Spiderman Projectiles Question

MJ has fallen from a balcony yet again, and Spiderman happened to be crouched on a tower block opposite. She fell from rest, and is accelerating under gravity. Spiderman must project himself downwards as well as across in order to have any hope of catching up to her before it's too late. The balcony is 70m from the ground.

Spiderman projects himself from a building horizontally opposite – but 30m away from – MJ's balcony, at an initial velocity of $15ms^{-1}$, at an angle of 45° below the horizontal.

Calculate the time it will take Spiderman to get directly below the balcony.

How far will he be below the balcony at this point? Will he reach her before she crashes to the ground?

How long a delay should Spiderman leave before jumping, to ensure he manages to intercept MJ?

Extension: If Spiderman is too eager, and jumps as soon as he sees MJ drop (assuming a spiderlike reaction time of 0.1 seconds), he will miss her, both physically and emotionally. The latter cannot easily be quantified, but could you calculate the distance for the former?

Solution

Since horizontal speed is constant, and the moment Spiderman will be directly below the balcony will be after he has travelled 30m horizontally, we need only look at horizontal motion:

$$Horizontally: \quad time = \frac{distance}{speed} \quad \Longrightarrow \quad t = \frac{30}{15\cos 45} = \mathbf{2\sqrt{2}\ seconds}$$

In order to calculate the time it takes for Spiderman to reach this point, we need to consider his vertical motion, under the constant acceleration due to gravity:

$$Vertically: \quad s = ? \quad u = 15\sin 45 \quad (v = ?) \quad a = 9.8 \quad t = 2\sqrt{2}$$

$$s = ut + \frac{1}{2}at^2 = 15\sin 45 \times 2\sqrt{2} + \frac{1}{2}(9.8) \times 8 = \mathbf{69.2m\ (ok - balcony\ is\ 70m\ high)}$$

Since Spiderman's path only crosses MJ's at this point, 69.2m below her balcony, and since this will happen $2\sqrt{2}$ seconds after he jumps, we need to calculate the time it would take MJ to fall this far, then subtract the time Spiderman will be in the air:

$$Vertically\ (MJ): \ s = 69.2 \ \ u = 0 \ \ (v = ?)\ a = 9.8 \ \ t = ?$$

$$s = ut + \frac{1}{2}at^2 \quad \Longrightarrow \quad 69.2 = 4.9t^2 \quad \Longrightarrow \quad t = \sqrt{\frac{692}{49}} = \frac{2\sqrt{173}}{7} = 3.76s\ to\ 3\ s.f.$$

$$Delay\ before\ jumping: \ \frac{2\sqrt{173}}{7} - 2\sqrt{2} = \mathbf{0.930s\ to\ 3\ s.f.}$$

Extension:
Since the drop will take Spiderman $2\sqrt{2}$ seconds, factoring in the 0.1 seconds reaction time we can calculate the distance she will have fallen by the time Spiderman arrives at his 69.2m rendezvous:

$$s = ? \ \ u = 0 \ \ (v = ?)\ a = 9.8 \ \ t = 2\sqrt{2} + 0.1$$

$$s = ut + \frac{1}{2}at^2 = 4.9\left(2\sqrt{2} + 0.1\right)^2 = 42.0m\ to\ 3\ s.f.$$

$$69.2 - 42.0 = \mathbf{27.2m\ to\ 3\ s.f.}$$