\sin\theta\cos\theta$ useful.

Horizontal motion:

$v = 70\cos\theta$
$x = 300$
$t = t$

$v = \frac{x}{t}$
$\Rightarrow \quad 70\cos\theta = \frac{300}{t}$
$\Rightarrow \quad t = \frac{300}{70\cos\theta}$

Vertical motion:

$s = 0$
$u = 70\sin\theta$
$v = -$
$a = -9.8$
$t = t$

$s = ut + \frac{1}{2}at^2$
$\Rightarrow \quad 0 = 70t\sin\theta - 4.9t^2$
$\Rightarrow \quad 70\left(\frac{300}{70\cos\theta}\right)\sin\theta = 4.9\left(\frac{300}{70\cos\theta}\right)^2$
$\Rightarrow \quad 300\sin\theta = \frac{90}{\cos\theta}$
$\Rightarrow \quad 10\sin\theta\cos\theta = 3$
$\Rightarrow \quad 5\sin 2\theta = 3$
$\Rightarrow \quad \sin 2\theta = 0.6$
$\Rightarrow \quad 2\theta = \sin^{-1}0.6 = 36.87° \ or \ 143.13°$
$\Rightarrow \quad \theta = 18.4° \ or \ 71.6°$
$From\ problem\ context: \boldsymbol{\theta = 71.6°\ to\ 3\ s.f.}$