

Days of Wonder game *Ticket To Ride Europe* Cities and distances based on Europe 1910

Each unit represents a 2 hour train journey (e.g. London to Edinburgh = 8 hours)





### Task A

Use a suitable method to find the smallest length of track required to link every city on the map. Give details of the routes you choose and the total length of track needed.

## Task B

Use a suitable method to find the city furthest from London, and give an estimate for the minimum travelling time.

# Task C

Use a suitable method to determine which of the following cities is the closest to Budapest, and give the distance and minimum route for each city:

Riga

Edinburgh

Barcelona

Erzurum

#### Task A SOLUTION

Kruskal's algorithm can be used, but since there are so many identically weighted edges, and identifying cycles becomes increasingly challenging as the complexity of the network increases, Prim's algorithm is probably easier to implement. One of the many possible minimum spanning trees is shown below. All minimum spanning trees will have the same total weighting, but edges may be different.



The total weighting of the minimum spanning tree is: 108 (that is, 216 hours).

#### Task B SOLUTION

The city furthest from London is Erzurum, at a distance of 22 (ie, 44 hours). Use Dijkstra's algorithm starting from London.



There are in fact two valid alternatives for the shortest route: from Bucuresti you can travel via Sevastopol (4 + 4 = 8) or via Constantinople and Angora (3 + 2 + 3 = 8).

Since Dijkstra's algorithm, as a side-effect of finding the shortest route to a particular vertex, also finds the shortest route to all other vertices along the way, not only do we know the distance from London to Erzurum, but to all the other cities too (see table at the end).

#### Task C SOLUTION

Dijkstra's algorithm, starting from Budapest, will calculate the shortest distance to any city, provided you continue far enough. Once the labelling procedure has encompassed the four cities of interest, you have your answer.



**Budapest to Riga is 10** (20 hours), via Wien, Warszawa and Danzig. **Budapest to Edinburgh is 13** (26 hours) – the furthest away, via Wien, Munchen, Frankfurt, Amsterdam and London. **Budapest to Barcelona is 11** (22 hours), via Wien, Munchen, Zurich and Marseille. Finally, **Budapest to Erzurum is 12** (24 hours), via Bucuresti, Constantinople and Angora *or* via Bucuresti and Sevastopol. *Note that, since Dijkstra's algorithm provides shortest distances from the starting point to all vertices if continued long enough, we have distances to all cities on the map, not just these four (see table at the end).* 

### Distances from London and from Budapest to each city

City	From London	From Budapest
Amsterdam	2	7
Angora	19	9
Athina	16	7
Barcelona	9	11
Berlin	7	3
Brest	4	11
Brindisi	12	8
Bruxelles	3	7
Bucuresti	14	4
Budapest	10	0
Cadiz	13	16
Constantinople	17	7
Danzig	11	7
Dieppe	2	9
Edinburgh	4	13
Erzurum	22	12
Essen	5	5
Frankfurt	4	5
Kharkov	20	10
Kobenhavn	8	8
Kyiv	16	6
Lisboa	13	16
London	0	9

Madrid	10	13
Marseille	7	7
Moskva	19	11
Munchen	6	3
Palermo	14	10
Pamplona	7	11
Paris	3	8
Petrograd	18	14
Riga	14	10
Roma	10	6
Rostov	22	12
Sarajevo	13	3
Sevastopol	18	8
Smolensk	17	9
Smyrna	18	9
Sochi	20	10
Sofia	15	5
Stockholm	11	11
Venezia	8	4
Warszawa	11	5
Wien	9	1
Wilno	14	8
Zagrab	10	2
Zurich	6	5