A-level Maths Pure Skills Checks
Pure Maths

What they're for

*These questions are designed to help you build **fluency** and **confidence** with some of the most common techniques and methods of A-level Maths.*
One of the most common reasons students struggle to achieve a decent grade is that they get stuck on the early parts of a question and then can't access the big ticket items that follow on. If you often get stuck at the start of a question, or don't feel confident with the key skills required within a topic, you should find these skills checks particularly beneficial.

How to use them

You're welcome to make use of these questions any way you see fit, but they are designed to fit into a little-and-often revision schedule. Research suggests you'll be more effective working in 25 minute bursts, with 5 minute breaks in between. Each skills check is designed to be easily doable within this time.

*First, **choose 5 skills checks** to focus on. If in doubt, go with the first five topics.*

During a 25 minute revision block, try the following:

- *Grab the **first skills check** from your set of 5 to tackle.*
- *Choose your **difficulty level** (a: easy, b: moderate, c: hard).*
- ***Complete all the questions** for your chosen difficulty level. **Time yourself**.*
- ***Check your answers**, make a note of any errors or misconceptions and look up (or ask a friend or teacher about) anything that doesn't make sense. Pay particular attention to common slips, where your method is valid, but you make a daft numerical or algebraic error. What can you do to avoid them, or find and fix them before finishing a question?*
- *Jot down your **score** (1 point for a substantially correct method, 2 points for correct method and fully correct answer for each question) and the **total time**.*
- *Put your skills check sheet at the **bottom of the pile** of 5 to revisit later.*

After you've worked your way through all five sheets, you'll start to revisit them:

- *Whenever you revisit a skills check, choose a **harder difficulty level**. Once you can confidently do all three (a, b and c), focus on speed and efficiency - start looking for ways to save time without compromising your accuracy.*

*Once you've mastered a particular skills check to your satisfaction, **set that sheet aside** and **add a brand new one** to your pile of five.*

Important:

These skills checks alone won't improve your problem solving or mathematical modelling skills – they are the building blocks you need, but on their own they are not enough. Once you feel you have mastered a particular topic's key skills, find some proper A-level Maths exam questions and work on applying your skills to them.

Contents

Pure Skills Check 1: Quadratics

Record your results here:

Date	***Level (a/b/c)***	***Score (/10)***	***Time (mins)***

1.

a)	b)	c)
Factorise: $x^2 - 9x - 36$	Factorise: $6x^2 + 11x - 7$	Factorise: $4x^2 - 9y^2$

2.

a)	b)	c)
Complete the square: $x^2 - 7x + 20$	Complete the square: $4x^2 + 6x - 5$	Complete the square: $-2x^2 + 3x$

3.

a)	b)	c)
Determine the number of roots of: $x^2 - 3x + 12 = 0$	Find k for which this has exactly one root: $2x^2 + kx + 3 = 0$	Find all k for which this has distinct roots: $k^2x^2 + kx + 1 = x$

4.

a)	b)	c)
Find the equation of a quadratic curve with roots 3 and -2 which passes through $(0,3)$.	Find the equation of a quadratic curve with roots -3 and $\frac{1}{2}$ which passes through $(1,6)$.	Find the equation of a quadratic curve with maximum point $(2,5)$ which passes through $(0,13)$.

5.

a)	b)	c)
Giving your answer in set notation, solve: $(x - 5)(x - 2) > 0$	Giving your answer in set notation, solve: $2x^2 - 5x - 12 \leq 0$	Giving your answer in set notation, solve: $-3x^2 + 2x \leq -5$

Answers Pure Skills Check 1: Quadratics **Answers**

1.

a)	b)	c)
$(x-12)(x+3)$	$(3x+7)(2x-1)$	$(2x-3y)(2x+3y)$

2.

a)	b)	c)
$\left(x-\frac{7}{2}\right)^2+\frac{31}{4}$	$4\left(x+\frac{3}{4}\right)^2-\frac{29}{4}$	$-2\left(x-\frac{3}{4}\right)^2+\frac{9}{8}$

3.

a)	b)	c)
$b^2-4ac=-39<0$ $\therefore\ 0\ roots$	$b^2-4ac=0$ $\Longrightarrow\ k^2-24=0$ $\Longrightarrow\ k=\pm 2\sqrt{6}$	$b^2-4ac>0$ $(k-1)^2-4k^2>0$ $-3k^2-2k+1>0$ $CVs:\ k=\frac{1}{3}\ or\ -1$ negative quadratic, ∩-shape $\therefore\ -1<k<\frac{1}{3}$

4.

a)	b)	c)
$y=k(x-3)(x+2)$ $3=k(0-3)(0+2)$ $3=-6k\ \ \therefore\ \ k=-\frac{1}{2}$ $\boldsymbol{y=-\frac{1}{2}(x-3)(x+2)}$	$y=k(x+3)\left(x-\frac{1}{2}\right)$ $6=k(1+3)\left(1-\frac{1}{2}\right)$ $6=2k\ \ \therefore\ \ k=3$ $\boldsymbol{y=3(x+3)\left(x-\frac{1}{2}\right)}$	$y=k(x-2)^2+5$ $13=k(0-2)^2+5$ $13=4k+5\ \ \therefore\ \ k=2$ $\boldsymbol{y=2(x-2)^2+5}$

5.

a)	b)	c)
Critical values: $5, 2$ Positive quadratic, U-shape $\{x: x<2\}\cup\{x: x>5\}$	Critical values: $4, -\frac{3}{2}$ Positive quadratic, U-shape $\left\{x: -\frac{3}{2}\le x\le 4\right\}$	$-3x^2+2x+5\le 0$ Critical values: $\frac{5}{3}, -1$ Negative quadratic, ∩-shape $\{x: x\le -1\}\cup\left\{x: x\ge\frac{5}{3}\right\}$

Pure Skills Check 2: Graphs

Record your results here:

Date	***Level (a/b/c)***	***Score (/8)***	***Time (mins)***

1.

a)	b)	c)
Sketch the given curve, showing any points of intersection with the coordinate axes: $y = x(x+3)(x-2)$	Sketch the given curve, showing any points of intersection with the coordinate axes: $y = (x-4)^2(x+2)$	Sketch the given curve, showing any points of intersection with the coordinate axes: $y = (x-2)^2(x+1)^2$

2.

a)	b)	c)
Sketch the given curve, showing any points of intersection with the coordinate axes and indicating the position of any asymptotes: $y = 3 + \frac{5}{x}$	Sketch the given curve, showing any points of intersection with the coordinate axes and indicating the position of any asymptotes: $y = 2 + \frac{1}{x-4}$	Sketch the given curve, showing any points of intersection with the coordinate axes and indicating the position of any asymptotes: $y = \frac{3}{(x+1)^2}$

3.

a)	b)	c)
Sketch the given circle, showing the coordinates of the centre: $x^2 + y^2 = 16$	Sketch the given circle, showing the coordinates of the centre: $(x-2)^2 + y^2 = 8$	Sketch the given circle, showing the coordinates of the centre: $x^2 + 2x + y^2 - y = -3$

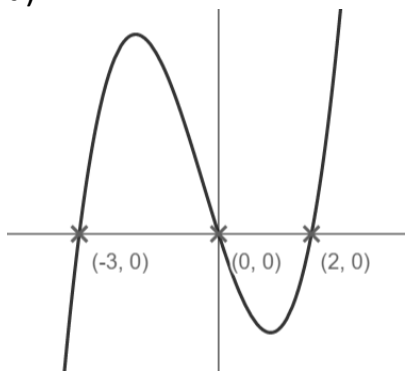
4.

a)	b)	c)
For $0 \leq x \leq 2\pi$, sketch: $y = \sin x$	For $0 \leq x \leq 2\pi$, sketch: $y = 2\cos(x) + 3$	For $0 \leq x \leq 2\pi$, sketch: $y = \tan\left(x + \frac{\pi}{4}\right)$

Answers Pure Skills Check 2: Graphs **Answers**

1.

a)



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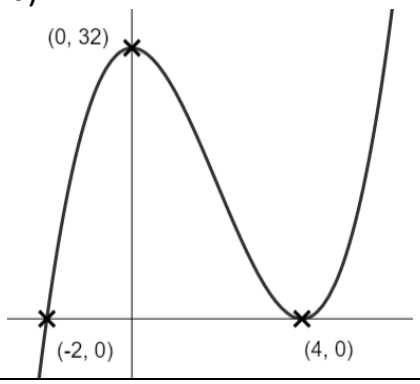

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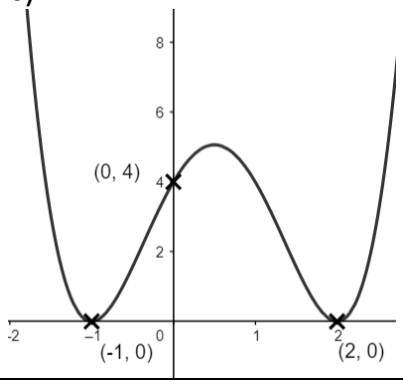

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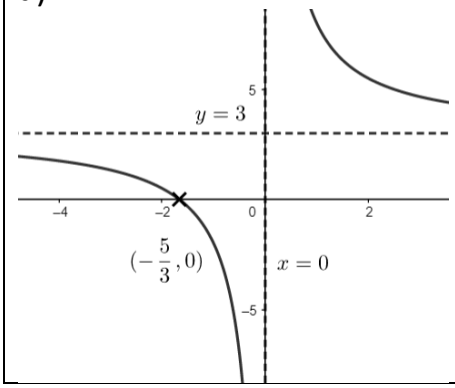
a)



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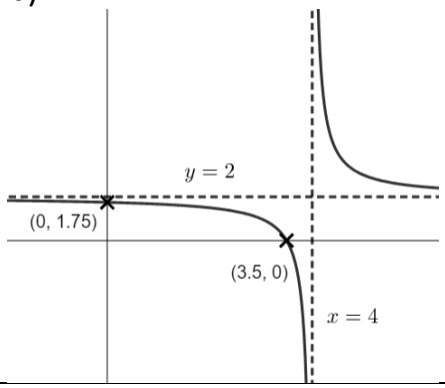

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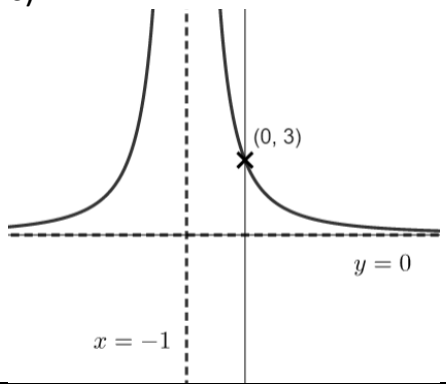
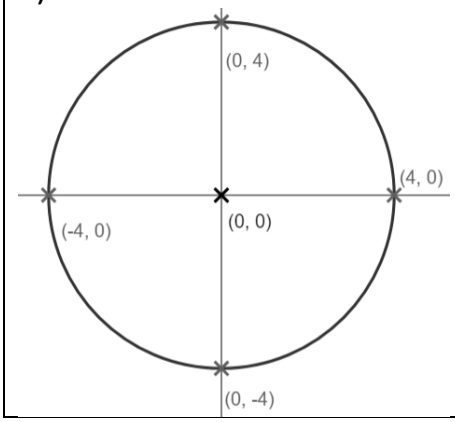


3.

a)



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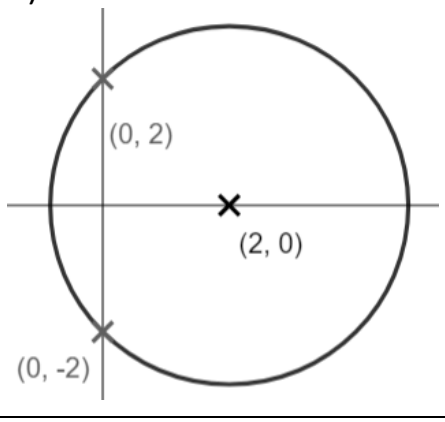
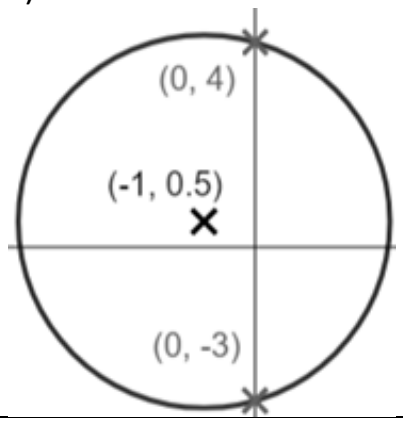
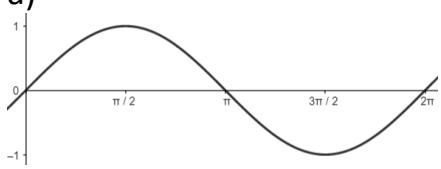


c)

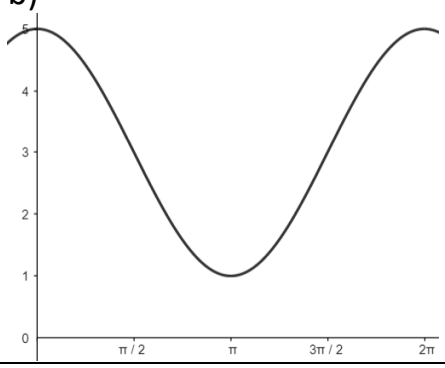


4.

a)



b)



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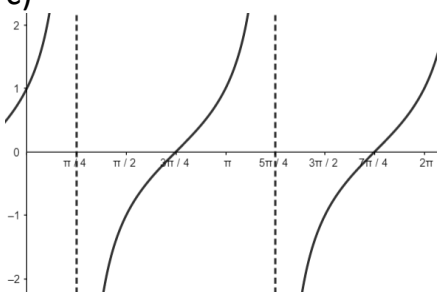


Pure Skills Check 3: Binomial Expansion

Record your results here:

Date	***Level (a/b/c)***	***Score (/10)***	***Time (mins)***

1.

a)	b)	c)
Expand fully: $(1+3x)^4$	Expand fully: $(5-2x)^3$	Find the coefficient of x^3 in the expansion of: $(1+x)^4(2-x)^5$

2. Find the first four non-zero terms of the expansion of:

a)	b)	c)
$(1+3x)^{-2}$	$(2-3x)^{\frac{3}{2}}$	$\frac{(2-3x)^2}{\sqrt{4-x}}$

3.

a)	b)	c)
For what values of x is this expansion valid: $(1+3x)^{-2}$	For what values of x is this expansion valid: $(2-3x)^{\frac{1}{5}}$	For what values of x is this expansion valid: $\frac{(2-3x)^2}{\sqrt{4-x}}$

4. Split the following into partial fractions:

a)	b)	c)
$\frac{15}{(x+3)(x-2)}$	$\frac{9}{(2x+1)(x-1)^2}$	$\frac{3x^2-4}{(x+1)(x+3)}$

5. For this question you may use the fact that $\sqrt{1+x} \approx 1+\frac{1}{2}x-\frac{1}{8}x^2$

a)	b)	c)
State a value of x that may be used to find an approximation for $\sqrt{\frac{3}{2}}$.	Use $x=0.25$ to find an approximation for $\sqrt{5}$. Give your answer to 3dp.	Use $x=0.08$ to find an approximation for $\sqrt{3}$. Give your answer to 3dp.

Answers Pure Skills Check 3: Binomial Expansion **Answers**

1.

a)	b)	c)
$1 + 12x + 54x^2 + 108x^3 + 81x^4$	$125 - 150x + 60x^2 - 8x^3$	-72

2.

a)	b)	c)
$1 - 6x + 27x^2 - 108x^3$	$2\sqrt{2} - \frac{9\sqrt{2}}{2}x + \frac{27\sqrt{2}x^2}{16} + \frac{27\sqrt{2}x^3}{64}$	$2 - \frac{23}{4}x + \frac{243}{64}x^2 + \frac{221}{512}x^3$

3.

a)	b)	c)
$\lvert x\rvert < \frac{1}{3}$	$\lvert x\rvert < \frac{2}{3}$	$\lvert x\rvert < 4$

4.

a)	b)	c)
$\frac{3}{(x-2)} - \frac{3}{(x+3)}$	$\frac{4}{2x+1} + \frac{3}{(x-1)^2} - \frac{2}{x-1}$	$3 - \frac{1}{2(x+1)} - \frac{23}{2(x+3)}$

5.

a)	b)	c)
$x = 0.5$	$LHS = \sqrt{1.25} = \sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{2}$ $RHS = 1.1328125$ $\therefore \frac{\sqrt{5}}{2} \approx 1.1328125$ $\sqrt{5} \approx 2.266\ to\ 3dp$	$LHS = \sqrt{1.08} = \sqrt{\frac{108}{100}} = \sqrt{\frac{27}{25}} = \frac{3\sqrt{3}}{5}$ $RHS = 1.0392$ $\therefore \frac{3\sqrt{3}}{5} \approx 1.0392$ $\therefore \sqrt{3} \approx 1.732\ to\ 3dp$

Pure Skills Check 4: Trigonometry 1

Record your results here:

Date	***Level (a/b/c)***	***Score (/10)***	***Time (mins)***

1. Triangle ABC has angle $A\hat{B}C = 90°$.

a)	b)	c)
If angle $B\hat{C}A$ is $30°$ and side AC is $10cm$ long, find the length of AB.	If angle $B\hat{C}A$ is $40°$ and side BC is $4cm$ long, find the length of AC.	If side AB is $4cm$ long and side AC is $7cm$ long, find angle $B\hat{C}A$.

2.

a)	b)	c)
Triangle ABC has angle $A\hat{B}C = 45°$. It has side $AC = 8cm$ and side $AB = 4cm$. Find the size of angle $B\hat{C}A$.	Triangle ABC has angle $A\hat{B}C = 60°$. It has side $AB = 5cm$ and side $BC = 6cm$. Find the length of side AC.	Find the exact area of a triangle with side lengths $7cm$, $8cm$ and $9cm$.

3. Find all values of x in the range $0 \leq x \leq 360°$ which satisfy the equation:

a)	b)	c)
$\sin x = 0.4$	$\sin(x - 70°) = -0.6$	$\sin(2x + 30°) = -0.5$

4. Find all values of x in the range $0 \leq x \leq 2\pi$ which satisfy the equation:

a)	b)	c)
$10\cos^2 x + \cos x = 3$	$2\cos^2 x + \sin x = 1$	$\tan^2 x + \sec x = 1$

5. Find the smallest value of x in the range $0 \leq x \leq 2\pi$ for which this function takes its maximum value:

a)	b)	c)
$y = 3\sin x$	$y = 4\,(\cos x + 1)^2$	$y = \dfrac{1}{\sin x - 3}$

Answers Pure Skills Check 4: Trigonometry 1 **Answers**

1.

a)

$$\sin 30 = \frac{x}{10}$$
$$\implies x = 10 \sin 30 = 5$$

b)

$$\cos 40 = \frac{4}{x}$$
$$\implies x = \frac{4}{\cos 40} \approx 5.22cm$$

c)

$$\sin\theta = \frac{4}{7}$$
$$\implies \theta = \sin^{-1}\left(\frac{4}{7}\right) \approx 34.8°$$

2.

a)

$$\frac{4}{\sin\theta} = \frac{8}{\sin 45}$$
$$\sin\theta = \frac{4\sin 45}{8}$$
$$\theta = \sin^{-1}\left(\frac{\sqrt{2}}{4}\right) = 20.7°$$

b)

$$x^2 = 5^2 + 6^2 - 2(5)(6)\cos 60$$
$$x = \sqrt{31}$$

c)

$$7^2 = 8^2 + 9^2 - 2(8)(9)\cos\theta$$
$$\cos\theta = \frac{49 - 64 - 81}{-144} = \frac{2}{3}$$
$$\sin\theta = \sqrt{1 - \cos^2\theta} = \frac{\sqrt{5}}{3}$$
$$\implies A = \frac{1}{2}(8)(9)\left(\frac{\sqrt{5}}{3}\right) = 12\sqrt{5}$$

3.

a)

$x = \sin^{-1}(0.4) = 23.6°$
&, by symmetry: $180 - 23.6$
$\therefore x = 23.6°, 156.4°$

b)

$-70° \le x - 70° \le 290°$
$x - 70° = \sin^{-1}(-0.6)$
$x - 70° = -36.9°$
&, by symmetry: $x - 70° = 216.9°$
$\therefore x - 70° = -36.9°, 216.9°$
$\therefore x = 33.1°, 286.9°$

c)

$30° \le 2x + 30° \le 750°$
$\sin 2x + 30° = -0.5$
$\implies 2x + 30° = -30°$
&, by symmetry: 210°
&, by period: 330°, 570°, 690°
$\therefore x = 90°, 150°, 270°, 330°$

4.

a)

$10\cos^2 x + \cos x - 3 = 0$
Let $C = \cos x$:
$10C^2 + C - 3 = 0$
Solving by calculator:
$C = \frac{1}{2}$ *or* $C = -\frac{3}{5}$
$\cos x = \frac{1}{2} \implies x = \frac{\pi}{3}, \frac{5\pi}{3}$
$\cos x = -\frac{3}{5} \implies x = 2.21, 4.07$
$\therefore x = \frac{\pi}{3}, 2.21, 4.07, \frac{5\pi}{3}$

b)

$2(1 - \sin^2 x) + \sin x = 1$
$-2\sin^2 x + \sin x + 1 = 0$
$\sin x = 1, \sin x = -0.5$
$\sin x = 1 \implies x = \frac{\pi}{2}$
$\sin x = -0.5 \implies x = -\frac{\pi}{6}$
and $\frac{7\pi}{6}$ and $\frac{11\pi}{6}$
$\therefore x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$

c)

$\tan^2 x + \sec x = 1$
$(\sec^2 x - 1) + \sec x = 1$
$\sec^2 x + \sec x - 2 = 0$
$\sec x = 1$ *or* $\sec x = -2$
$\cos x = 1$ *or* $\cos x = -\frac{1}{2}$
$\cos x = 1 \implies x = 0, 2\pi$
$\cos x = -\frac{1}{2} \implies x = \frac{2\pi}{3}, \frac{4\pi}{3}$
$\therefore x = 0, \frac{2\pi}{3}, \frac{4\pi}{3}, 2\pi$

5.

a)

Max is $3(1)$:
$3\sin x = 3 \implies \sin x = 1$
$x = \sin^{-1}(1) = \frac{\pi}{2}$
$\therefore$ *smallest in range is* $\frac{\pi}{2}$

b)

Max is $4(1+1)^2 = 0$
$\cos x = 1 \implies 0$ or 2π
$\therefore$ *smallest in range is* 0

c)

Max is $\frac{1}{-1-3} = -\frac{1}{4}$
$\sin x = -1 \implies x = -\frac{\pi}{2}$ or $\frac{3\pi}{2}$
$\therefore$ *smallest in range is* $\frac{3\pi}{2}$

Pure Skills Check 5: Differentiation 1

Record your results here:

Date	***Level (a/b/c)***	***Score (/10)***	***Time (mins)***

1. Find $\frac{dy}{dx}$ when:

a)	b)	c)
$y = 5x^2 - 4x^7$	$y = 3\sqrt{x} - \frac{7}{x^2}$	$y = \left(2 + \sqrt{x}\right)^2 - \frac{2+x}{x^3}$

2. Find all stationary points on the curve:

a)	b)	c)
$y = 5x^3 - 3x$	$y = x^2 + \frac{4}{x}$	$y = x^3 + 3x^2 + 3x + 1$

3. Determine the nature of each stationary point on the curve:

a)	b)	c)
$y = 5x^3 - 3x$	$y = x^2 + \frac{4}{x}$	$y = x^3 + 3x^2 + 3x + 1$

4. Find the equation of the ...

a)	b)	c)
… tangent to the curve $y = 3x^2 - 4x + 1$ where $x = 2$.	… normal to the curve $y = x^3 - 7x^2$ where $x = -2$.	… tangent to the curve $y = x^3 - x$ at each of its roots.

