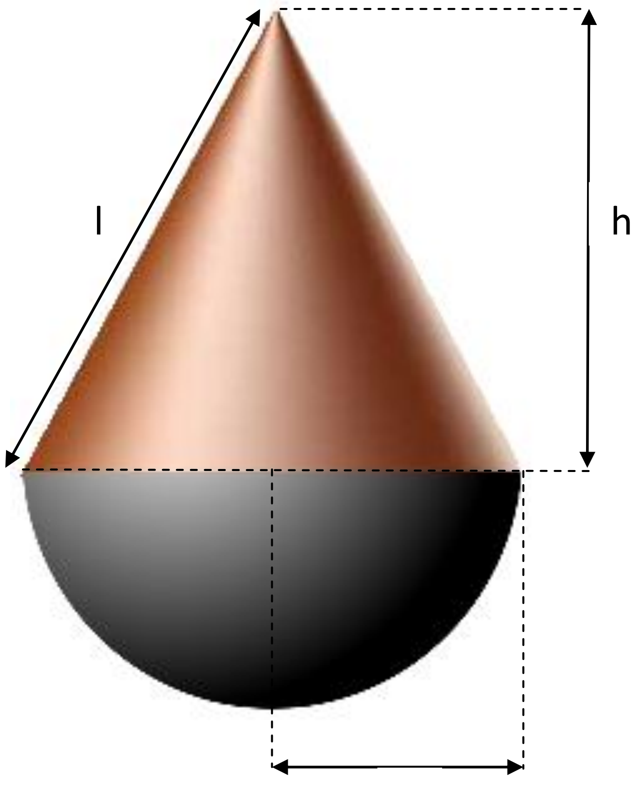
Creating your own formula – Combining a sphere and a cone

Sphere		
	Volume:	$V = \frac{4}{3}\pi r^3$ *where **r** is the radius.*
	Surface Area:	$SA = 4\pi r^2$ *where **r** is the radius.*
Cone		
	Volume:	$V = \frac{1}{3}\pi r^2 \mathrm{h}$ *where **r** is the radius of the circular base and **h** is the perpendicular height.*
	Surface Area: (of curved surface)	$SA = \pi r l$ *where **r** is the radius of the circular base and **l** is the distance from the rim of the base to the tip of the cone.*

A new 3-D shape has been made by attaching a hemisphere to the base of a cone of the same radius.

1) **Using the formulae listed above, produce an expression for the volume of this new shape.**



2) **Produce an expression for the surface area of this shape, and hence show that the surface area is the same as that of an entire sphere of radius r when $h = \sqrt{3}r$**

Combining a sphere and a cone – hint sheet 1

1) Find the total volume by adding the volume of a cone to that of a hemisphere (half a sphere).

2) Start by finding the surface area of the curved surface of a cone in terms of **h** and **r** (recall Pythagoras: **h** and **r** represent two sides of a right-angle triangle with **l** as the hypotenuse).
   Next, add this expression to that of half a sphere. Simplify.
   Finally, substitute $\sqrt{3}r$ into the equation for h and check that it is equivalent to that of a sphere.

Combining a sphere and a cone – hint sheet 2

1) Volume of a cone + ½ Volume of a sphere = $\frac{1}{3}\pi r^2\mathrm{h} + \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$. Simplify this for the final expression (multiply out brackets, take out common factors).

2) By Pythagoras, $a^2 + b^2 = c^2$ where c is the hypotenuse of a right-angle triangle.
   Substitute in the values of **h**, **r** and **l**, and rearrange to make **l** the subject.
   Next, substitute this expression into the formula for the surface area of the curved surface of a cone.
   Adding to the formula for the surface area of half a sphere gives:

$$\pi r\sqrt{h^2 + r^2} + \frac{1}{2}(4\pi r^2)$$

   This can be simplified slightly. Then $\sqrt{3}r$ may be substituted into the equation for h and you can check that it is equivalent to the formula for the surface area of a sphere.