5. For which values of x is this curve *decreasing*?

a)	b)	c)
$y = 6x^3 + 7x - 2$	$y = 4x + \frac{2}{x}$	$y = 5 + \frac{3}{\sqrt{x}}$

Answers Pure Skills Check 5: Differentiation 1 **Answers**

1.

a)

$$\frac{dy}{dx} = 10x - 28x^6$$

b)

$$y = 3x^{\frac{1}{2}} - 7x^{-2}$$
$$\therefore \ \frac{dy}{dx} = \frac{3}{2}x^{-\frac{1}{2}} + 14x^{-3}$$

c)

$$y = 4 + 4x^{\frac{1}{2}} + x - 2x^{-3} - x^{-2}$$
$$\therefore \ \frac{dy}{dx} = 2x^{-\frac{1}{2}} + 1 + 6x^{-4} + 2x^{-3}$$

2.

$St\ Pt \implies \frac{dy}{dx} = 0$

a)

$$\frac{dy}{dx} = 15x^2 - 3$$
$$\therefore \ 15x^2 - 3 = 0 \implies x^2 = \frac{1}{5} \implies x = \pm\frac{\sqrt{5}}{5}$$
$$x = \frac{1}{\sqrt{5}} \implies y = 5\left(\frac{1}{\sqrt{5}}\right)^3 - 3\left(\frac{1}{\sqrt{5}}\right) = -\frac{2}{\sqrt{5}}$$
$$x = -\frac{1}{\sqrt{5}} \implies y = \frac{2}{\sqrt{5}}$$
$$\therefore \ \left(\pm\frac{\sqrt{5}}{5}, \mp\frac{2\sqrt{5}}{5}\right)$$

b)

$$y = x^2 + 4x^{-1}$$
$$\frac{dy}{dx} = 2x - 4x^{-2}$$
$$\therefore \ 2x^3 - 4 = 0$$
$$\therefore \ x = 2^{\frac{1}{3}} \implies y = 2^{\frac{2}{3}} + \frac{4}{2^{\frac{1}{3}}}$$
$$\therefore \ \left(2^{\frac{1}{3}}, 2^{\frac{2}{3}} + 2^{\frac{5}{3}}\right)$$

c)

$$\frac{dy}{dx} = 3x^2 + 6x + 3$$
$$\therefore \ 3x^2 + 6x + 3 = 0$$
$$\therefore \ x = -1$$
$$x = -1 \implies y = 0$$
$$\therefore \ (-1, 0)$$

3.

a)

$$\frac{d^2y}{dx^2} = 30x$$
$$\frac{d^2y}{dx^2}\Big|_{x=\frac{1}{\sqrt{5}}} > 0 \implies min$$
$$\frac{d^2y}{dx^2}\Big|_{x=-\frac{1}{\sqrt{5}}} < 0 \implies max$$

b)

$$\frac{d^2y}{dx^2} = 2 + 8x^{-3}$$
$$\frac{d^2y}{dx^2}\Big|_{x=2^{\frac{1}{3}}} > 0 \implies min$$

c)

$$\frac{d^2y}{dx^2} = 6x + 6$$
$$\frac{d^2y}{dx^2}\Big|_{x=-1} = 0$$

Inconclusive, so investigate either side:
$\frac{dy}{dx}$ is a positive quadratic with repeated roots, so +ve to +ve gradient:
$point\ of\ inflection$

4.

a)

$$\frac{dy}{dx} = 6x - 4$$
$$m = 6(2) - 4 = 8$$
$$x = 2 \implies y = 5$$
$$y - 5 = 8(x - 2)$$

b)

$$\frac{dy}{dx} = 3x^2 - 14x$$
$$m = 3(-2)^2 - 14(-2) = 40$$
$$m_{norm} = -\frac{1}{40}$$
$$x = -2 \implies y = -36$$
$$y + 36 = -\frac{1}{40}(x + 2)$$

c)

$$\frac{dy}{dx} = 3x^2 - 1$$

Roots: $x^3 - x = 0$
$$\implies x(x+1)(x-1) = 0$$
$$\implies x = -1, 0, 1$$
$$\frac{dy}{dx}\Big|_{x=-1} = 2 \implies y = 2(x+1)$$
$$\frac{dy}{dx}\Big|_{x=0} = -1 \implies y = -x$$
$$\frac{dy}{dx}\Big|_{x=1} = 2 \implies y = 2(x-1)$$

5.

a)

$$\frac{dy}{dx} = 18x^2 + 7$$
$$\frac{dy}{dx} \le 0 \implies x^2 \le -\frac{7}{18}$$
$$\therefore \ never\ since\ x^2 \ge 0$$

b)

$$\frac{dy}{dx} = 4 - 2x^{-2}$$
$$\frac{dy}{dx} \le 0 \implies 4 - \frac{2}{x^2} \le 0$$
$$4 \le \frac{2}{x^2} \implies x^2 \le \frac{1}{2}$$
$$\implies -\frac{1}{2} \le x \le \frac{1}{2}, x \ne 0$$

(as y is undefined at $x = 0$)

c)

$$\frac{dy}{dx} = -\frac{3}{2}x^{-\frac{3}{2}}$$
$$\frac{dy}{dx} \le 0 \implies -\frac{3}{2}x^{-\frac{3}{2}} \le 0$$
$$\implies \frac{1}{x^{\frac{3}{2}}} \ge 0 \implies x^{\frac{3}{2}} \ge 0$$
$$\implies x \ge 0$$

(& y is only defined for $x \ge 0$)

Pure Skills Check 6: Integration 1

Record your results here:

Date	***Level (a/b/c)***	***Score (/8)***	***Time (mins)***

1. Evaluate the following indefinite integral:

a)	b)	c)
$\int 3x + 7 - 2x^4 \, dx$	$\int 5\sqrt{x} - \frac{4}{x^2} \, dx$	$\int (2 + 3x)^2 \, dx$

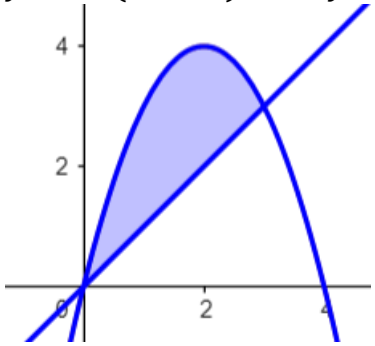
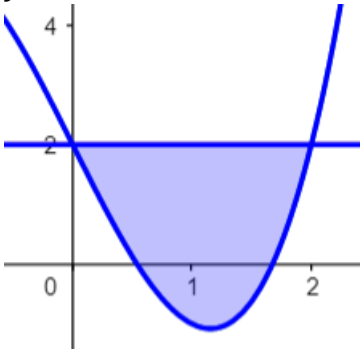
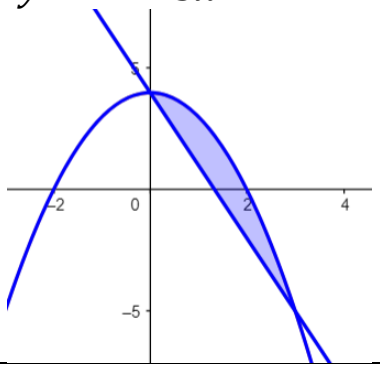
2. Evaluate the following definite integral (show your working, although you may use the calculator to verify your answer):

a)	b)	c)
$\int_2^4 4 + 3x^3 \, dx$	$\int_1^2 \frac{3x - x^3}{x} \, dx$	$\int_1^4 \frac{4x^{\frac{1}{2}} + x^6}{3x} \, dx$

3. Making use of a sketch, determine the total area enclosed by this curve, the x-axis and the limits given:

a)	b)	c)
$y = x^2 + 5x + 10$ from $x = 2$ to $x = 5$.	$y = x(1 + x)(1 - x)$ from $x = 1$ to $x = 2$.	$y = x(x^2 + 5x + 6)$ from $x = -3$ to $x = 0$.

4. Find the area of the shaded region between the two functions:

a)	b)	c)
$y = x(4 - x)$ and $y = x$	$y = x^3 - 4x + 2$ and $y = 2$	$y = 4 - x^2$ and $y = 4 - 3x$
		

Answers Pure Skills Check 6: Integration 1 **Answers**

1.

a)

$$I = \frac{3x^2}{2} + 7x - \frac{2x^5}{5} + C$$

b)

$$I = \frac{10}{3}x^{\frac{3}{2}} + 4x^{-1} + C$$

c)

$$I = \int 4 + 12x + 9x^2\, dx$$
$$= 4x + 6x^2 + 3x^3 + C$$

2.

a)

$$I = \left[4x + \frac{3x^4}{4}\right]_2^4 = 188$$

b)

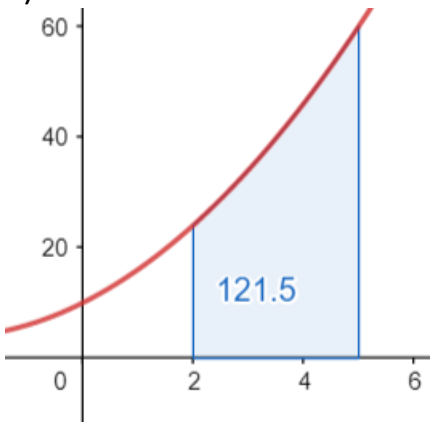
$$I = \left[3x - \frac{x^3}{3}\right]_1^2 = \frac{2}{3}$$

c)

$$I = \int_1^4 \frac{4}{3}x^{-\frac{1}{2}} + \frac{1}{3}x^5\, dx$$
$$= \left[\frac{8}{3}x^{\frac{1}{2}} + \frac{1}{18}x^6\right]_1^4 = \frac{1381}{6}$$

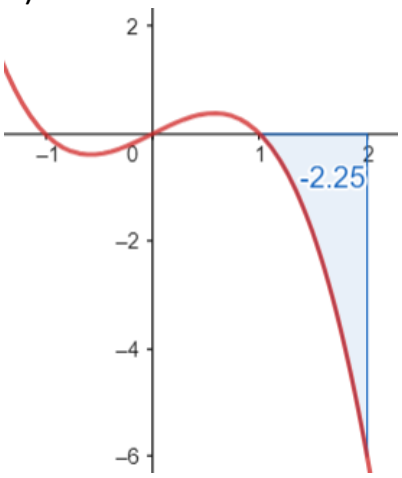
3.

a)



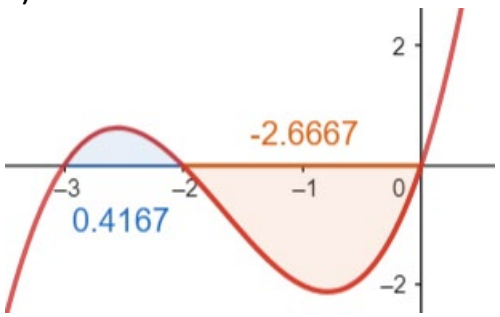
Area: 121.5

b)



Area: $|-2.25| = 2.25$

c)



$$Area = \left|\int_{-3}^{-2} y\, dx\right| + \left|\int_{-2}^{0} y\, dx\right|$$
$$= \left|\frac{5}{12}\right| + \left|-\frac{32}{12}\right| = \frac{37}{12}$$

4.

a)

Line crosses curve at:

$$x = x(4 - x)$$
$$x\big(1 - (4 - x)\big) = 0$$
$$x = 0 \;\&\; x = 3$$
$$Curve = \int_0^3 4x - x^2\, dx = 9$$
$$Line = \frac{1}{2}(3)(3) = \frac{9}{2}$$
$$Shaded\ Area = 9 - \frac{9}{2} = \frac{9}{2}$$

b)

Line crosses curve at:

$$2 = x^3 - 4x + 2$$
$$x^3 - 4x = 0$$
$$\Rightarrow\ x = 0, x = \pm 2$$
$$Line - Curve =$$
$$\int_0^2 2 - (x^3 - 4x + 2)\, dx$$
$$= \int_0^2 -x^3 + 4x\, dx = 4$$

c)

Line crosses curve at:

$$4 - x^2 = 4 - 3x$$
$$x^2 - 3x = 0$$
$$\Rightarrow\ x = 0 \;\&\; x = 3$$
$$Curve - Line =$$
$$\int_0^3 (4 - x^2) - (4 - 3x)\, dx$$
$$= \int_0^3 3x - x^2\, dx = \frac{9}{2}$$

Pure Skills Check 7: Exponentials and Logs

Record your results here:

Date	***Level (a/b/c)***	***Score (/10)***	***Time (mins)***

1. Sketch the following curve, indicating the location of any asymptotes, and the points of intersection with either coordinate axis:

a)	b)	c)
$y = 3^x$	$y = 0.8^x$	$y = 5 \times 2^{-x}$

2. Sketch the following curve, indicating the location of any asymptotes, and the points of intersection with either coordinate axis:

a)	b)	c)
$y = 3e^x + 4$	$y = 2 - e^x$	$y = 5 - 3e^{-2x}$

3. Simplify the following expression, giving your answer as a single logarithm:

a)	b)	c)
$\log_4 3 + \log_4 6$	$5 \log_4 3$	$5 \log_4 3 - 2 \log_4 6$

4. Solve the following equation, giving answers in exact form.

a)	b)	c)
$3^{2x} = 30$	$4^{2x} - 5(4^x) + 6 = 0$	$3^{x+2} = 4^x$

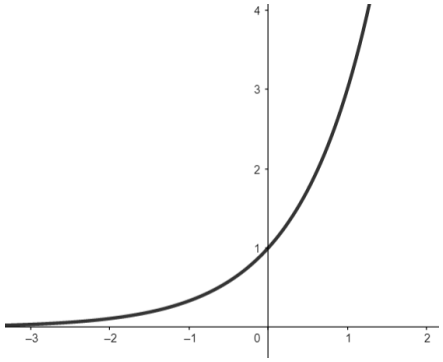
5. If $P = 40 \times e^{0.3t}$, find…

a)	b)	c)
… the value of P when $t = 10$.	… the value of t when $P = 400$.	… the values of t and P when $\frac{dP}{dt} = 100$.

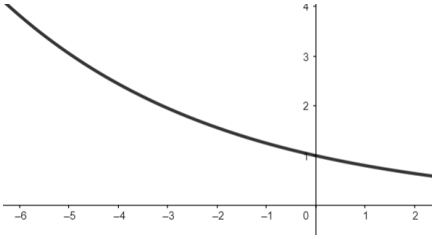
Answers Pure Skills Check 7: Exponentials and Logs **Answers**

1.

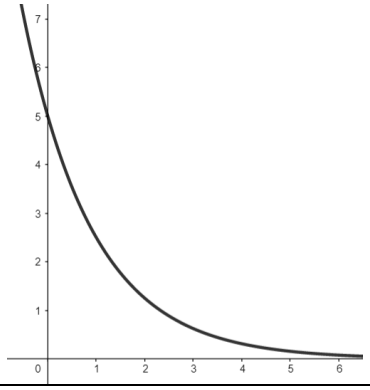
a)



b)

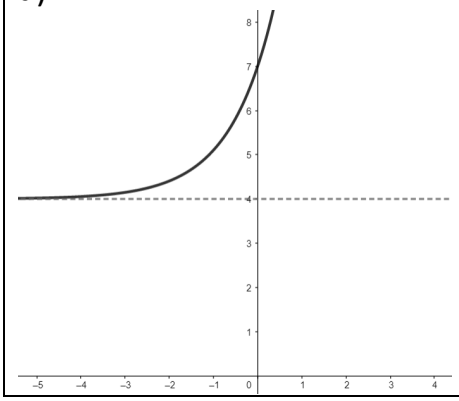


c)

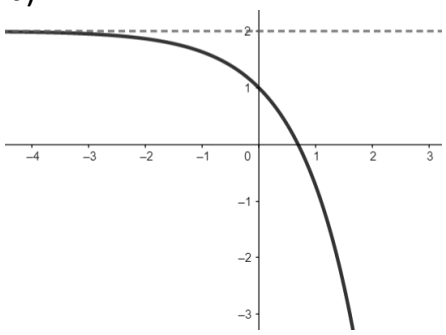


2.

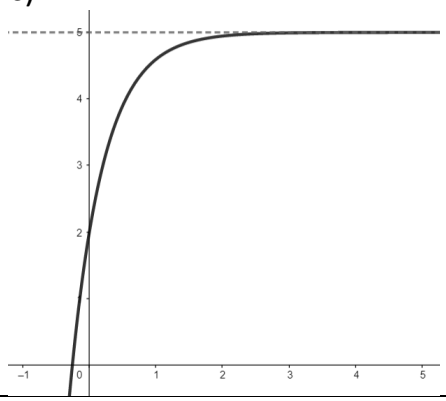
a)



b)



c)



3.

a) $\log_4 18$

b) $\log_4 3^5 = \log_4 243$

c) $\log_4 \frac{3^5}{6^2} = \log_4 \left(\frac{27}{4}\right)$

4.

a)

$2x = \log_3 30$

$x = \frac{1}{2}\log_3 30$

b)

$(4^x - 2)(4^x - 3) = 0$

$4^x = 2 \implies x = \log_4 2 = \frac{1}{2}$

Or $4^x = 3 \implies x = \log_4 3$

c)

$\ln(3^{x+2}) = \ln(4^x)$

$(x+2)\ln 3 = x \ln 4$

$x \ln 3 + 2 \ln 3 = x \ln 4$

$x(\ln 3 - \ln 4) = -2 \ln 3$

$x = \frac{2 \ln 3}{\ln 4 - \ln 3}$ *or* $\frac{2 \ln 3}{\ln \frac{4}{3}}$

5.

a)

$40 \times e^{0.3 \times 10} = 803$ *to* $3sf$

b)

$400 = 40 \times e^{0.3t}$

$e^{0.3t} = 10 \implies t = \frac{\ln 10}{0.3}$

$\therefore \ t = 7.68$ *to* $3sf$

c)

$\frac{dP}{dt} = 40 \times e^{0.3t} \times 0.3$

$100 = 12e^{0.3t} \implies t = 7.07$

and $P = 333$ *to* $3sf$

Pure Skills Check 8: Algebra and Functions

Record your results here:

Date	***Level (a/b/c)***	***Score (/10)***	***Time (mins)***

1.

a)	b)	c)
Divide $x^3 - 2x^2 + 4$ by $(x + 3)$.	Divide $3x^4 - 2x^3 + 5x^2 + 5x$ by $(3x + 1)$.	Divide $4x^4 - 1$ by $(x^2 + x)$.

2. Use the remainder theorem to find the remainder when…

a)	b)	c)
$2x^3 + 3x^2 + 4x + 5$ is divided by $(x - 2)$.	$x^4 - x^3 + x^2 - x + 1$ is divided by $(2x - 1)$.	$3x^4 + 4x^3 - x$ is divided by $(3x + 2)$.

3. State the range of the function:

a)	b)	c)
$f(x) = 2x^2,$ $-1 \leq x < 2$	$f(x) = (x - 3)^2 + 4,$ $x > 0$	$f(x) = 5 - (x + 1)^2,$ $-2 < x < 2$

4. If $f(x) = 4x + 1, x \in \mathbb{R}$ and $g(x) = \sqrt{x}, x \geq 0$, find an expression for the function below, stating the domain of your function.

a)	b)	c)
$f^{-1}(x)$	$gf(x)$	$ffg(x)$

5. Sketch the function given below, indicating the coordinates of any vertex and the points of intersection with the coordinate axes:

a)	b)	c)
$y = \|x\| + 2$	$y = 2\|x + 3\|$	$y = \|3x + 4\| - 5$

Answers Pure Skills Check 8: Algebra and Functions **Answers**

1.

a)

$$x^3 - 2x^2 + 4 = (x+3)(x^2 - 5x + 15) - 41$$

$$\therefore \frac{x^3 - 2x^2 + 4}{x+3} = x^2 - 5x + 15 - \frac{41}{x+3}$$

b)

$$3x^4 - 2x^3 + 5x^2 + 5x = (3x+1)(x^3 - x^2 + 2x + 1) - 1$$

$$\therefore \frac{3x^4 - 2x^3 + 5x^2 + 5x + 1}{3x+1} = x^3 - x^2 + 2x + 1 - \frac{1}{3x+1}$$

c)

$$4x^4 - 1 = (x^2 + x)(4x^2 - 4x + 4) - 4x - 1$$

$$\therefore \frac{4x^4 - 1}{x^2 + x} = 4x^2 - 4x + 4 - \frac{4x+1}{x^2+x}$$

2.

a)

$$R = f(2) = 2(2)^3 + 3(2)^2 + 4(2) + 5 = 41$$

b)

$$R = f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^4 - \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right) + 1 = \frac{11}{16}$$

c)

$$R = f\left(-\frac{2}{3}\right) = 3\left(-\frac{2}{3}\right)^4 + 4\left(-\frac{2}{3}\right)^3 - \left(-\frac{2}{3}\right) = \frac{2}{27}$$

3.

a)

Ends: $f(-1) = 2$, $f(2) = 8$

Extrema: $f(0) = 0$ (min)

$\therefore Range: \ 0 \leq x < 8$

b)

Ends: $f(0) = 13$, $f(k) \to \infty \ as \ k \to \infty$

Extrema: $f(3) = 4$ (min)

$\therefore Range: \ x \geq 4$

c)

Ends: $f(-2) = 4$, $f(2) = -4$

Extrema: $f(-1) = 5$ (max)

$\therefore Range: \ -4 < x \leq 5$

4.

a)

$$y = 4x + 1 \implies x = \frac{y-1}{4}$$

$$y \leftrightarrow x \ gives \ y = \frac{x-1}{4}$$

$$\therefore \ f^{-1}(x) = \frac{x-1}{4}$$

$$x \in \mathbb{R}$$

b)

$$gf(x) = g[f(x)] = g[4x+1]$$

$$\therefore \ gf(x) = \sqrt{4x+1}$$

$$4x + 1 \geq 0 \implies x \geq -\frac{1}{4}$$

c)

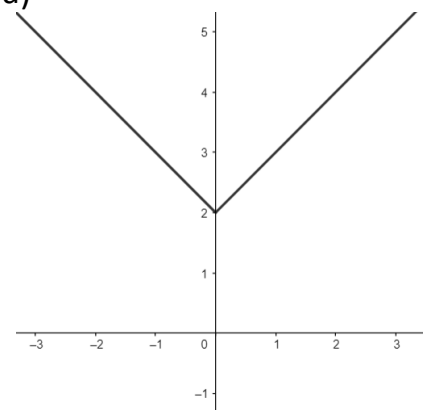
$$ffg(x) = 4\left(4\sqrt{x} + 1\right) + 1$$

$$ffg(x) = 16\sqrt{x} + 5$$

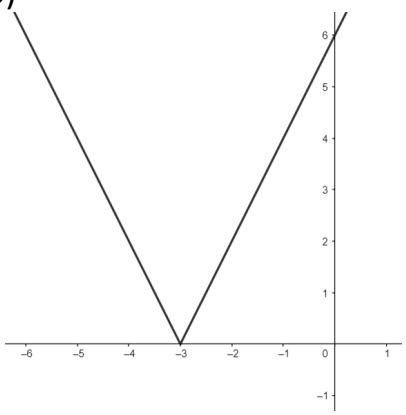
$$x \geq 0$$

5.

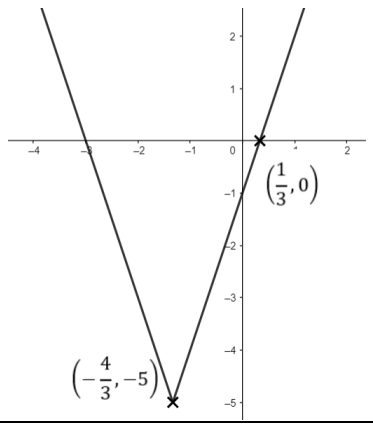
a)



b)



c)



Pure Skills Check 9: Sequences and Series

Record your results here:

Date	***Level (a/b/c)***	***Score (/10)***	***Time (mins)***

1. An arithmetic sequence has first term 4 and common difference 2. Find:

a)	b)	c)
The fifth term in the sequence, U_5.	The sum of the first ten terms in the sequence, S_{10}.	The sum of terms 8 to 16 inclusive: $\sum_{r=8}^{16} U_r$.

2. A geometric series has first term 8 and common ratio 1.4. Find, to 3dp:

a)	b)	c)
The fifth term in the sequence, U_5.	The sum of the first ten terms in the sequence, S_7.	The sum of terms 8 to 16 inclusive: $\sum_{r=4}^{9} U_r$.

3. A geometric series has first term 10 and common ratio 0.8.

a)	b)	c)
Find S_4, the sum of the first four terms.	Find the smallest integer k for which $S_k > 49$.	Find $\sum_{r=4}^{\infty} U_r$.

4.

a)	b)	c)
If $U_{n+1} = 2U_n + 4$, and $U_1 = 4$, find U_4.	If $U_{n+1} = kU_n + 3$, where k is an integer, $U_1 = 5$ and $U_3 = 201$, find the value of k.	If $U_{n+1} = 0.6U_n + 7$, and $U_1 = 12$, find the limit of U_n as $n \to \infty$.

5. The sequence starting $4, 1, 7, 4, 1, 7, 4, \ldots$ is periodic with order 3.

a)	b)	c)
Find the 50^{th} term, U_{50}.	Find the sum of the first 100 terms, S_{100}.	Find the value of: $\sum_{r=20}^{40} U_r$

Answers Pure Skills Check 9: Sequences and Series **Answers**

1.

a)	b)	c)
$U_n = a + (n-1)d$ $U_5 = 4 + (4)2 = 12$	$S_n = \frac{n}{2}(2a + (n-1)d)$ $S_{10} = \frac{10}{2}\big(2(4) + (9)(2)\big)$ $S_{10} = 130$	$\sum_{r=8}^{16} U_r = S_{16} - S_7$ $S_{16} = \frac{16}{2}\big(8 + 15(2)\big) = 304$ $S_7 = \frac{7}{2}\big(8 + 6(2)\big) = 70$ $S_{16} - S_7 = 304 - 70 = 234$

2.

a)	b)	c)
$U_n = ar^{n-1}$ $U_5 = 8 \times 1.4^4 = \frac{19208}{625}$ $= 30.733\ to\ 3dp$	$S_n = \frac{a(1-r^n)}{1-r}$ $S_7 = \frac{8(1-1.4^7)}{1-1.4}$ $= 190.827\ to\ 3dp$	$\sum_{r=4}^{9} U_r = S_9 - S_3$ $= 393.220 \ldots - 34.88$ $= 358.341\ to\ 3dp$

3.

a)	b)	c)
$S_n = \frac{a(1-r^n)}{1-r}$ $S_4 = \frac{10(1-0.8^4)}{1-0.8} = 29.52$	$\frac{10(1-0.8^k)}{1-0.8} = 49$ $0.8^k = \frac{1}{50}$ $k = \log_{0.8}\frac{1}{50} = 17.53 \ldots$ $\therefore\ smallest\ k = 18$	$\sum_{r=4}^{\infty} U_r = 10 \times 0.8^3 +$ $10 \times 0.8^4 + 10 \times 0.8^5 + \cdots$ $S_\infty = \frac{(10 \times 0.8^3)}{1-0.8} = 25.6$

4.

a)	b)	c)
$U_2 = 2(4) + 4 = 12$ $U_3 = 2(12) + 4 = 28$ $U_4 = 2(28) + 4 = 60$	$U_2 = 5k + 3$ $U_3 = 201 = k(5k+3) + 3$ $5k^2 + 3k - 198 = 0$ $k = 6\ or\ -6.6 \implies k = 6$	$U_n \to L\ and\ U_{n+1} \to L$ $L = 0.6L + 7$ $0.4L = 7 \implies L = 17.5$

5.

a)	b)	c)
Period 3, so: $U_1 = U_4 = \cdots = U_{49}$ $U_{49} = 4 \implies U_{50} = 1$	$S_3 = 12$ $\implies S_{99} = 33(12) = 396$ $\implies S_{100} = S_{99} + 4 = 400$	$\sum_{r=20}^{40} U_r = U_{20} + \cdots + U_{40}$ $20 = 3(6) + 2 \implies U_{20} = U_2 = 1$ $40 = 3(13) + 1 \implies U_{40} = U_1 = 4$ $1 + 7 + 4 + \cdots + 4$ $= (1 + 7 + 4) \times 7 = 84$

Pure Skills Check 10: Trigonometry 2

Record your results here:

Date	***Level (a/b/c)***	***Score (/10)***	***Time (mins)***

1. Indicating any asymptotes, and all points of intersection with the coordinate axes, sketch the following:

a)	b)	c)
$y = \sin x$ and, on the same set of axes, $y = \cos x$, both for $-\pi \leq x \leq 2\pi$.	$y = \sec x$ and, on a separate set of axes, $y = \operatorname{cosec} x$, both for $-\pi \leq x \leq 2\pi$.	$y = \tan x$ and, on the same set of axes, $y = \cot x$, both for $-\pi \leq x \leq 2\pi$.

2. Sketch the function, indicating clearly the location of any end points, points of intersection with the coordinate axes, and any asymptotes.

a)	b)	c)
$y = \arcsin x$ $-1 \leq x \leq 1$	$y = \arccos x$ $-1 \leq x \leq 1$	$y = \arctan x$ $x \in \mathbb{R}$

3. For a circle with radius $8cm$, find:

a)	b)	c)
The area of a sector with angle $1.5\ rad$.	The perimeter of a sector with angle $1.5\ rad$.	The perimeter of the segment formed with angle $1.5\ rad$, to 3sf.

4. Solve, giving solutions either in exact form or to 2dp in the range $0 \leq \theta \leq 2\pi$:

a)	b)	c)
$\sec x = 4$	$1 + 2\operatorname{cosec} 3x = 4$	$2\operatorname{cosec} x - 3 = 5\sin x$

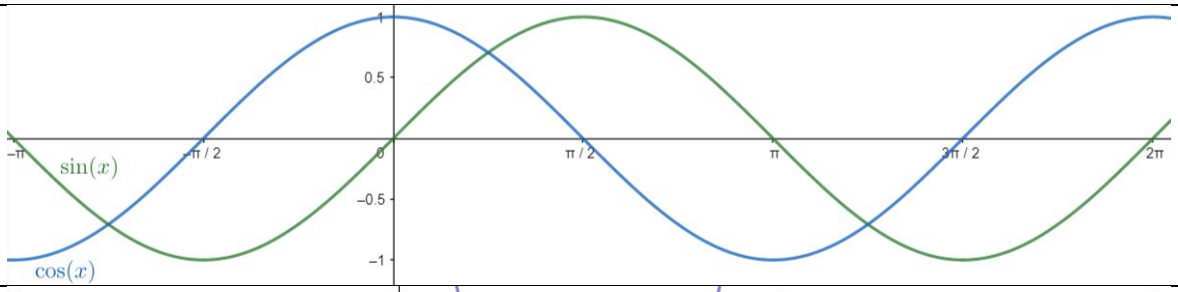
5. Write down the standard identity linking these two trigonometric functions:

a)	b)	c)
$\sin\theta$ and $\cos\theta$	$\tan\theta$ and $\sec\theta$	$\cot\theta$ and $\operatorname{cosec}\theta$

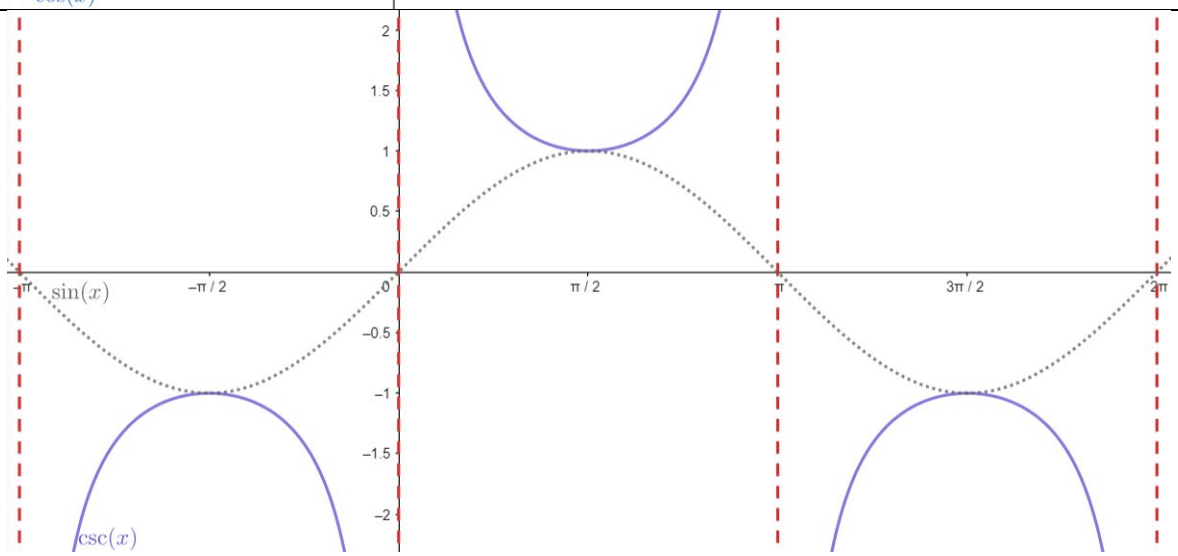
Answers Pure Skills Check 10: Trigonometry 2 **Answers**

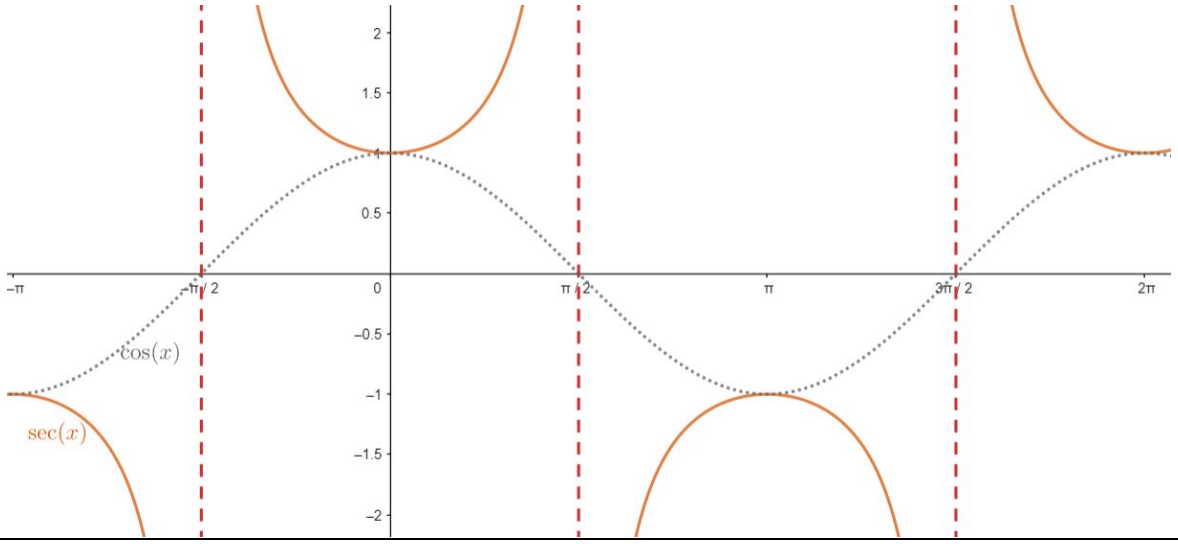
1.

a)

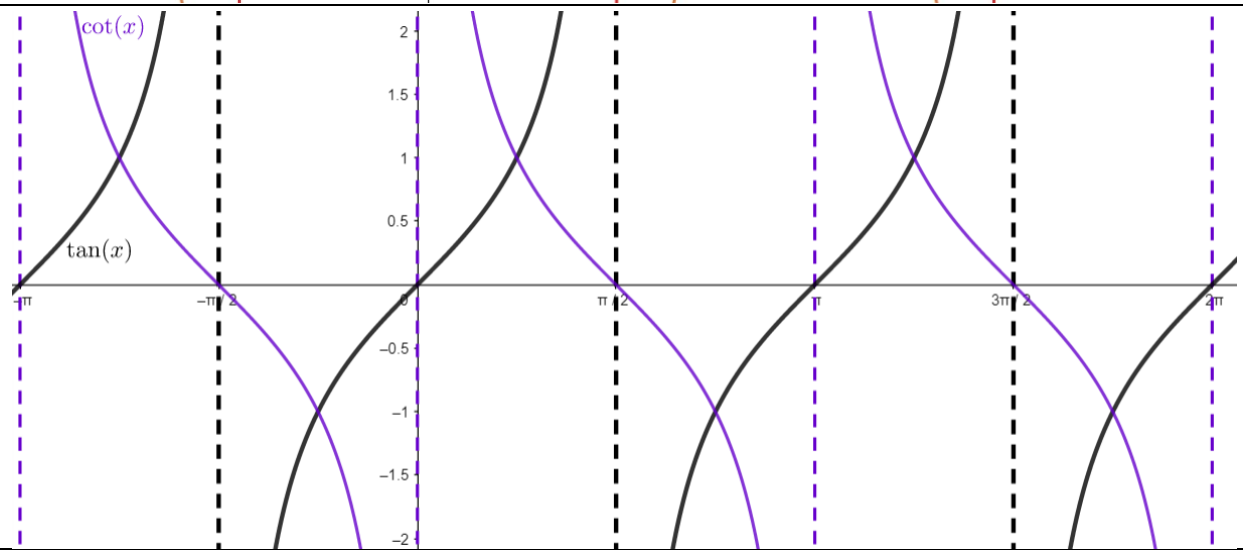


b)



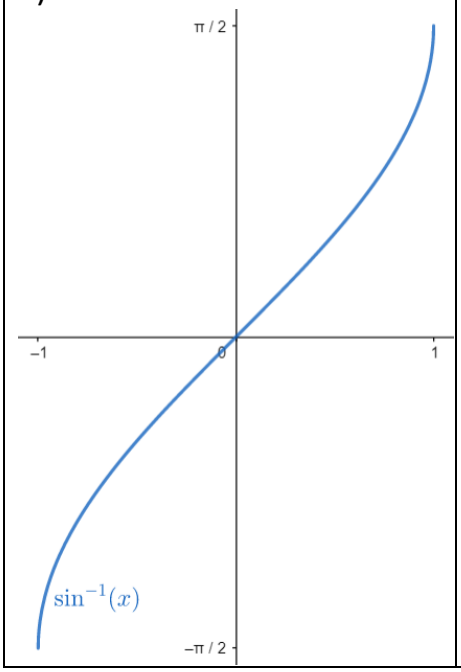


c)

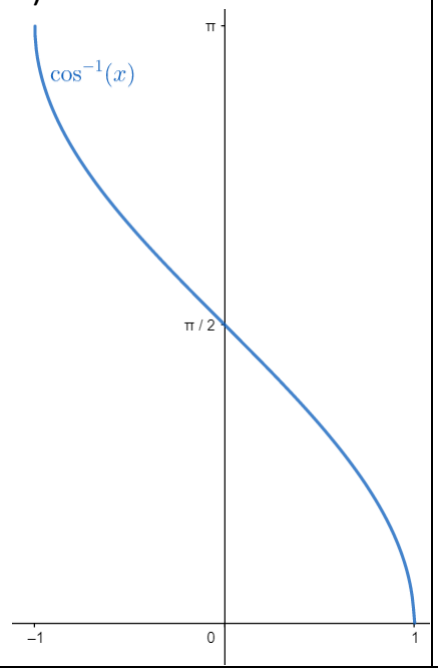


2.

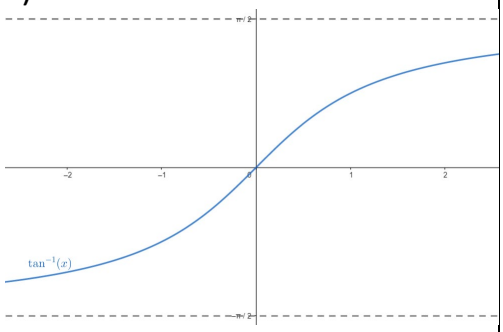
a)



b)



c)



3.

a)

$$A = \frac{1}{2}r^2\theta = \frac{1}{2}(8)^2(1.5)$$
$$= 48cm^2$$

b)

$$l = r\theta = 8 \times 1.5 = 12cm$$
$$P = l + 2r = 28cm$$

c)

$$l = r\theta = 8 \times 1.5 = 12cm$$

Chord length: (cosine rule):

$$a^2 = 8^2 + 8^2 - 2(8)^2 \cos 1.5$$
$$\Rightarrow \quad a = 10.906 \ldots$$
$$\therefore \quad P = 22.9cm$$

4.

a)

$$\cos x = \frac{1}{4}$$
$$x = \text{acos}\left(\frac{1}{4}\right) = 1.318 \ldots$$
$$or\ x = 2\pi - 1.318 \ldots$$
$$\therefore \quad x = 1.32\ or\ 4.97$$

b)

$$0 \leq 3x \leq 6\pi \approx 18.8$$
$$\text{cosec}\, 3x = \frac{3}{2}$$
$$\sin 3x = \frac{2}{3}$$
$$3x = \text{asin}\left(\frac{2}{3}\right) = 0.729 \ldots$$
$$or\ x = \pi - 0.729 \ldots$$

Adding 2π repeatedly gives:

$$x = 0.73, 2.41, 7.01,$$
$$8.70, 13.30, 15.98$$

c)

$$\frac{2}{\sin x} - 3 = 5 \sin x$$
$$2 - 3 \sin x = 5 \sin^2 x$$
$$5 \sin^2 x + 3 \sin x - 2 = 0$$
$$\sin x = \frac{2}{5} \ \ or \ \ \sin x = -1$$
$$\sin x = \frac{2}{5} \quad \Rightarrow \quad x = 0.41, 2.73$$
$$\sin x = -1 \quad \Rightarrow \quad -\frac{\pi}{2}, \frac{3\pi}{2}$$
$$\therefore x = 0.41, 2.73, \frac{3\pi}{2}$$

5.

a)

$$\sin^2\theta + \cos^2\theta \equiv 1$$

b)

$$\tan^2\theta + 1 \equiv \sec^2\theta$$

c)

$$1 + \cot^2\theta \equiv \text{cosec}^2\theta$$

Pure Skills Check 11: Trigonometry 3

Record your results here:

Date	***Level (a/b/c)***	***Score (/10)***	***Time (mins)***

1. With reference to the formula book if needed, give an identity involving:

a)	b)	c)
$\sin(2\theta)$ in terms of $\sin\theta$ and $\cos\theta$.	$\tan(2\theta)$ in terms of $\tan\theta$.	$\cos(2\theta)$ in terms of $\sin\theta$ and $\cos\theta$, and hence in terms of only $\cos\theta$.

2. Use the standard trigonometry results for $30°$, $45°$ and $60°$, find exact values of the trigonometric expression given. Show your working.

a)	b)	c)
$\sin(105)$	$\cos(15)$	$\tan(75)$

3. Write down the range of this trigonometric function:

a)	b)	c)
$f(x) = 8\sin x$ $x \in \mathbb{R}$	$f(x) = 4 - 3\cos x$ $x \in \mathbb{R}$	$f(x) = \dfrac{2}{5 + \sin 2x}$ $x \in \mathbb{R}$

4. Write down the maximum value of this trigonometric expression:

a)	b)	c)
$3\sin x + 4\cos x$	$5\cos x - 12\sin x$	$4\sin x - \cos x$

5. Rewrite each expression in the form required, stating the exact value of R and giving α in radians to $3sf$.

a)	b)	c)
Write $3\sin x + 4\cos x$ in the form: $R\cos(x - \alpha)$	Write $5\cos x - 12\sin x$ in the form: $R\cos(x + \alpha)$	Write $4\sin x - \cos x$ in the form: $R\sin(x - \alpha)$

Answers Pure Skills Check 11: Trigonometry 3 **Answers**

1.

a)	b)	c)
$\sin 2\theta \equiv 2\sin\theta\cos\theta$	$\tan 2\theta \equiv \frac{2\tan\theta}{1-\tan^2\theta}$	$\cos 2\theta \equiv \cos^2\theta - \sin^2\theta$ Or: $\cos 2\theta \equiv 2\cos^2\theta - 1$

2.

a)	b)	c)
$\sin(60+45)$ $= \sin 60\cos 45 + \cos 60\sin 45$ $= \frac{\sqrt{3}}{2}\left(\frac{\sqrt{2}}{2}\right) + \frac{1}{2}\left(\frac{\sqrt{2}}{2}\right)$ $= \frac{\sqrt{6}+\sqrt{2}}{4}$	$\cos(60-45)$ $= \cos 60\cos 45 + \sin 60\sin 45$ $= \frac{1}{2}\left(\frac{\sqrt{2}}{2}\right) + \frac{\sqrt{3}}{2}\left(\frac{\sqrt{2}}{2}\right)$ $= \frac{\sqrt{2}+\sqrt{6}}{4}$	$\tan(30+45)$ $= \frac{\tan 30 + \tan 45}{1-\tan 30\tan 45}$ $= \frac{\frac{\sqrt{3}}{3}+1}{1-\frac{\sqrt{3}}{3}(1)}$ $= \frac{\sqrt{3}+3}{3-\sqrt{3}}$ $= 2+\sqrt{3}$

3.

a)	b)	c)
$-8 \le f(x) \le 8$	$1 \le x \le 7$	$Extremes: \frac{2}{5+1}\ and\ \frac{2}{5-1}$ $\therefore \frac{1}{3} \le f(x) \le \frac{1}{2}$

4.

a)	b)	c)
$R = \sqrt{3^2+4^2} = 5$	$R = \sqrt{5^2+(-12)^2} = 13$	$R = \sqrt{4^2+(-1)^2} = \sqrt{17}$

5.

a)	b)	c)
$R = 5$ $3\sin x + 4\cos x$ $= 5(\cos x\cos\alpha + \sin x\sin\alpha)$ $3 = 5\sin\alpha\ \&\ 4 = 5\cos\alpha$ $\therefore \alpha = 0.644$ $\therefore 5\cos(x-0.644)$	$R = 13$ $5\cos x - 12\sin x$ $= 13(\cos x\cos\alpha - \sin x\sin\alpha)$ $5 = 13\cos\alpha\ \&12 = 13\sin\alpha$ $\therefore\ \alpha = 1.18$ $\therefore 13\cos(x+1.18)$	$R = \sqrt{17}$ $4\sin x - \cos x$ $= \sqrt{17}(\sin x\sin\alpha - \cos x\cos\alpha)$ $4 = \sqrt{17}\sin\alpha\ \&\ 1 = \sqrt{17}\cos\alpha$ $\therefore \alpha = 1.33$ $\therefore \sqrt{17}\sin(x-1.33)$

Pure Skills Check 12: Parametrics

Record your results here:

Date	***Level (a/b/c)***	***Score (/10)***	***Time (mins)***

1. Make t the subject of the equation.

a)	b)	c)
$x = 4e^{2t+1}$	$x = 3\ln(2-t)$	$x = \dfrac{2t+3}{1+t}$

2. Find the Cartesian circle equation described by the parametric equations:

a)	b)	c)
$x = \cos t$ $y = \sin t$	$x = 2 + \cos t$ $y = \sin t - 5$	$x = 5\cos t + 2$ $y = 3 - 5\sin t$

3. Use trigonometric identities, including the double angle identities, to convert the parametric equations into a Cartesian equation:

a)	b)	c)
$x = \sin t$ $y = \cos 2t$	$x = \sin 2t$ $y = 3\cos t$	$x = 1 + \tan t$ $y = 2 + \cos t$

4. State the domain and range of the curve defined parametrically by:

a)	b)	c)
$x = t^2 - 1$ $y = 2 + t$ for $0 \leq t \leq 2$	$x = \cos t$ $y = 1 - \sin t$ for $0 \leq t \leq \pi$	$x = 2 + 3\sin t$ $y = 3 - 2\cos t$ for $t \in \mathbb{R}$

5. Sketch the curve defined parametrically by the equations given, indicating the coordinates of any points of intersection with the coordinate axes. :

a)	b)	c)
$x = t^2 - 1$ $y = 2 + t$ for $0 \leq t \leq 2$	$x = \cos t$ $y = 1 - \sin t$ for $0 \leq t \leq \pi$	$x = 2 + 3\sin t$ $y = 3 - 2\cos t$ for $t \in \mathbb{R}$

Answers Pure Skills Check 12: Parametrics **Answers**

1.

a)

$$\frac{x}{4} = e^{2t+1}$$
$$\ln\left(\frac{x}{4}\right) = 2t + 1$$
$$t = \frac{1}{2}\left(\ln\left(\frac{x}{4}\right) - 1\right)$$

b)

$$\frac{x}{3} = \ln(2 - t)$$
$$e^{\frac{x}{3}} = 2 - t$$
$$t = 2 - e^{\frac{x}{3}}$$

c)

$$x(1 + t) = 2t + 3$$
$$x + xt = 2t + 3$$
$$t(x - 2) = 3 - x$$
$$t = \frac{3 - x}{x - 2}$$

2.

a)

$$x^2 = \cos^2 t$$
$$y^2 = \sin^2 t$$
$$\therefore x^2 + y^2 = 1$$

b)

$$(x - 2)^2 = \cos^2 t$$
$$(y + 5)^2 = \sin^2 t$$
$$\therefore (x - 2)^2 + (y + 5)^2 = 1$$

c)

$$\frac{x - 2}{5} = \cos t \quad \& \quad \frac{3 - y}{5} = \sin t$$
$$\therefore \frac{(x - 2)^2}{25} + \frac{(3 - y)^2}{25} = 1$$
$$(x - 2)^2 + (y - 3)^2 = 25$$

3.

a)

$$y = 1 - 2\sin^2 t$$
$$\therefore y = 1 - 2x^2$$

b)

$$x = 2\sin t \cos t$$
$$x^2 = 2(1 - \cos^2 t)\cos^2 t$$
$$\cos^2 t = \left(\frac{y}{3}\right)^2$$
$$\therefore\ x^2 = 2\left(1 - \left(\frac{y}{3}\right)^2\right)\left(\frac{y}{3}\right)^2$$
$$81x^2 = 2y^2(9 - y^2)$$

c)

$$\tan^2 t = x - 1$$
$$\sec^2 t = \frac{1}{y - 2}$$
$$\therefore (x - 1) + 1 = \frac{1}{y - 2}$$
$$x = \frac{1}{y - 2}$$

4.

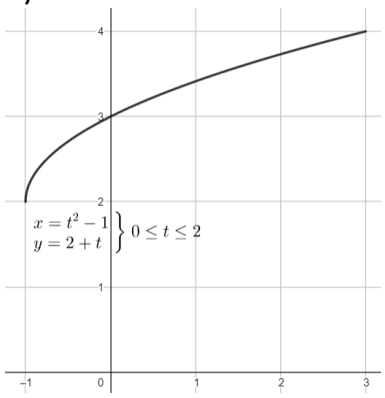
a)
Domain: $-1 \le x \le 3$
Range: $2 \le y \le 4$

b)
Domain: $-1 \le x \le 1$
Range: $0 \le y \le 1$

c)
Domain: $-1 \le x \le 5$
Range: $1 \le y \le 5$

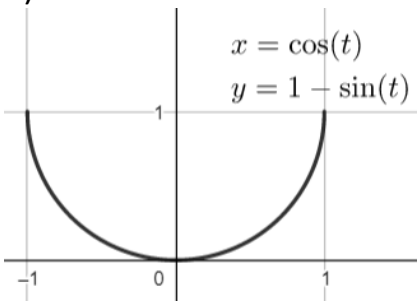
5.

a)



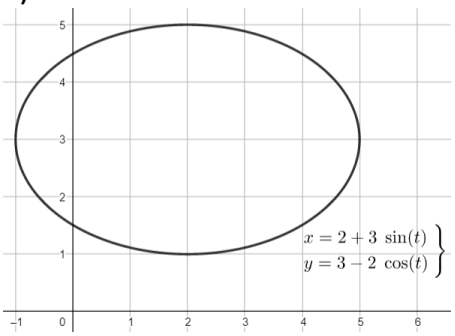
Intercept: $(0,1)$

b)



Intercept: $(0,0)$

c)



Intercepts: $x = 0 \Longrightarrow$

$$t = \text{asin}\left(-\frac{2}{3}\right) \Longrightarrow \ y = 1.51$$

$$t = \pi - \text{asin}\left(-\frac{2}{3}\right) \Longrightarrow \ y = 4.49$$

Pure Skills Check 13: Differentiation 2

Record your results here:

Date	***Level (a/b/c)***	***Score (/10)***	***Time (mins)***

1. Find the derivative of the function…

a)	b)	c)
$f(x) = e^x + 5x^2$	$f(x) = e^{2x} - 5$	$f(x) = e^{3-x}$

2. Find the derivative of the function…

a)	b)	c)
$y = 5^x$	$y = 2(3^x) + 4$	$y = 7^{4x}$

3. Find the derivative of the function…

a)	b)	c)
$f(x) = \ln x$	$f(x) = 3\ln(4x)$	$f(x) = 2\ln(3x + 4)$

4. Find the derivative of the function…

a)	b)	c)
$y = \sin x$	$y = \cos(3x)$	$y = 2\sin x - 3\cos 4x$

5. Find the derivative of the function…

a)	b)	c)
$f(x) = e^{2x} + \sin 3x$	$f(x) = \ln 3x - 4(5^x)$	$f(x) = \ln\left(4(2^x)\right)$

Answers Pure Skills Check 13: Differentiation 2 **Answers**

1.

a)	b)	c)
$f'(x) = e^x + 10x$	$f'(x) = 2e^{2x}$	$f'(x) = -e^{3-x}$

2.

a)	b)	c)
$\frac{dy}{dx} = 5^x \ln 5$	$\frac{dy}{dx} = 2(3^x \ln 3)$	$\frac{dy}{dx} = 7^{4x}(4 \ln 7)$

3.

a)	b)	c)
$f'(x) = \frac{1}{x}$	$f'(x) = \frac{12}{4x}$	$f'(x) = \frac{8}{3x+4}$

4.

a)	b)	c)
$\frac{dy}{dx} = \cos x$	$\frac{dy}{dx} = -3 \sin 3x$	$\frac{dy}{dx} = 2 \cos x + 12 \sin 4x$

5.

a)	b)	c)
$f'(x) = 2e^{2x} + 3 \cos 3x$	$f'(x) = \frac{1}{x} - 4(5^x \ln 5)$	$f(x) = \ln 4 + \ln 2^x$ $\Rightarrow f(x) = \ln 4 + x \ln 2$ $\Rightarrow f'(x) = \ln 2$

Pure Skills Check 14: Differentiation 3

Record your results here:

Date	***Level (a/b/c)***	***Score (/10)***	***Time (mins)***

1. Use the chain rule to differentiate…

a)	b)	c)
$y = e^{3x^2+5}$	$y = \cos(2e^x)$	$y = \ln(\sin(2x))$

2. Use the product rule to differentiate…

a)	b)	c)
$f(x) = e^x \cos x$	$f(x) = x^2 \sin x$	$f(x) = \cos(3x)\sin(2x)$

3. Use the quotient rule to differentiate…

a)	b)	c)
$y = \dfrac{x^2}{\sin x}$	$y = \dfrac{e^x}{1 - \ln x}$	$y = \tan x$

4. Differentiate both sides of the given equation implicitly with respect to x:

a)	b)	c)
$x^2 + 2y^2 - 3y = 4$	$5x - 2xy = 3e^y$	$\sin x \cos y = \ln(y)$

5. Find an expression in terms of t for $\frac{dy}{dx}$ for the parametric curve:

a)	b)	c)
$x = \dfrac{3}{t}$ $y = 2t^2 + 1$	$x = 3\cos 2t$ $y = 4\sin 5t$	$x = e^{3t}$ $y = \ln(1 + 2t)$

Answers Pure Skills Check 14: Differentiation 3 **Answers**

1.

a)	b)	c)
$\frac{dy}{dx} = (6x)e^{3x^2+5}$	$\frac{dy}{dx} = -2e^x \sin(2e^x)$	$\frac{dy}{dx} = \frac{2\cos 2x}{\sin 2x} = 2\cot 2x$

2.

a)	b)	c)
$f'(x) = e^x(-\sin x) + e^x(\cos x)$ $= e^x(\cos x - \sin x)$	$f'(x)$ $= x^2 \cos x + 2x \sin x$ $= x(x\cos x + 2\sin x)$	$f'(x)$ $= \cos(3x)\,(2\cos(2x))$ $+ (-3\sin 3x)(\sin(2x))$ $= 2\cos 3x \cos 2x$ $- 3\sin 3x \sin 2x$

3.

a)	b)	c)
$\frac{dy}{dx}$ $= \frac{2x\sin x - x^2\cos x}{\sin^2 x}$	$\frac{dy}{dx}$ $= \frac{(1-\ln x)e^x - e^x\left(-\frac{1}{x}\right)}{(1-\ln x)^2}$ $= \frac{\left(1-\ln x+\frac{1}{x}\right)e^x}{(1-\ln x)^2}$	$y = \frac{\sin x}{\cos x}$ $\Rightarrow \frac{dy}{dx}$ $= \frac{\cos x\,(\cos x) - \sin x\,(-\sin x)}{\cos^2 x}$ $= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$ $= \sec^2 x$

4.

a)	b)	c)
$2x + 4y\frac{dy}{dx} - 3\frac{dy}{dx} = 0$	$5 - 2\left(x\frac{dy}{dx} + y\right) = 3e^y\frac{dy}{dx}$	$\sin x\left(-\sin y\frac{dy}{dx}\right) + \cos x\cos y = \frac{1}{y}\frac{dy}{dx}$

5.

a)	b)	c)
$\frac{dx}{dt} = -3t^{-2}$ $\frac{dy}{dt} = 4t$ $\Rightarrow \frac{dy}{dx} = \frac{4t}{-3t^{-2}} = -\frac{4t^3}{3}$	$\frac{dx}{dt} = -6\sin 2t$ $\frac{dy}{dt} = 20\cos 5t$ $\Rightarrow \frac{dy}{dx} = -\frac{10\cos 5t}{3\sin 2t}$	$\frac{dx}{dt} = 3e^{3t}$ $\frac{dy}{dt} = \frac{2}{1+2t}$ $\Rightarrow \frac{dy}{dx} = \frac{2}{3e^{3t}(1+2t)}$

Pure Skills Check 15: Numerical Methods

Record your results here:

Date	***Level (a/b/c)***	***Score (/8)***	***Time (mins)***

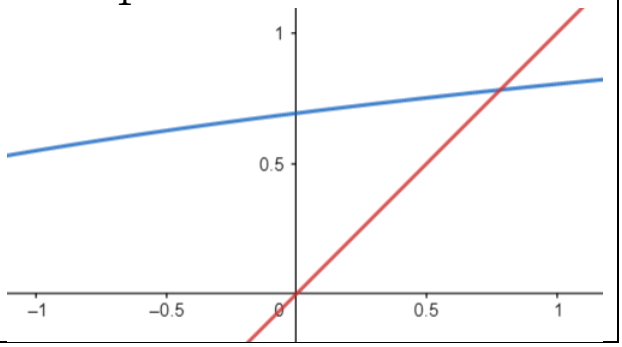
1. For the function $f(x) = \sin x - \ln x$:

a)	b)	c)
Show that f has a root in the interval $[2, 3]$.	If $f(\alpha) = 0$, show that α is 2.2, correct to $2sf$.	If $f(\alpha) = 0$, show that α is 2.22, correct to $3sf$.

2. Use a graphical method to determine the total number of roots for:

a)	b)	c)
$\sin x - \ln x = 0$	$x^3 - \cos x = 0$	$x^2 - 6x + 9 - \frac{1}{x} = 0$

3. For the equation $e^{2x} - x - 4 = 0$:

a)	b)	c)
Show that the equation can be rewritten in the form $x = \ln(g(x))$ where $g(x)$ is a function to be found.	Use the formula $x_{n+1} = \ln(\sqrt{x_n + 4})$ with first approximation $x_1 = -2$ to find the values of x_2 and x_3 to 3 decimal places.	Use this diagram to explain why $x_{n+1} = \ln(\sqrt{x_n + 4})$ can be used with $x_1 = -1$ to find a root: 

4. Using the Newton-Raphson process, with formula $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$:

a)	b)	c)
Starting with $x_1 = 3$, find, to $3sf$, the next approximation for a root of $f(x) = \sin x - \ln x$.	Starting with $x_1 = -4$, find, to $3sf$, x_3, the third approximation for a root of $f(x) = \tan x - e^x$.	Explain the issue with using this method with a value close to a stationary point.

Answers Pure Skills Check 15: Numerical Methods **Answers**

1.

a)

$f(2) = 0.21 \ldots > 0$

$f(3) = -0.95 \ldots < 0$

Change of sign, and f is continuous,

$\therefore$ root in $[2, 3]$.

b)

$f(2.15) = 0.071 \ldots > 0$

$f(2.25) = -0.032 \ldots < 0$

Change of sign, and f is continuous,

$\therefore \alpha = 2.2 \; to \; 3sf.$

c)

$f(2.215) = 0.004 \ldots > 0$

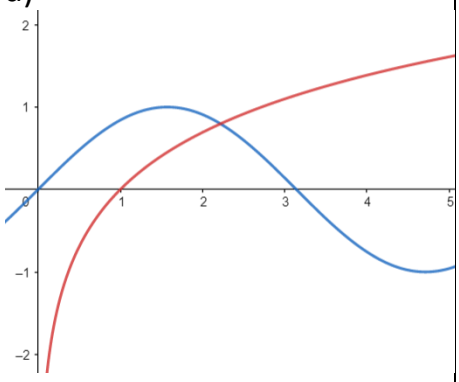
$f(2.225) = -0.006 \ldots < 0$

Change of sign, and f is continuous,

$\therefore \alpha = 2.22 \; to \; 3sf.$

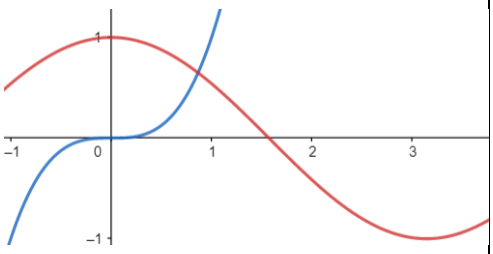
2.

a)



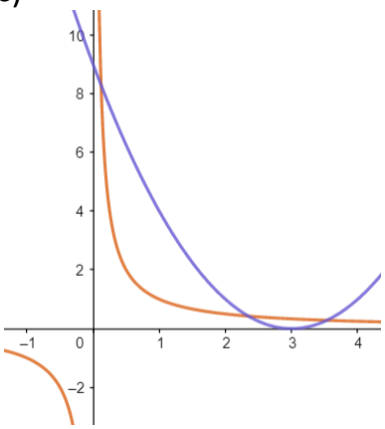
Roots correspond to crossing points of $y = \sin x$ and $y = \ln x$. According to the graph, they cross exactly once, so exactly one root.

b)



Roots correspond to crossing points of $y = x^3$ and $y = \cos x$. According to the graph, they cross exactly once, so exactly one root.

c)



Since $y = (x-3)^2$ and $y = \frac{1}{x}$ cross three times, there must be 3 roots.

3.

a)

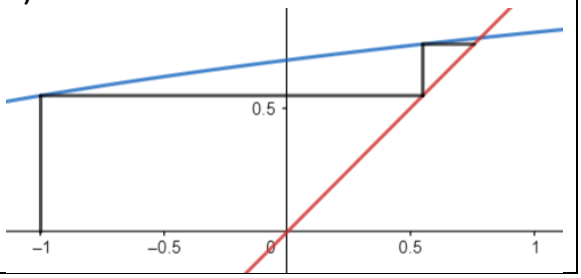
$$e^{2x} = x + 4$$
$$2x = \ln(x+4)$$
$$x = \frac{1}{2}\ln(x+4)$$
$$x = \ln\left(\sqrt{x+4}\right)$$

b)

$$x_2 = \ln\left(\sqrt{-2+4}\right) = 0.347 \; to \; 3dp$$

$$x_3 = \ln\left(\sqrt{0.347 \ldots + 4}\right) = 0.735$$

c)



4.

a)

$$f'(x) = \cos x - \frac{1}{x}$$
$$x_2 = 3 - \frac{f(3)}{f'(3)}$$
$$= 3 - \frac{-0.95 \ldots}{-1.32 \ldots}$$
$$x_2 = 2.28 \; to \; 3sf$$

b)

$$f'(x) = \sec^2 x - e^x$$
$$x_2 = -4 - \frac{f(-4)}{f'(-4)}$$
$$= -4 - \frac{-1.17 \ldots}{2.32 \ldots} = -3.49 \ldots$$
$$x_3 = (-3.49 \ldots) - \frac{f(-3.49 \ldots)}{f'(-3.49 \ldots)}$$
$$= -3.13 \; to \; 3sf$$

c)

If Newton-Raphson is used close to a stationary point, the tangent line will be close to horizontal, meaning its x-axis intercept may be a long way from any root.

Pure Skills Check 16: Integration 2

Record your results here:

Date	***Level (a/b/c)***	***Score (/10)***	***Time (mins)***

1. Evaluate the indefinite integral:

a)	b)	c)
$\int x^2 + \sin x \ dx$	$\int e^x - \cos 4x \ dx$	$\int \frac{1}{x} + e^{3x} \ dx$

2. Evaluate the indefinite integral:

a)	b)	c)
$\int 3\tan 4x \ dx$	$\int 2\sec^2\left(\frac{x}{3}\right) \ dx$	$\int \cot 3x - 2\sec x \ dx$

3. Find the exact value of:

a)	b)	c)
$\int_0^{\frac{\pi}{4}} \tan^2 x \ dx$	$\int_{\pi}^{2\pi} \cos^2 x \ dx$	$\int_0^{\frac{\pi}{3}} \sin x \cos x \ dx$

4. Evaluate the indefinite integral:

a)	b)	c)
$\int (4x+5)^6 \ dx$	$\int \sqrt{1-2x} \ dx$	$\int \frac{3}{\sqrt{5x+2}} - \frac{2}{x^2} \ dx$

5. Evaluate the indefinite integral:

a)	b)	c)
$\int e^{6x+7} \ dx$	$\int \sin 2x - \cos 3x \ dx$	$\int \frac{1}{3-2x} + 2^x \ dx$

Answers Pure Skills Check 16: Integration 2 **Answers**

1.

a)
$$I = \frac{x^3}{3} - \cos x + C$$

b)
$$I = e^x - \frac{1}{4}\sin 4x + C$$

c)
$$I = \ln|x| + \frac{1}{3}e^{3x} + C$$

2.

a)
$$I = \frac{3}{4}\ln|\sec(4x)| + C$$

b)
$$I = 6\tan\left(\frac{x}{3}\right) + C$$

c)
$$I = \frac{1}{3}\ln|\sin(3x)| - 2\ln|\sec x + \tan x| + C$$

3.

a)
$$I = \int_0^{\frac{\pi}{4}} \sec^2 x - 1 \, dx$$
$$I = [\tan x - x]_0^{\frac{\pi}{4}}$$
$$I = \left(1 - \frac{\pi}{4}\right) - (0) = 1 - \frac{\pi}{4}$$

b)
$$I = \int_{\pi}^{2\pi} \frac{1}{2}\cos 2x + \frac{1}{2} \, dx$$
$$I = \left[\frac{1}{4}\sin 2x + \frac{1}{2}x\right]_{\pi}^{2\pi}$$
$$I = (\pi) - \left(\frac{\pi}{2}\right) = \frac{\pi}{2}$$

c)
$$I = \int_0^{\frac{\pi}{3}} \frac{1}{2}\sin 2x \, dx$$
$$I = \left[-\frac{1}{4}\cos 2x\right]_0^{\frac{\pi}{3}}$$
$$I = \left(\frac{1}{8}\right) - \left(-\frac{1}{4}\right) = \frac{3}{8}$$

4.

a)
$$I = \frac{(4x+5)^7}{7} \times \frac{1}{4} + C$$
$$I = \frac{(4x+5)^7}{28} + C$$

b)
$$I = \int (1-2x)^{\frac{1}{2}} \, dx$$
$$I = \frac{(1-2x)^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} \times \frac{1}{-2} + C$$
$$I = \frac{-(1-2x)^{\frac{3}{2}}}{3} + C$$

c)
$$I = \int 3(5x+2)^{-\frac{1}{2}} - 2x^{-2} \, dx$$
$$I = \frac{3(5x+2)^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} \times \frac{1}{5} - \frac{2x^{-1}}{-1} + C$$
$$I = \frac{6}{5}(5x+2)^{\frac{1}{2}} + 2x^{-1} + C$$

5.

a)
$$I = \frac{e^{6x+7}}{6} + C$$

b)
$$I = -\frac{1}{2}\cos 2x - \frac{1}{3}\sin 3x + C$$

c)
$$I = -\frac{1}{2}\ln|3-2x| + \frac{2^x}{\ln 2} + C$$

Pure Skills Check 17: Integration 3

Record your results here:

Date	***Level (a/b/c)***	***Score (/8)***	***Time (mins)***

1. Use partial fractions to evaluate:

a)	b)	c)
$\int \frac{6}{(x+2)(x-1)}\,dx$	$\int \frac{x+1}{(2x+1)x^2}\,dx$	$\int \frac{1}{x^2-1}\,dx$

2. Use inspection or substitution to evaluate:

a)	b)	c)
$\int \frac{2x+3}{x^2+3x+7}\,dx$	$\int \frac{2\sin x}{\cos x-4}\,dx$	$\int \cos x \sin^3 x\ dx$

3. Use the substitution given to evaluate the indefinite integral:

a)	b)	c)
$\int x\sqrt{3-x}\,dx$	$\int \frac{x}{\sqrt{2x+1}}\,dx$	$\int \frac{1}{\sqrt{1-x^2}}\,dx$
Using $u = 3-x$	Using $u = 2x+1$	Using $x = \cos u$

4. Use Integration by Parts to evaluate the indefinite integral:

a)	b)	c)
$\int x \sin x\ dx$	$\int x^3 \ln x\ dx$	$\int 2xe^{3x}\,dx$

Answers Pure Skills Check 17: Integration 3 **Answers**

1.

a)

$$\frac{3}{(x+2)(x-1)} \equiv \frac{A}{x+2} + \frac{B}{x-1}$$

$$3 \equiv A(x-1) + B(x+2)$$

$$x = 1 \implies 3 = 3B$$

$$x = -2 \implies 3 = -3A$$

$$I = \int -\frac{1}{x+2} + \frac{1}{x-1}\, dx$$

$$= -\ln|x+2| + \ln|x-1| + C$$

$$I = \ln\left|\frac{x-1}{x+2}\right| + C$$

b)

$$\frac{x+1}{(2x+1)(x-2)^2} \equiv \frac{A}{2x+1} + \frac{B}{x} + \frac{C}{x^2}$$

$$x + 1 \equiv Ax^2 + Bx(2x+1) + C(2x+1)$$

$$x = 0 \implies 1 = C$$

$$x = -\frac{1}{2} \implies \frac{1}{2} = \frac{A}{4}$$

$$x^2\ coeffs:\ 0 = A + 2B$$

$$I = \int \frac{2}{2x+1} - \frac{1}{x} + \frac{1}{x^2}\, dx$$

$$I = \ln|2x+1| - \ln|x| + \frac{x^{-1}}{-1} + C$$

$$I = \ln\left|\frac{2x+1}{x}\right| - \frac{1}{x} + C$$

c)

$$\frac{1}{(x+1)(x-1)} \equiv \frac{A}{x+1} + \frac{B}{x-1}$$

$$1 \equiv A(x-1) + B(x+1)$$

$$x = 1 \implies 1 = 2B$$

$$x = -1 \implies 1 = -2A$$

$$I = \int \frac{\left(-\frac{1}{2}\right)}{x+1} + \frac{\left(\frac{1}{2}\right)}{x-1}\, dx$$

$$I = \frac{1}{2}(\ln|x-1| - \ln|x+1|) + C$$

$$I = \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| + C$$

2.

a)

$$I = \ln|x^2 + 3x + 7| + C$$

b)

$$I = 2\ln|\cos x - 4| + C$$

c)

$$I = \frac{1}{4}\sin^4 x + C$$

3.

a)

$$\frac{du}{dx} = -1 \implies dx = -du$$

$$I = -\int (3-u)\sqrt{u}\, du$$

$$I = \int u^{\frac{3}{2}} - 3u^{\frac{1}{2}}\, du$$

$$I = \frac{u^{\frac{5}{2}}}{\left(\frac{5}{2}\right)} - \frac{3u^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + C$$

$$I = \frac{2}{5}(3-x)^{\frac{5}{2}} - 2(3-x)^{\frac{3}{2}} + C$$

b)

$$\frac{du}{dx} = 2 \implies dx = 2\, du$$

$$I = \int \frac{\left(\frac{u-1}{2}\right)}{\sqrt{u}}\, 2\, du$$

$$I = \int u^{\frac{1}{2}} - u^{-\frac{1}{2}}\, du$$

$$I = \frac{2}{3}u^{\frac{3}{2}} - 2u^{\frac{1}{2}} + C$$

$$I = \frac{2}{3}(2x+1)^{\frac{2}{3}} - 2(2x+1)^{\frac{1}{2}} + C$$

c)

$$\frac{dx}{du} = -\sin u$$

$$I = \int \frac{-\sin u}{\sin u}\, du$$

$$I = \int -1\, du$$

$$I = -u + C$$

$$I = -\arccos x + C$$

4.

a)

$$u = x \quad \frac{dv}{dx} = \sin x$$

$$\frac{du}{dx} = 1 \quad v = -\cos x$$

$$I = -x\cos x - \int -\cos x\ dx$$

$$I = -x\cos x + \sin x + C$$

b)

$$u = \ln x \quad \frac{dv}{dx} = x^3$$

$$\frac{du}{dx} = \frac{1}{x} \quad v = \frac{x^4}{4}$$

$$I = \frac{x^4}{4}\ln x - \int \frac{x^3}{4}\ dx$$

$$I = \frac{x^4}{4}\ln x - \frac{x^4}{16} + C$$

c)

$$u = 2x \quad \frac{dv}{dx} = e^{3x}$$

$$\frac{du}{dx} = 2 \quad v = \frac{1}{3}e^{3x}$$

$$I = \frac{2}{3}xe^{3x} - \int \frac{2}{3}e^{3x}\, dx$$

$$I = \frac{2}{3}xe^{3x} - \frac{2}{9}e^{3x} + C$$

Pure Skills Check 18: Vectors

Record your results here:

Date	***Level (a/b/c)***	***Score (/8)***	***Time (mins)***

1. Calculate:

a)	b)	c)
The distance between $(3, 2, -4)$ and $(2, -1, 5)$.	The magnitude of vector $\overrightarrow{OA} = \begin{pmatrix} 4 \\ -3 \\ 7 \end{pmatrix}$.	The magnitude of $\overrightarrow{AB}$, if: $\overrightarrow{OA} = 3\boldsymbol{i} + \boldsymbol{j} - 2\boldsymbol{k}$ $\overrightarrow{OB} = 2\boldsymbol{i} - \boldsymbol{k}$

2. Find:

a)	b)	c)
$\boldsymbol{a} + 2\boldsymbol{b}$ where $\boldsymbol{a} = 3\boldsymbol{i} - 7\boldsymbol{j} + \boldsymbol{k}$ $\boldsymbol{b} = 2\boldsymbol{j} + 3\boldsymbol{k}$	Find p and q such that: $\begin{pmatrix} -5 \\ 3 \\ p \end{pmatrix} - 3\begin{pmatrix} q \\ -2 \\ 4 \end{pmatrix} = \begin{pmatrix} -2 \\ 9 \\ -8 \end{pmatrix}$	Find p and q such that: $\begin{pmatrix} p \\ q \\ 3 \end{pmatrix}$ is parallel to $\begin{pmatrix} 3 \\ -1 \\ -6 \end{pmatrix}$

3.

a)	b)	c)
Find the unit vector $\hat{\boldsymbol{a}}$ in the direction of: $\boldsymbol{a} = \begin{pmatrix} -2 \\ 9 \\ -5 \end{pmatrix}$	Find the magnitude of the acceleration experienced by a mass of $4kg$ under the influence of these forces: $\boldsymbol{T} = \begin{pmatrix} 8 \\ 4 \\ 20 \end{pmatrix}$ $\boldsymbol{W} = \begin{pmatrix} 0 \\ 0 \\ -40 \end{pmatrix}$	Find the angle made by the vector $\begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$ with the positive $\boldsymbol{j}$ direction in degrees to 1 decimal place.

4. What quadrilateral would $ABCD$ be if:

a)	b)	c)
$\lvert\overrightarrow{AB}\rvert = \lvert\overrightarrow{BC}\rvert$ and $\lvert\overrightarrow{CD}\rvert = \lvert\overrightarrow{DA}\rvert$	$\lvert\overrightarrow{AB}\rvert = k\lvert\overrightarrow{CD}\rvert$	$\overrightarrow{AB} = \overrightarrow{DC}$

Answers Pure Skills Check 18: Vectors **Answers**

1.

a)
$$\sqrt{(3-2)^2+(2--1)^2+(-4-5)^2}$$
$$=\sqrt{91}$$

b)
$$\left|\overrightarrow{AB}\right|$$
$$=\sqrt{4^2+(-3)^2+7^2}$$
$$=\sqrt{74}$$

c)
$$\overrightarrow{AB}=\begin{pmatrix}3\\1\\-2\end{pmatrix}-\begin{pmatrix}2\\0\\-1\end{pmatrix}$$
$$\overrightarrow{AB}=\begin{pmatrix}1\\1\\-1\end{pmatrix}$$
$$\therefore\ \left|\overrightarrow{AB}\right|=\sqrt{3}$$

2.

a)
$$\boldsymbol{a}+2\boldsymbol{b}$$
$$=\begin{pmatrix}3\\-7\\1\end{pmatrix}+2\begin{pmatrix}0\\2\\3\end{pmatrix}$$
$$=\begin{pmatrix}3\\-3\\7\end{pmatrix}$$

b)
$$-5-3q=-2$$
$$3+6=9$$
$$p-12=-8$$
$$\therefore\ q=-1$$
$$p=4$$

c)
$$\begin{pmatrix}p\\q\\3\end{pmatrix}=k\begin{pmatrix}3\\-1\\-6\end{pmatrix}\Rightarrow\ \begin{matrix}p=3k\\q=-k\\3=-6k\end{matrix}$$
$$\therefore\ k=-\frac{1}{2}\Rightarrow\ p=-\frac{3}{2}\ \ and\ \ q=\frac{1}{2}$$

3.

a)
$$|\boldsymbol{a}|$$
$$=\sqrt{(-2)^2+9^2+(-5)^2}$$
$$=\sqrt{110}$$
$$\therefore\ \widehat{\boldsymbol{a}}=\begin{pmatrix}-\frac{2}{\sqrt{110}}\\\frac{9}{\sqrt{110}}\\-\frac{5}{\sqrt{110}}\end{pmatrix}$$

b)
$$\boldsymbol{F}=\boldsymbol{T}+\boldsymbol{W}=\begin{pmatrix}8\\4\\-20\end{pmatrix}$$
$$\boldsymbol{F}=m\boldsymbol{a}\Rightarrow\boldsymbol{a}=\begin{pmatrix}2\\1\\-5\end{pmatrix}$$
$$\therefore\ |\boldsymbol{a}|=$$
$$\sqrt{2^2+1^2+(-5)^2}$$
$$=\sqrt{30}\ ms^{-2}$$

c)
$$|\boldsymbol{a}|\cos(\theta_y)=-2$$
$$\cos(\theta_y)=-\frac{2}{\sqrt{2^2+(-2)^2+1^2}}$$
$$\cos(\theta_y)=-\frac{2}{3}\Rightarrow\ \theta_y=131.8°$$

4.

a)
Two pairs of adjacent equal-length sides: Kite

b)
One pair of parallel sides: Trapezium

c)
Opposite sides equal and parallel: Parallelogram.
Note: the other two sides are automatically also equal and parallel in this case.

A-level Maths Mechanics Skills Checks
Applied Maths: Mechanics

What they're for

*These questions are designed to help you build **fluency** and **confidence** with some of the most common techniques and methods of A-level Maths.*
One of the most common reasons students struggle to achieve a decent grade is that they get stuck on the early parts of a question and then can't access the big ticket items that follow on. If you often get stuck at the start of a question, or don't feel confident with the key skills required within a topic, you should find these skills checks particularly beneficial.

How to use them

You're welcome to make use of these questions any way you see fit, but they are designed to fit into a little-and-often revision schedule. Research suggests you'll be more effective working in 25 minute bursts, with 5 minute breaks in between. Each skills check is designed to be easily doable within this time.

*First, **choose 5 skills checks** to focus on. If in doubt, go with the first five topics.*

During a 25 minute revision block, try the following:

- *Grab the **first skills check** from your set of 5 to tackle.*
- *Choose your **difficulty level** (a: easy, b: moderate, c: hard).*
- ***Complete all the questions** for your chosen difficulty level. **Time yourself**.*
- ***Check your answers**, make a note of any errors or misconceptions and look up (or ask a friend or teacher about) anything that doesn't make sense. Pay particular attention to common slips, where your method is valid, but you make a daft numerical or algebraic error. What can you do to avoid them, or find and fix them before finishing a question?*
- *Jot down your **score** (1 point for a substantially correct method, 2 points for correct method and fully correct answer for each question) and the **total time**.*
- *Put your skills check sheet at the **bottom of the pile** of 5 to revisit later.*

After you've worked your way through all five sheets, you'll start to revisit them:

- *Whenever you revisit a skills check, choose a **harder difficulty level**. Once you can confidently do all three (a, b and c), focus on speed and efficiency - start looking for ways to save time without compromising your accuracy.*

*Once you've mastered a particular skills check to your satisfaction, **set that sheet aside** and **add a brand new one** to your pile of five.*

Important:

These skills checks alone won't improve your problem solving or mathematical modelling skills – they are the building blocks you need, but on their own they are not enough. Once you feel you have mastered a particular topic's key skills, find some proper A-level Maths exam questions and work on applying your skills to them.

Contents

Mechanics Skills Check 1: Modelling with Mechanics

Record your results here:

Date	***Level (a/b)***	***Score (/8)***	***Time (mins)***

1.

a)	b)
In the context of a mechanics question, what does it mean for something to be modelled as a 'particle'?	What are the implications of modelling an object as a 'particle' in the context of a problem involving motion?

2.

a)	b)
In the context of a mechanics question, what does it mean for something to be modelled as 'smooth'?	What are the implications of modelling the pulley as 'smooth' in the context of a connected particles problem?

3.

a)	b)
In the context of a mechanics question, what does it mean for something to be modelled as 'light'?	What are the implications of modelling the string as 'light' in the context of a connected particles problem?

4.

a)	b)
In the context of a mechanics question, what does it mean for an object to be modelled as a 'uniform rod'?	What are the implications of modelling the ladder as a 'uniform rod' in the context of a moments problem?

****Answers** Mechanics Skills Check 1: Modelling with Mechanics**Answers****

1.

a)	b)
The dimensions (length, width and height) of the particle are ignored, or considered to be negligible compared to the other distances involved in the problem.	Any forces acting on the object can be considered to all act at the same point (therefore there is no moment on the object), and if the object comes into contact with a surface (eg dropping a ball onto the ground) it is assumed that the centre of mass reaches the surface.

2.

a)	b)
Friction is ignored, or assumed to be zero.	The tension in the string can be assumed to be the same on both sides, whereas if friction were acting, it would resist motion, increasing the tension required for motion on one side of the pulley.

3.

a)	b)
The mass (and therefore weight) of an object is ignored, or assumed to be zero.	If the mass of the string is ignored, we can assume that the only forces causing motion are the weights of the objects suspended by the strings, so acceleration is constant on each side (rather than increasing as more string ends up on one side of the pulley).

4.

a)	b)
The object is assumed to have length only (no width or height), and its mass is assumed to be evenly distributed throughout its length.	The centre of mass can be assumed to be in the centre of the object. For instance, a ladder of length 4 metres can be assumed to have its centre of mass 2 metres from each end.

Mechanics Skills Check 2: Constant Acceleration

Record your results here:

Date	*Level (a/b)*	*Score (/8)*	*Time (mins)*

1.

A train starts from rest, accelerates at a rate of $0.8ms^{-2}$ for $30s$, continues at a constant speed for a further $10s$ and then decelerates to rest over $20s$.

a)	b)
Sketch the velocity-time graph for the motion described.	Calculate the acceleration experienced by the train during the final 20 seconds of motion, and the total distance covered during motion.

2.

A ball is thrown in the air from ground level with an initial speed of $20ms^{-1}$. Assuming air resistance is negligible, and taking $g = 10ms^{-2}$,

a)	b)
Sketch the velocity-time graph for the first 3 seconds of motion.	Calculate the total distance travelled by the ball during this time.

3.

A motorbike, initially travelling at $22ms^{-1}$, accelerates forwards at a rate of $4ms^{-2}$ for 3 seconds.

a)	b)
How fast is it moving at the end of the 3 seconds?	How far does it travel during the 3 seconds?

4.

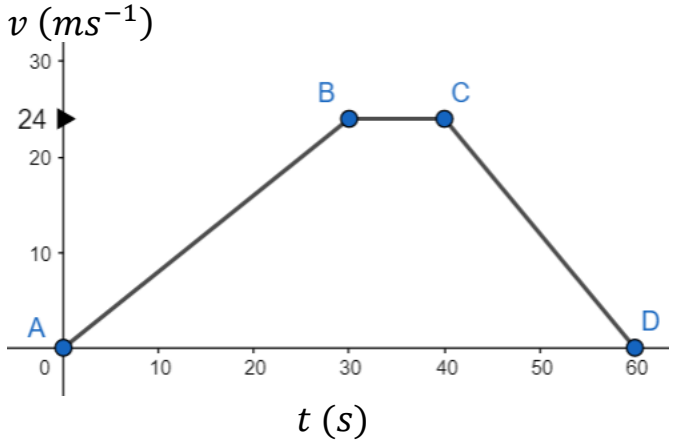
A football is kicked vertically upwards at $30ms^{-1}$. Taking $g = 9.8ms^{-2}$,

a)	b)
How long does it take for the ball to return to its initial height?	What is the maximum height reached by the ball?

****Answers** Mechanics Skills Check 2: Constant Acceleration **Answers****

1.

a)



b)

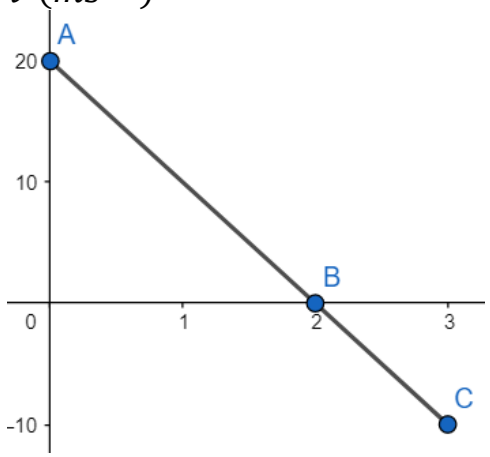
Speed decreases from $24ms^{-1}$ (since $0.8ms^{-2} \times 30s = 24ms^{-1}$) to $0ms^{-1}$ (since 'decelerates to rest') in $20s$, therefore acceleration is $\frac{-24}{20} = -1.2ms^{-2}$.

Displacement is given by the area under the graph (and since all in the forwards direction, distance = displacement, so:

$$s = \frac{1}{2}(30 \times 24) + (10 \times 24) + \frac{1}{2}(20 \times 24)$$
$$= \mathbf{840m}$$

2.

a)



b)

Displacement during the first 2 seconds:

$$\frac{1}{2}(2 \times 20) = 20m$$

Displacement during the next 1 second:

$$\frac{1}{2}(1 \times -10) = -5m$$

Separating motion into phases where direction doesn't change allows us to find the sum of the separate distances (the magnitude of displacement): $|20| + |-5| = \mathbf{25m.}$

3.

a)

$s =$	$v = u + at$
$u = 22$	$v = 22 + 4 \times 3$
$v =?$	$\boldsymbol{v = 34ms^{-1}}$
$a = 4$	
$t = 3$	

b)

$s =?$	$s = ut + \frac{1}{2}at^2$
$u = 22$	$s = 22 \times 3 + \frac{1}{2}(4)(3)^2$
$v =$	$\boldsymbol{s = 84m}$
$a = 4$	
$t = 3$	

4.

a)

$s = 0$	$s = ut + \frac{1}{2}at^2$
$u = 30$	$0 = 30t - 4.9t^2$
$v =$	$t(30 - 4.9t) = 0$
$a = -9.8$	$t = 0\ (start\ of\ motion)$
$t =?$	$or\ \ t = \frac{30}{4.9} = \boldsymbol{6.1ms^{-1}\ to\ 2sf}$

b)

$s =?$	$v^2 = u^2 + 2as$
$u = 30$	$0 = 900 - 19.6s$
$v = 0$	$s = \frac{900}{19.6} = \boldsymbol{46m\ to\ 2sf}$
$a = -9.8$	
$t =$	

Mechanics Skills Check 3: Forces and Motion

Record your results here:

Date	***Level (a/b)***	***Score (/8)***	***Time (mins)***

1.

a)	b)
A lorry of mass 3 tonnes accelerates along a horizontal road at a rate of $1.5ms^{-2}$. If the lorry experiences resistive forces totalling $4000N$, what is the motive forwards force provided by the engine?	A lift is pulled upwards by a cable with tension $5000N$. If the resulting acceleration is $0.5ms^{-2}$, what is the mass of the lift?

2. A ski-lift chair of mass $200g$ is initially pulled along by a cable inclined at an angle of $15°$ to the vertical.

a)	b)
What is the tension in the cable?	What is the initial forwards acceleration of the chair?

3. An object is at rest on a rough inclined plane angled at $\theta°$ to the horizontal.

a)	b)
If the object is on the point of slipping, and $\theta = 35°$, find μ to 3sf.	If the object has mass $1kg$, and is slipping down the slope which is inclined at an angle of $40°$, when $\mu = 0.4$, find the acceleration.

4. The force F is defined by the vector $\boldsymbol{F} = 5\boldsymbol{i} - 2\boldsymbol{j}$ where $\boldsymbol{i}$ and $\boldsymbol{j}$ represent unit vectors in the directions East and North respectively.

a)	b)
Find the magnitude of $\boldsymbol{F}$.	Find the direction of the force $\boldsymbol{F}$, giving your answer as a bearing measured clockwise from North.

Answers Mechanics Skills Check 3: Forces and Motion **Answers**

1.

a)

$$m = 3000\ kg$$
$$a = 1.5\ ms^{-2}$$
$$F = M_f - 4000$$

$$F = ma \implies M_f - 4000 = 3000 \times 1.5$$
$$M_f = 4000 + 4500 = \mathbf{8500N}$$

b)

$$F = 5000 - mg$$
$$m = m$$
$$a = 0.5$$

$$F = ma \implies 5000 - mg = 0.5m$$
$$5000 = (0.5 + g)m$$
$$\implies m = \frac{5000}{0.5 + g} \approx \mathbf{490kg\ to\ 2sf}$$

2.

a)

Resolving vertically ↕:

$$T\cos 15 = 200g$$
$$T = \frac{200g}{\cos 15} \approx \mathbf{2000N\ to\ 2sf}$$

b)

Resolving horizontally →:

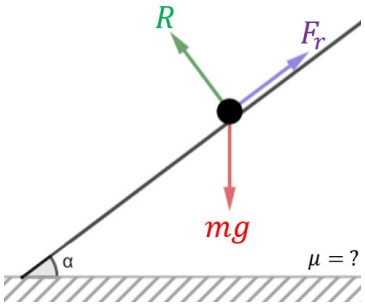
$$F = ma$$
$$T\sin 15 = 200a$$

(using result from a):

$$\frac{200g}{\cos 15}\sin 15 = 200a$$
$$a = g\tan 15 \approx \mathbf{2.6ms^{-2}}$$

3.

a)



Resolving perpendicular to the slope:

$$R = mg\cos 35$$

Resolving parallel to the slope:

$$F_r = mg\sin 35$$

Limiting equilibrium, so $F_r = \mu R$:

$$\mu = \frac{F_r}{R} = \frac{mg\sin 35}{mg\cos 35} = \tan 35$$
$$\approx \mathbf{0.700\ to\ 3sf}$$

b)

Resolving perpendicular to the slope:

$$R = 1g\cos 40$$

Resolving *down* the slope:

$$1g\sin 40 - F_r = ma$$

$F_r = 0.4R$ so:

$$g\sin 40 - 0.4g\cos 40 = 2a$$

$$\therefore\ \ \mathbf{a = 1.65\ ms^{-2}}$$

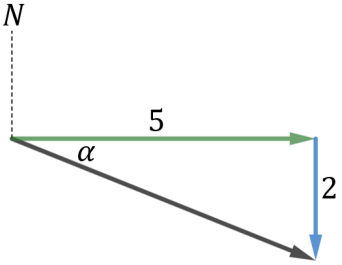
4.

a)

$$|\boldsymbol{F}| = \sqrt{5^2 + (-2)^2}$$

$$= \sqrt{\mathbf{29}}\ \boldsymbol{Newtons}$$

b)



$$\alpha = \tan^{-1}\left(\frac{2}{5}\right)$$

$$bearing = 90 + \alpha$$
$$= \mathbf{111.8°}$$

Mechanics Skills Check 4: Moments

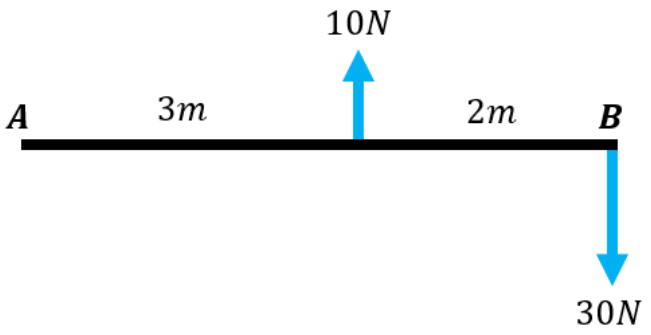
Record your results here:

Date	*Level (a/b)*	*Score (/6)*	*Time (mins)*

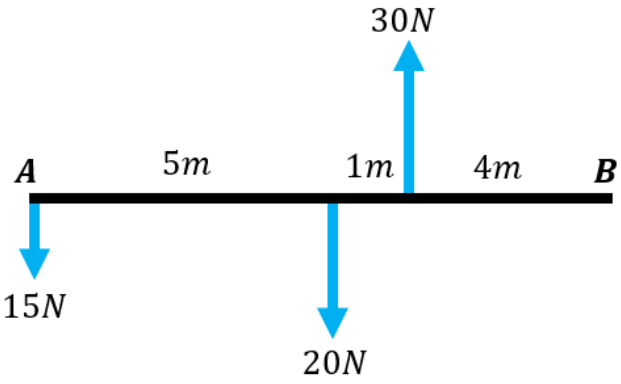
Drawing force diagram for a basic ladder scenario
Linking forces through moments and resolving for said ladder

1. Find the net anti-clockwise moment about the point A in the diagram:

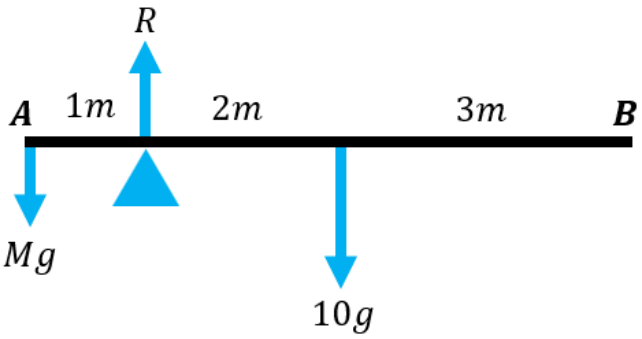
a)



b)



2.
A uniform rod of mass $10kg$ and length $6m$ is supported $1m$ from one end, and held in equilibrium by a counter-weight of mass M on that end as shown.



a)
Find the value of M.

b)
Find the value of R.

3. A uniform ladder of mass m kg and length $2a$ metres leans against a smooth vertical wall, resting on rough horizontal ground at $75°$ to the horizontal.

a)
Draw a complete force diagram for the ladder.

b)
Calculate the minimum coefficient of friction required for the ladder to remain in equilibrium.

Answers Mechanics Skills Check 4: Moments **Answers**

1.

a)

$$Moment = \perp force \times distance$$
$$= 10 \times 3 - 30 \times 5 = \mathbf{-120Nm}$$

b)

$$Moment = \perp force \times distance$$
$$= 30 \times 6 - 20 \times 5 = \mathbf{80Nm}$$

2.

a)

Taking moments about the support point:

$$Mg \times 1 = 10g \times 2$$
$$\boldsymbol{M = 20\ kg}$$

b)

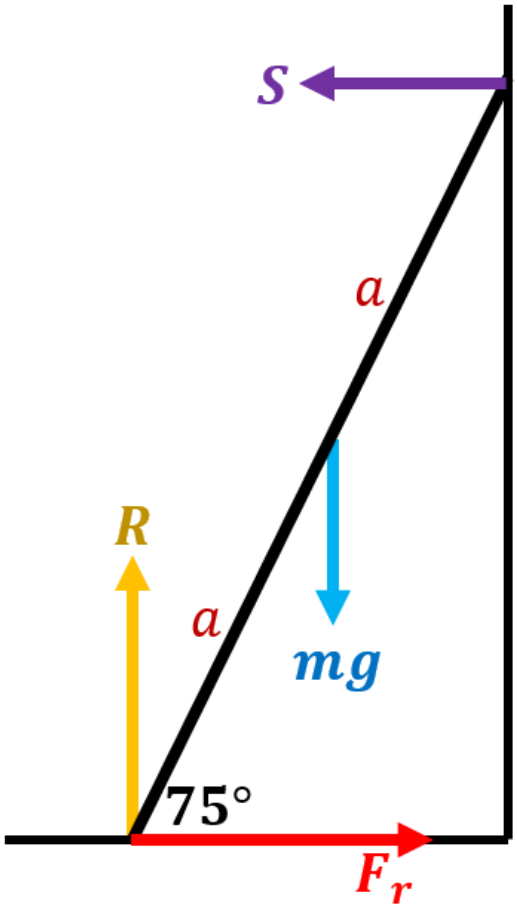
Taking moments about the end point A:

$$R \times 1 = 10g \times 3$$
$$\boldsymbol{R = 30g\ Newtons}$$

Note: If you already know the value of M, you can resolve forces vertically and use $R = Mg + 10g$.

3.

a)



b)

Taking moments about the base:

$$S(2a \sin 75) = mg(a \cos 75)$$
$$S = \frac{mg}{2 \tan 75}$$

Resolving horizontally:

$$F_r = S$$

Resolving vertically:

$$R = mg$$

If the ladder is on the point of slipping:

$$F_r = \mu R$$

$$F_r = S = \frac{mg}{2 \tan 75} \quad and \quad R = mg$$

$$\frac{mg}{2 \tan 75} = \mu mg$$

$$\mu = \frac{1}{2 \tan 75} = \mathbf{0.14\ to\ 2sf}$$

Mechanics Skills Check 5: Projectiles

Record your results here:

Date	***Level (a/b)***	***Score (/8)***	***Time (mins)***

1. A toy car is launched horizontally from a tabletop which is $0.8m$ high.

a)	b)
If the initial speed is $10ms^{-1}$, calculate the time of flight, and the horizontal distance covered by the car during motion.	If the horizontal range is $3m$, calculate the initial speed of projection.

2. A ballistic missile is launched from level horizontal ground with initial velocity $200ms^{-1}$ at an angle of $50°$ to the horizontal.

a)	b)
Calculate the overall horizontal range of the missile.	Calculate the maximum height reached by the missile, and state its speed at this moment.

3. An arrow is fired forwards and downwards with a speed of $40ms^{-1}$ at an angle of $30°$ below the horizontal, from a high building. Given that the arrow lands exactly 4 seconds later, calculate:

a)	b)
The height of the building.	The speed and direction of the arrow at the instant just before it lands.

4. A football, a golf ball and a frisbee are all projected from a given point with the same initial speed and in the same direction.

a)	b)
What are the key modelling assumptions made when treating the motion of these objects as projectiles?	Which of the three objects is most likely to follow a trajectory closely matching perfect projectile motion? Which is least likely to do so? Justify your answers.

Answers Mechanics Skills Check 5: Projectiles **Answers**

1.

a) Vertical motion:

$s = -0.8$
$u = 0$
$v =$
$a = -g$
$t =$

$$s = ut + \frac{1}{2}at^2$$
$$-0.8 = -\frac{g}{2}t^2$$
$$t = \sqrt{\frac{1.6}{g}} = \mathbf{0.40s\ to\ 2sf}$$

Horizontal motion:

$v = 10$
$x =$
$t = \sqrt{\frac{1.6}{g}}$

$$x = vt$$
$$x = \mathbf{4.0m\ to\ 2sf}$$

b) Vertical motion:

(see part a: working is identical, leading to $t = \sqrt{\frac{1.6}{g}}$)

Horizontal motion:

$v =$
$x = 3$
$t = \sqrt{\frac{1.6}{g}}$

$$v = \frac{x}{t} = \mathbf{7.4ms^{-1}\ to\ 2sf}$$

2.

a) Vertical motion:

$s = 0$
$u = 200\sin 50$
$v =$
$a = -g$
$t =$

$$s = ut + \frac{1}{2}at^2$$
$$0 = 200\sin 50\, t - \frac{g}{2}t^2$$
$$t = \frac{400\sin 50}{g}$$

Horizontal motion:

$v = 200\cos 50$
$x =$
$t = \frac{400\sin 50}{g}$

$$x = vt$$
$$x = 4019.6\ldots$$
$$= \mathbf{4.0km\ to\ 2sf}$$

b) Vertical motion:

$s = h$
$u = 200\sin 50$
$v = 0$
$a = -g$
$t =$

$$v^2 = u^2 + 2as$$
$$0 = (200\sin 50)^2 - 2gh$$
$$h = \mathbf{1200m\ to\ 2sf}$$

Since, at this point, vertical speed is zero, all speed is horizontal:

$$speed = 200\cos 50 \approx \mathbf{130ms^{-1}}$$

3.

a)

$s = -h$
$u = -40\sin 30 = -20$
$v =$
$a = -g$
$t = 4$

$$s = ut + \frac{1}{2}at^2$$
$$-h = -80 - 8g$$
$$h = 80 + 8g$$
$$= 158.4\ or\ \mathbf{160m\ to\ 2sf}$$

b)

$s = -80 - 8g$
$u = -40\sin 30 = -20$
$v =$
$a = -g$
$t = 4$

$$v = u + at = -20 - 4g = -59.2$$
$$speed = \sqrt{(-59.2)^2 + (-20)^2} = \mathbf{62ms^{-1}\ to\ 2sf}$$
$$angle = \tan^{-1}\left(\frac{59.2}{20}\right)$$
$$\approx \mathbf{71^\circ\ below\ the\ horizontal\ (to\ 2sf)}$$

4.

a)

Gravity is the only force acting. Ie, no air resistance, wind, spin, etc. The object is modelled as a particle (zero size).

b)

The golf ball will behave most like a true projectile – it's small, dense and designed to be aerodynamic, so air resistance, spin and wind will have the least effect. The frisbee will be least like a projectile – it is large and light, designed for lift, and will be affected more by the air.

Mechanics Skills Check 6: Variable Acceleration

Record your results here:

Date	***Level (a/b)***	***Score (/6)***	***Time (mins)***

1. The position r of a particle at time t seconds is given by $r = 12t - t^3$, $t \geq 0$.

a)	b)
Find the velocity, $v\ ms^{-1}$, at time t seconds, and hence determine when the particle will be instantaneously at rest.	Find the acceleration, $a\ ms^{-2}$, at time t, and hence determine, if the mass of the object is $20kg$, when the force experienced will first exceed $100N$.

2. A particle starts from rest, and is subjected to a variable acceleration in a straight line given by: $a = 3\cos t - \sin t\,,\ t \geq 0$.

a)	b)
Find the velocity of the particle after π seconds have elapsed.	Find the displacement of the particle from its starting point after 2π seconds have elapsed.

3. The velocity of an object at time t seconds after motion begins is given by:

$$\boldsymbol{v} = \begin{bmatrix} 3t + 2t^2 \\ 4 - \sqrt{t} \end{bmatrix}$$

a)	b)
Find the acceleration of the particle after 16 seconds have elapsed.	If the particle is at the point A at the start of motion, and at the point B 5 seconds later, find the distance AB.

Answers Mechanics Skills Check 6: Variable Acceleration **Answers**

1.

a)

$$r = 12t - t^3$$

$$v = \frac{dr}{dt} = 12 - 3t^2$$

$v = 0 \ when \ 12 - 3t^2 = 0 \implies t = \pm 2$

Since $t \geq 0$, it will be at rest when $\boldsymbol{t = 2.}$

b)

$$a = \frac{dv}{dt} = \frac{d^2r}{dt^2}$$

$$v = 12 - 3t^2 \implies a = -6t$$

$$F = ma = 20 \times (-6t)$$

$$100 = |-120t|$$

$$\implies t = \frac{100}{120} = \frac{5}{6} = \boldsymbol{0.833s \ to \ 3sf}$$

2.

a)

$$a = 3\cos t - \sin t$$

$$v = \int a \, dt = 3\sin t + \cos t + C$$

$t = 0 \implies v = 0 \ so$:

$$0 = 3\sin 0 + \cos 0 + C \implies C = -1$$

$$\therefore \ \boldsymbol{v = 3\sin t + \cos t - 1}$$

b)

$$v = 3\sin t + \cos t - 1$$

$$x = \int v \, dt = -3\cos t + \sin t - t + C$$

Since we want displacement relative to its starting position, let $x = 0$ at $t = 0$. Then:

$$0 = -3\cos 0 + \sin 0 - 0 + C \implies C = 3$$

$$\boldsymbol{x = -3\cos t + \sin t - t + 3}$$

3.

a)

$$\boldsymbol{a} = \frac{d\boldsymbol{v}}{dt} = \begin{bmatrix} 3 + 4t \\ -\frac{1}{2}t^{-\frac{1}{2}} \end{bmatrix}$$

$$t = 16 \implies a = \begin{bmatrix} 3 + 4(16) \\ -\frac{1}{2}(16)^{-\frac{1}{2}} \end{bmatrix}$$

$$\approx \begin{bmatrix} \boldsymbol{67} \\ \boldsymbol{-0.125} \end{bmatrix} \ \boldsymbol{ms^{-2}}$$

b)

$$\boldsymbol{x} = \int \boldsymbol{v} \, dt = \begin{bmatrix} \frac{3t^2}{2} + \frac{2t^3}{3} + C_1 \\ 4t - \frac{2}{3}t^{\frac{3}{2}} + C_2 \end{bmatrix}$$

Let A be the starting point. Then:

At B: $t = 5 \implies x = \begin{bmatrix} \frac{3(5)^2}{2} + \frac{2(5^3)}{3} \\ 4(5) - \frac{2}{3}(5)^{\frac{3}{2}} \end{bmatrix}$

$$\overrightarrow{AB} = \begin{bmatrix} \frac{725}{6} \\ \frac{60 - 10\sqrt{5}}{3} \end{bmatrix} \implies \left|\overrightarrow{AB}\right| = \boldsymbol{121m \ to \ 3sf}$$

A-level Maths Statistics Skills Checks
Applied Maths: Statistics

What they're for

*These questions are designed to help you build **fluency** and **confidence** with some of the most common techniques and methods of A-level Maths.*
One of the most common reasons students struggle to achieve a decent grade is that they get stuck on the early parts of a question and then can't access the big ticket items that follow on. If you often get stuck at the start of a question, or don't feel confident with the key skills required within a topic, you should find these skills checks particularly beneficial.

How to use them

You're welcome to make use of these questions any way you see fit, but they are designed to fit into a little-and-often revision schedule. Research suggests you'll be more effective working in 25 minute bursts, with 5 minute breaks in between. Each skills check is designed to be easily doable within this time.

*First, **choose 5 skills checks** to focus on. If in doubt, go with the first five topics.*

During a 25 minute revision block, try the following:

- *Grab the **first skills check** from your set of 5 to tackle.*
- *Choose your **difficulty level** (a: easy, b: moderate, c: hard).*
- ***Complete all the questions** for your chosen difficulty level. **Time yourself**.*
- ***Check your answers**, make a note of any errors or misconceptions and look up (or ask a friend or teacher about) anything that doesn't make sense. Pay particular attention to common slips, where your method is valid, but you make a daft numerical or algebraic error. What can you do to avoid them, or find and fix them before finishing a question?*
- *Jot down your **score** (1 point for a substantially correct method, 2 points for correct method and fully correct answer for each question) and the **total time**.*
- *Put your skills check sheet at the **bottom of the pile** of 5 to revisit later.*

After you've worked your way through all five sheets, you'll start to revisit them:

- *Whenever you revisit a skills check, choose a **harder difficulty level**. Once you can confidently do all three (a, b and c), focus on speed and efficiency - start looking for ways to save time without compromising your accuracy.*

*Once you've mastered a particular skills check to your satisfaction, **set that sheet aside** and **add a brand new one** to your pile of five.*

Important:

These skills checks alone won't improve your problem solving or mathematical modelling skills – they are the building blocks you need, but on their own they are not enough. Once you feel you have mastered a particular topic's key skills, find some proper A-level Maths exam questions and work on applying your skills to them.

A-level Maths Skills Checks
Statistics Maths

Contents

Statistics Skills Check 1: Data collection

Record your results here:

Date	***Level (a/b)***	***Score (/5)***	***Time (mins)***

1. Know key terms in data collection

a)	b)
Give definitions for the following: population, census, sampling frame, sample.	State an advantage and a disadvantage of a census and a sample.

2. Know and describe sampling techniques

a)	b)
List the 3 random and the 2 non-random sampling techniques on your spec.	Describe each of the 5 sampling techniques .

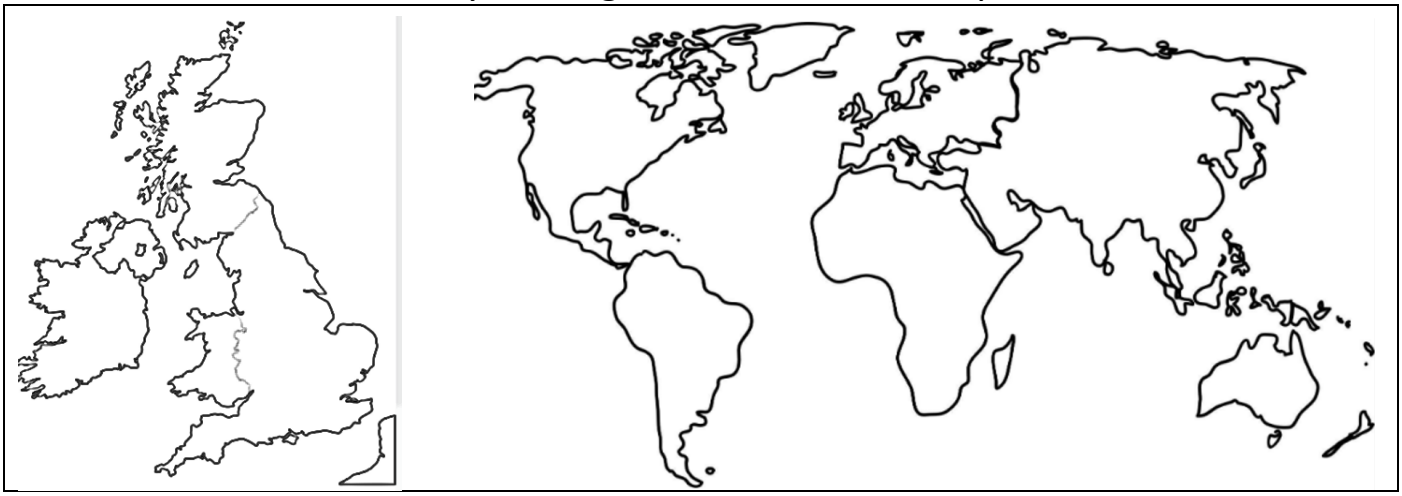
3. Know types of data

a)	b)
Is the following data quantitative or qualitative? Red, orange, green, blue.	Give one similarity and one difference between discrete and continuous data

4. Remember measures from Large Data Set

a)	b)
What are the measures recorded for the non-UK countries?	What do the following units measure? oktas, kn, Dm

5. Place the 8 locations on your large data set on the map below



Answers Statistics Skills Check 1: Data collection **Answers**

1.

a)	b)
A population is the whole set of items that are of interest. A census observes every member of a population. A sampling frame is a list of individually named/numbered sampling units. A sample is a selection of observations taken from a subset of the population.	Advantages: Census should give an accurate result. Sample is less time consuming and less expensive. Disadvantages: Census can be time consuming and expensive. Sample may not be representative of population.

2.

a)	b)
Random: simple random, systematic, stratified. Non-random: quota, opportunity.	Simple random: Allocate each population member a number then use a random number generator to generate desired sample size then choose member that corresponds to numbers generated. Systematic: select members of population at regular intervals Stratified: population is divided into groups and a random sample is taken from each

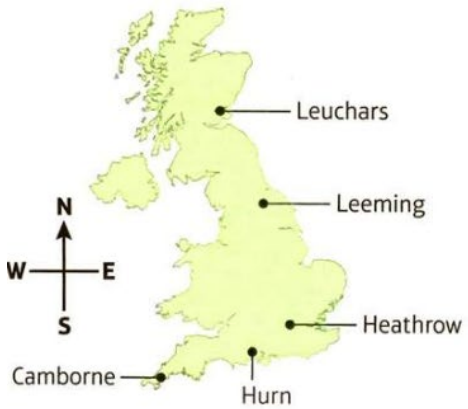
3.

a)	b)
Qualitative, does not involve numbers	Similarity: both quantitative Difference: discrete can only take certain values whereas continuous can take any value

4.

a) Daily Mean Air temperature, Rainfall (24 hour total), Daily mean pressure (hPa), Daily mean windspeed, Daily mean windspeed (Beaufort conversion)	b) oktas daily mean total cloud kn knots daily mean windspeed Dm decametre daily mean visibility

5.



Statistics Skills Check 2: Measures of location and spread

Record your results here:

Date	***Level (a/b/c)***	***Score (/4)***	***Time (mins)***

1. Recall the measures of location and spread and how to calculate them

a) List three measures of location and three measures of spread	b) Give the formula for calculating mean	c) How do you calculate lower quartile and upper quartile?

2. Understand IQR and IPR and what variance measures

a) Describe the difference between interquartile range and interpercentile range	b) Find the formula for calculating standard deviation in the formula booklet and describe what variance is

3. Use and understand coding: be able to calculate measures of coded data

a)

i) Code the data below using the coding $y = \frac{x-7}{80}$

807 967 727 167 207 767

ii) Calculate the mean of the coded data values

iii) Use your answer to part ii to calculate the mean of the original data

b) 20 giant ibises were caught for ringing. Their wingspans (y cm) were recorded to the nearest centimetre and the data coded using $z = \frac{y-5}{10}$. The following summary statistics were obtained from the coded data:

$\sum z = 104, S_{zz} = 1.8$

Work out the mean and standard deviation of the wingspans of the giant ibises.

4. Use interpolation to find MOL and MOS

a) Rachel records the number of CDs in the collections of students in her year. The results are in the table below.

Number of CDs	Frequency
35	3
36	17
37	29
38	34
39	12

Find Q_1, Q_2 and Q_3

b) A hotel is worried about the reliability of its lift. It keeps a weekly record of the number of times it breaks down over a period of 26 weeks. The data collected is summarised in the table below. Use interpolation to estimate the median number of breakdowns.

Number of breakdowns	Frequency
0–1	18
2–3	7
4–5	1

Answers Statistics Skills Check 2: **Answers**

1.

a) MOL: mean, mode, median MOS: standard deviation, variance, interquartile range, range	b) $\bar{x} = \frac{\sum x}{n}$ or $\bar{x} = \frac{\sum xf}{f}$ if in a frequency table	c) For listed data, divide n by 4 to find lower quartile and multiply n by ¾ for upper quartile

2.

a) IQR is upper quartile minus lower quartile whereas interpercentile range is the difference between two percentiles	b) standard deviation = $\sqrt{\frac{S_{xx}}{n}}$ or $\sqrt{\frac{\sum x^2}{n} - \bar{x}^2}$ Variance measures how far each number in the dataset is from the mean (and therefore every other number in the dataset)

3.

a) i) Using the coding given, data becomes 10 12 9 2 2.5 9.5 ii) $\bar{x} = \frac{10+12+9+2+2.5+9.5}{6} = 7.5$ iii) Code is subtract 7 from original data then divide by 80. So the original mean would be reversing that process $7.5 \times 80 + 7 = 607$	b) Coded mean $= \frac{104}{20} = 5.2$ Original mean $= 5.2 \times 10 + 5 = 57cm$ Coded standard deviation $= \sqrt{\frac{1.8}{20}} = 0.3$ Standard deviation $= 0.3 \times 10 = 3$

4.

a) $Q_1 = 24th$ value so 37 $Q_2 = 48th$ value so 37 $Q_3 = 72nd$ value so 38	b) median value is the 13th value, this is in the first class $\frac{m-0}{1.5-0} = \frac{13-0}{18-0}$ m = 1.08

Statistics Skills Check 3: Representations of data

Record your results here:

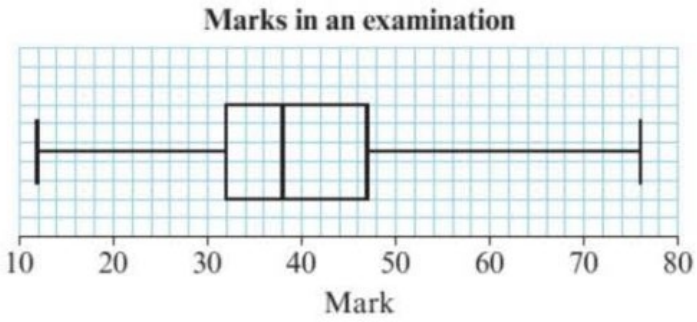
Date	***Level (a/b/c)***	***Score (/5)***	***Time (mins)***

1. Define what an outlier is

a) In your own words, define an outlier.	b) If the exam question doesn't define what an outlier is, what is the typical definition?	c) What is the difference between an outlier and an anomalous result?

2. Be able to draw, interpret boxplots and know how to represent outliers

a)



i) Write down the upper and lower quartile and the median

ii) Calculate the IQR

iii) Calculate the range

b)

Data for the maximum daily gust (in knots) in Camborne in September 1987 is taken from the large data set:

13 17 19 20 21 21 22 23 24
25 25 25 26 26 26 27 29 30 30 30
33 35 38 46 78

An outlier is defined as a value which lies either 1.5 x IQR above the upper quartile of 1.5 x IQR below the lower quartile

Draw a boxplot for this data.

3. Be able to draw and interpret cumulative frequency diagrams

a) The table shows the masses, in kilograms, of 120 Coulter pine cones.

Mass, m (kg)	Frequency
$1.0 \leqslant m < 1.2$	7
$1.2 \leqslant m < 1.4$	18
$1.4 \leqslant m < 1.6$	34
$1.6 \leqslant m < 1.8$	41
$1.8 \leqslant m < 2.0$	15
$2.0 \leqslant m < 2.2$	5

Draw a cumulative frequency diagram for this data

b)
The table shows the lengths, in cm, of 70 earthworms,

Length, l (cm)	Frequency
$6.0 \leqslant l < 6.5$	3
$6.5 \leqslant l < 7.0$	13
$7.0 \leqslant l < 7.5$	14
$7.5 \leqslant l < 8.0$	26
$8.0 \leqslant l < 8.5$	10
$8.5 \leqslant l < 9.0$	4

i) Draw a cumulative frequency diagram for this data.
ii) Estimate how many earthworms are
a) longer than 8.2cm
b) shorter than 7.3 cm

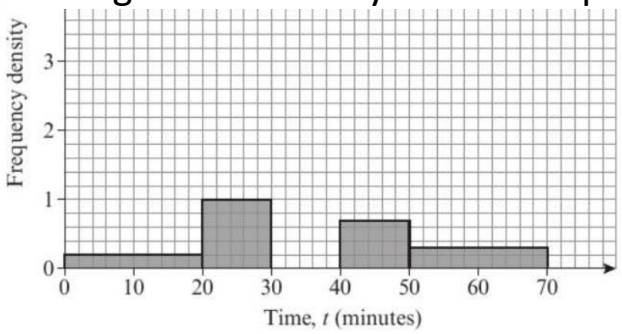
4. Be able to draw and interpret histograms and frequency polygons

a) The data show the mass, in pounds, of 50 adult puffer fish

Mass, m (pounds)	Frequency
$10 \leqslant m < 15$	4
$15 \leqslant m < 20$	12
$20 \leqslant m < 25$	23
$25 \leqslant m < 30$	8
$30 \leqslant m < 35$	3

i) Draw a histogram for this data
ii) On the same set of axes, draw a frequency polygon.

b) The partially completed histogram shows the time, in minutes, that passengers were delayed at an airport.



i) Complete the table

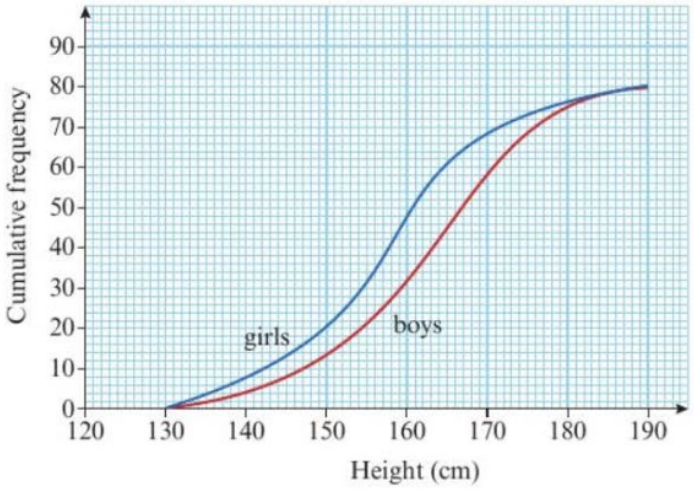
Time, t (min)	Frequency
$0 \leqslant t < 20$	4
$20 \leqslant t < 30$	
$30 \leqslant t < 35$	15
$35 \leqslant t < 40$	25
$40 \leqslant t < 50$	
$50 \leqslant t < 70$	

ii) Complete the histogram
iii) Estimate the number of passengers that were delayed for between 25 and 38 minutes
iv) Explain why a histogram was appropriate for this data

5. Understand how to compare two sets of data (include a MOL, a MOS and a reason why, all in the context of the question)

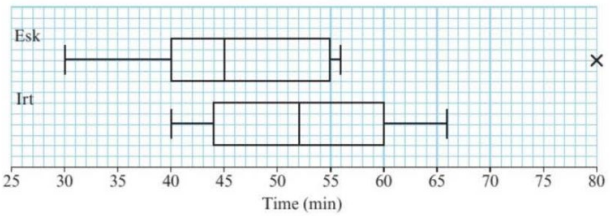
a)

The cumulative frequency diagram shows the distribution of heights of 80 boys and 80 girls in a basketball club. Compare the heights of boys and girls in the club.



b)

Fell runners from the Esk club and the Irt club were keen to see which club had the faster runners overall. They decided that all the members from both clubs would take part in a fell run. The time each runner took to complete the run was recorded. The results are summarised in the box plot.



Compare and contrast these two box plots

Answers Statistics Skills Check 3: **Answers**

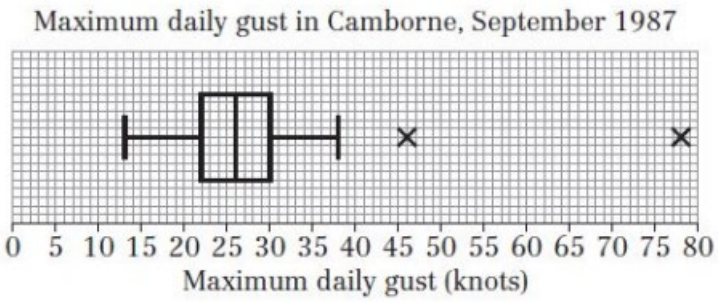
1.

a)	b)	c) Outliers could be legitimate values albeit extreme whereas anomalies are errors and should be removed from the data (known as cleaning the data)
An outlier is an extreme value that lies outside the overall pattern of the data.	Unless defined in the question otherwise Outlier is Either greater than $Q_3 + 1.5 \times IQR$ Or less than $Q_1 - 1.5 \times IQR$	

2.

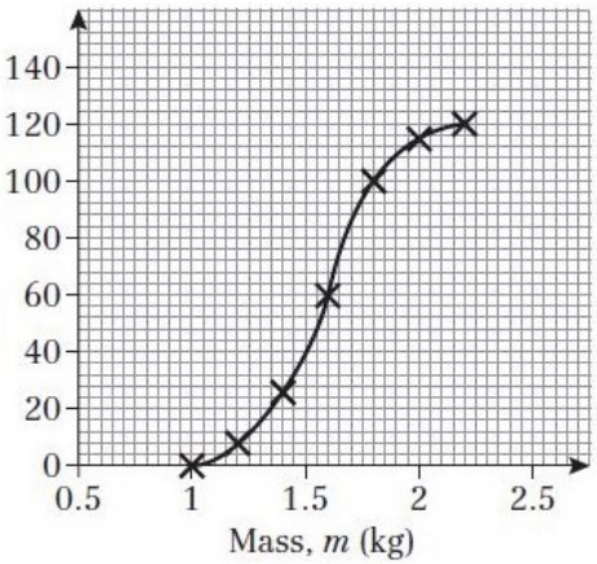
a)
i) UQ = 47 marks
LQ = 32 marks
ii) IQR = 15 marks
iii) Range = 64 marks

b)

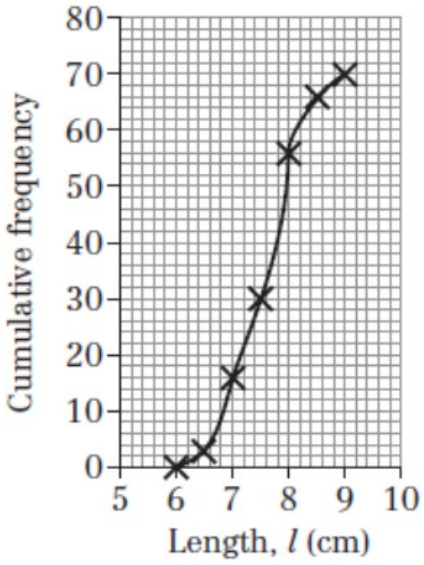


3.

a)



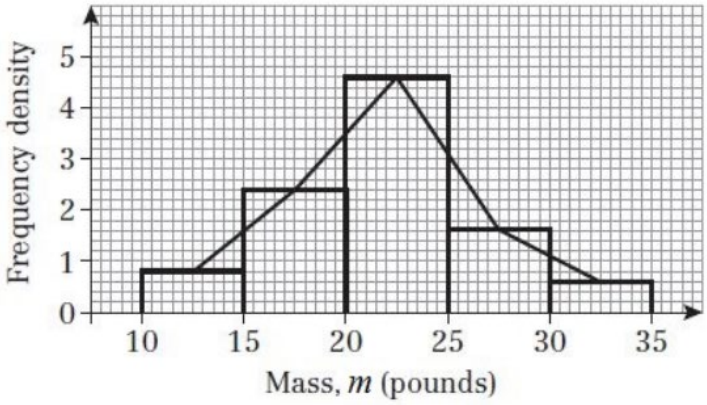
b)



a) 8
b) 24

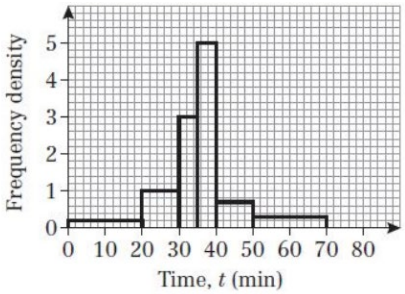
4.

a)



b)

Time, t (min)	Frequency	Class width	Frequency density
$0 \leqslant t < 20$	4	20	0.2
$20 \leqslant t < 30$	$10 \times 1 = 10$	10	1
$30 \leqslant t < 35$	15	5	3
$35 \leqslant t < 40$	25	5	5
$40 \leqslant t < 50$	$10 \times 0.7 = 7$	10	0.7
$50 \leqslant t < 70$	$20 \times 0.3 = 6$	20	0.3



iii) $\left(\frac{5}{10} \times 10\right) + 15 + \left(\frac{3}{5} \times 25\right) =$ 35 passengers

iv) Time is a continuous variable so histogram is appropriate

5.

a) The median height for boys is 163cm which is higher than the median heigh for girls at 158cm showing that on average, boys are taller. The boys IQR is 16.5cm which is higher than that of the girls at 15cm which shows the spread of the boys height is greater.

b) The Irt club had a higher median at 60mn compared to that of the Esk club at 45mn which means Irt club overall had the slowest runners.
The IQR ranges for both clubs were about the same with the Irt club being slightly more spread out. Esk club also had an outlier whereas Irt club did not.

Statistics Skills Check 4: Statistical distributions

Record your results here:

Date	***Level (a/b/c)***	***Score (/4)***	***Time (mins)***

1. Recall key characteristics of a probability distribution

a) What must be true for a discrete uniform probability distribution?	b) What can you say about the sum of probabilities for any probability distribution?	c) A fair dice is thrown four times and the number of times it falls with a 6 on the top, Y is noted. Write down the sample space of Y.

2. Know characteristics of a binomial distribution

a) What are the parameters of a binomial distribution?	b) List the 4 conditions needed to be met in order to model using a binomial distribution.

3. Be able to calculate binomial probabilities (binomial PD)

a) $X \sim B\left(8, \frac{1}{3}\right)$ i) $P(X = 2)$ ii) $P(X = 5)$	b) A balloon manufacturer claims that 95% of his balloons will not burst when blown up. If you have 20 of these balloons to blow up for a birthday party: Find the probability that exactly 2 balloons burst.

4. Be able to calculate cumulative binomial probabilities (binomial CD)

a) $X \sim B(37, 0.65)$ i) $P(X > 20)$ ii) $P(X \leq 26)$ iii) $P(15 \leq X < 20)$	b) A factory produces a component for the motor trade and 5% of the components are defective. A quality control officer regularly inspects a random sample of 50 components. Find the probability that the next sample contains fewer than 2 defectives.

Answers Statistics Skills Check 4: **Answers**

1.

a) All probabilities of all events must be equal	b) Probabilities sum to 1	c) {0,1,2,3,4}

2.

a) number of trials n probability of success p	b) Fixed number of trials, fixed probability of success, two outcomes: success or failure, trials are independent.

3.

a) Use binomial PD on your calculator or formula given in booklet i) 0.2731 ii) 0.0683	b) Let X be the number of balloons that pop $X \sim B(20, 0.05) \implies P(X = 2) = 0.1887$ Or: Let Y be the number of balloons that don't pop: $Y \sim B(20, 0.95) \implies P(Y = 18) = 0.1887$

4.

a) i) $1 - P(X \leq 20) = 0.8882$ ii) $P(X \leq 26) = 0.7992$	b) Let X be the number of defects in 50 components $X \sim B(50, 0.05)$ $P(X < 2) = P(X \leq 1) = 0.2794$

Statistics Skills Check 5: Hypothesis testing for binomial

Record your results here:

Date	***Level (a/b/c)***	***Score (/4)***	***Time (mins)***

1. Understand hypothesis testing and be able to set up suitable hypotheses

a)	b)
Dmitri wants to see whether a dice is biased towards the value of 6. He throws the dice 60 times and counts the number of sixes he gets. i) Describe the test statistic ii) Write down suitable null and alternative hypotheses.	In a manufacturing process the proportion of faulty articles has been found from long experience to be 0.1. A sample of 100 articles from a new manufacturing process is tested and 8 are found to be faulty. The manufacturer wishes to test at the 5% significance level whether or not there has been a reduction in the proportion of faulty articles. i) Describe the test statistic ii) Write down suitable null and alternative hypotheses iii) Explain the condition under which the null hypothesis is rejected

2. Be able to find critical regions and understand the implication of that

a) A test statistic has distribution $B(10, p)$. Given that: $H_0: p = 0.2$ $H_1 : p > 0.2$ Find the critical region for the rest using a 5% significance level.	b) A mechanical component fails, on average, 3 times out of every 10. An engineer designs a new system of manufacture that he believes reduces the likelihood of failure. He tests a sample of 20 components made using his new system. i) State suitable null and alternative hypotheses ii) Using a 5% significance level, find the critical region for a test to check his belief, ensuring the probability is as close as possible to 0.05 iii) Write down the actual significance level of the test

3. Perform a one tailed hypothesis test (in full)

a)	b)
A single observation, x, is taken from a binomial distribution $B(10, p)$ and a value of 5 is obtained. Use this observation to test $H_0: p = 0.25$ against $H_1: p > 0.25$ using a 5% significance level.	A polling organisation claims that the support for a particular candidate is 35%. It is revealed that the candidate will pledge to support local charities if elected. The polling organisation think that the level of support will go up as a result. It takes a new poll of 50 voters and 28 are found to support the candidate. Using a 5% level of significance, perform a hypothesis test to check whether the polling organisation was correct.

4. Perform a two tailed hypothesis test (in full)

a) A random variable has distribution $X \sim B(50, p)$. A single observation of $x = 4$ is taken from this distribution. Test, at the 2% significance level, $H_0: p = 0.02$ against $H_1: p \neq 0.02$.	b) The national proportion of people experiencing complications after having a particular operation in hospitals is 20%. A hospital decides to take a sample size of 20 from their records. They found that 8 out of their 20 patient experienced complications. By finding the critical region and ensuring the significance level is as close to 5% as possible, test whether this hospital's proportion of complications differs from the national proportion.

Answers Statistics Skills Check 5: **Answers**

1.

a) i) The number of sixes rolled in the 60 trials ii) $H_0: p = \frac{1}{6}$ iii) $H_1: p > \frac{1}{6}$	b) i) The number of faulty articles found in a sample of 100 ii) $H_0: p = 0.1\ H_1: p < 0.1$ iii) If the probability of that number being 7 is 2% or more the null hypothesis is accepted.

2.

a) $P(X \geq 4) = 0.1209$ $P(X \geq 5) = 0.0328$ Critical value is 5 so critical region is $X \geq 5$	b) $H_0: p = 0.3, H_1: p < 0.3$ $P(X \leq 2) = 0.03555$ $P(X \leq 3) = 0.107$ 0.0355 is closer to 0.05 so critical region is $X \leq 2$ Actual significance level = 3.55%

3.

a) $P(X \geq 5) = 0.0781$ 0.0781 > 0.05 so there is insufficient evidence to reject H_0	b) Let X be the number of people who support the candidate $H_0: p = 0.35, H_1: p > 0.35$ $X \sim B(50, 0.35)$ $P(X \geq 23) = 0.071$ $P(X \geq 24) = 0.0396$ Critical region is $X \geq 24$ 28 lies in the critical region, so there is evidence to reject the H_0 and hence evidence that the candidate's level of popularity has increased.

4.

a) $P(X \geq 4) = 0.01775 > 0.01$ So there is insufficient evidence to reject H_0	b) $H_0: p = 0.20, H_1: p \neq 0.20$ $X \sim B(20, 0.20)$ $P(X \leq 1) = 0.0692$ $P(X \leq 0) = 0.0115$ (closer to 0.025) Critical value is 0 $P(X \geq 9) = 0.01$ $P(X \geq 8) = 0.0321$ (closer to 0.025) Critical value is 8 so critical region is $0 \leq X \leq 8$ 8 lies in the critical region, so there is evidence to reject the H_0 and hence evidence that the hospital's proportion of complications differs from the national figure.

Statistics Skills Check 6: Regression, correlation and hypothesis testing for zero correlation

Record your results here:

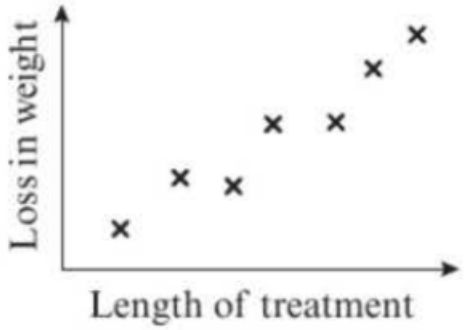
Date	*Level (a/b/c)*	*Score (/5)*	*Time (mins)*

1. Be able to draw and interpret scatter diagrams

a) Some research was done into the effectiveness of a weight-reducing drug. Seven people recorded their weight loss and this was compared with the length of time for which they had been treated. A scatter diagram was drawn to represent this data.

i) describe the type of correlation show

ii) interpret the correlation in context



b)

Eight students were asked to estimate the mass of a bag of sweets in grams. First, they were asked to estimate the mass without touching the bag and then they were told to pick the bag up and estimate the mass again. The results are shown n the table below.

Student	A	B	C	D	E	F	G	H
not touching bag (g)	25	18	32	27	21	35	28	30
holding bag (g)	16	11	20	17	15	26	22	20

i) explain why a scatter diagram is appropriate

ii) draw a scatter diagram

iii) describe and interpret the correlation

2. Be able to interpret gradient and y-intercept of regression line

a) The relationship between the number of coats of paint (x) applied to a boat and the resulting weather resistance in years

b) The table shows the ages of some chickens and the number of eggs that they laid in a month. Robin calculates the regression line of n on a as $n = 16.1 + 0.063a$

Age of chicken, a (months)	18	32	44	60	71	79	99	109	118	140
Number of eggs laid in a month, n	16	18	13	7	12	7	11	13	6	9

(y) was tested in a laboratory. The equation of the regression line is $y = 2.93 + 14.5x$ Interpret the meaning of the figures 2.93 and 1.45	Without further calculation, explain why Robin's regression equation is incorrect.

3. Be able to code exponential/polynomial graphs to transform into a linear one

a) Data are coded using $Y = \log y$ and $X = \log x$ to give a linear relationship. The equation of the regression line for the coded data is $Y = 1.2 + 0.4X$ i) State whether y and x form an exponential or polynomial graph ii) Find the values of the constants and an equation between y and x	b) Data are coded using $Y = \log y$ and $X = x$ to give a linear relationship. The equation of the regression line for the coded data is $Y = 0.4 + 1.6X$. The relationship between the original data sets is modelling by an equation of the form $y = ab^x$. Find the values of a and b.

4. Understand PMCC (Product Moment Correlation Coefficient)

a) State what PMCC is	b) What values can PMCC take?	c) What does PMCC allow us to do? And how can you calculate it?

5. Be able to perform a hypothesis test for zero correlation

a) n = 40 PMCC = 0.3275 Significance level = 5% Perform the test to see if there is evidence of a difference between the 2 variables	b) The following table shows the marks attained by 8 students in English and Maths.

Student	A	B	C	D	E	F	G	H
English	25	18	32	27	21	35	28	30
Mathematics	16	11	20	17	15	26	32	20

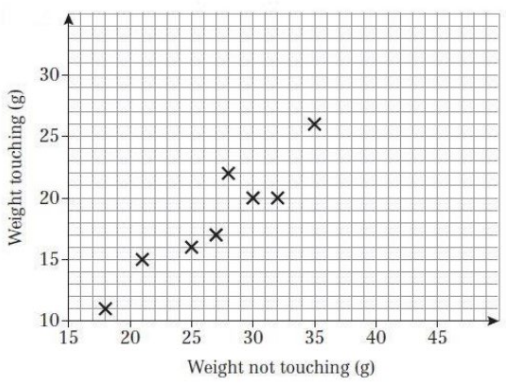
Test at the 5% significance level whether these results show evidence of a linear relationship between English and Maths marks.

Answers Statistics Skills Check 6: **Answers**

1.

a) i) Positive correlation
ii) the longer the treatment the greater the loss of weight

b)
i) bivariate data



iii) Positive correlation, each student's tendency to guess higher or lower was the same in both tests.

2.

a) For every coat of paint added, it adds another 1.45 years of weather protection on. With no coat of paint, there will 2.93 years of weather protection

b) The data shows negative correlation so the gradient should be negative

3.

a) i) Polynomial
ii) $Y = 1.2 + 0.4X$
$\log y = 1.2 + 0.4 \log x$
$y = 10^{1.2+0.4 \log x}$
$y = 10^{1.2} \times x^{0.4}$
$y = 15.8 \times x^{0.4}$

b)
$Y = 0.4 + 1.6X$
$\log y = 0.4 + 1.6x$
$y = 10^{0.4+1.6x}$
$y = 10^{0.4} \times 10^{1.6x}$
$y = 2.51 \times 39.8^{x}$

4.

a) PMCC measures the strength of the linear relationship between two variables

b) PMCC can take any value between and including -1 and 1

c) PMCC gives us a better measure of correlation strength, rule of thumb is ≥ 0.7 is strong positive whereas anything less is weak positive and similarly for negative correlation.
Calculator: stats mode > optn > regression calc > r=

5.

a) $H_0: \rho = 0, H_1: \rho \neq 0$
Look critical value up in formula booklet, PMCC correlation coefficients, use correct n and sig level. Critical value is ± 0.3120
Reject H_0 as 0.3275 > 0.3120 and hence lies in the critical region, there is evidence to suggest there is a correlation between the two variables.

b)
use calculator to get PMCC r = 0.68556
$H_0: \rho = 0, H_1: \rho \neq 0$
Critical value is 0.6215
Reject H_0 as there is reason to believe that there is a linear correlation between English and Maths marks.

Statistics Skills Check 7: Venn diagrams, tree diagrams and conditional probability

Record your results here:

Date	***Level (a/b/c)***	***Score (/5)***	***Time (mins)***

1. Understand key terminology and be able to calculate probabilities

a) Define the following:
sample space
event

b) Two coins are tossed, state the sample space

c) The lengths, in cm, of 240 koalas are recorded in a table.

Length, l (cm)	Frequency (male)	Frequency (female)
$65 \leqslant l < 70$	4	14
$70 \leqslant l < 75$	20	15
$75 \leqslant l < 80$	24	32
$80 \leqslant l < 85$	47	27
$85 \leqslant l < 90$	31	26

One koala is chosen at random
i) Find the probability that the koala is female
ii) find the probability that the koala is under 72cm long

2. Draw Venn diagrams and find probabilities

a) A group of 275 people at a music festival were asked if they play guitar, piano or drums.
1 person plays all 3 instruments
65 people play guitar and piano
10 people play piano and drums
30 people play guitar and drums
15 people play piano only
20 people play guitar only
35 people play drums only

i) draw a Venn diagram to represent this information

b) (Continued from (a) left)
ii) A festival goer is chosen at random from the group. Find the probability that the person chosen:
a) plays the piano
b) plays at two of guitar, piano, drums
c) plays exactly one instrument
d) plays none of the instruments

3. Understand mutually exclusive events and also independent events

a) Explain what two events A and B being mutually exclusive means	b) $P(A) = 0.5$ and $P(B) = 0.3$. Given that events A and B are independent, find $P(A \text{ and } B)$

4. Be able to draw tree diagrams and find probabilities

a) A bag contains three red beads and five blue beads. A bead is chosen at random from the bag, the colour is recorded and the bead is replaced. A second bead is chosen and the colour recorded. i) Draw a tree diagram	b) A bag contains 13 tokens, 4 blue, 3 red, 6 yellow. Two tokens are drawn from the bag without replacement. i) find the probability that both tokens are yellow A third token is drawn from the bag. ii) write down the probability that the third token is yellow given that the first two are yellow

5. Be able to use and understand union, intersection and complement notation (Hint: draw a fully labelled Venn diagram)

a) A and B are two events, P(A)=0.5, P(B)=0.2 and $P(A \cap B) = 0.1$ Find i) $P(A \cup B)$ ii) $P(B')$ iii) $P(A \cap B')$	b) A, B and C are three events with P(A)=0.55, P(B)=0.35 and P(C)=0.4. $P(A \cap C) = 0.2$. Given that A and B re mutually exclusive and B and C are independent. Find i) $P(A' \cap B')$ ii) $P(A \cup (B \cap C'))$ iii) $P((A \cap C)' \cup B')$

6. Be able to calculate conditional probabilities

a) A and B are two events, P(A)=0.4, P(B)=0.35 and $P(A \cap B) = 0.2$ Find i) $P(B\|A)$ ii) P(A'|B)	b) In a tennis match, the probability that Anne wins the first set against Colin is 0.7. If Anne wins the first set, the probability that she wins the second set is 0.8. If Anne loses the first set, the probability that she wins the second set is 0.4. A match is won when one player wins two sets. Find the probability that Anne wins given that the game is over after two sets.

Answers Statistics Skills Check 7: **Answers**

1.

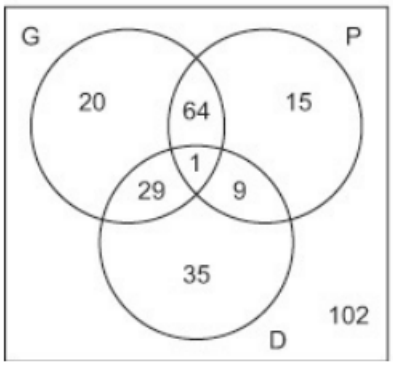
a) a sample space is the set of all possible outcomes
an event is a collection of one or more outcomes

b) {HH, TT, HT, TH}

c) i) $\frac{14+15+32+27+26}{240}$
19/40
ii) Male: $4 + \frac{72-70}{75-70} \times 20 = 12$
Females: $14 + \frac{72-70}{75-70} \times 15 = 20$
$\frac{12+20}{240}$ = 2/15

2.

a)



b)
a) 89/275
b) 103/275
c) 14/55
d) 102/275

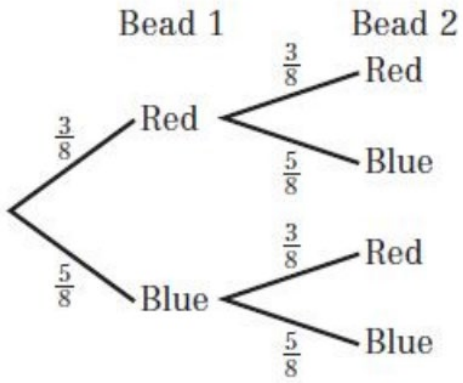
3.

a) Mutually exclusive means both events cannot happen at the same time i.e. $P(A \text{ and } B) = 0$

b) $P(A \text{ and } B) = 0.5 \times 0.3 = 0.15$

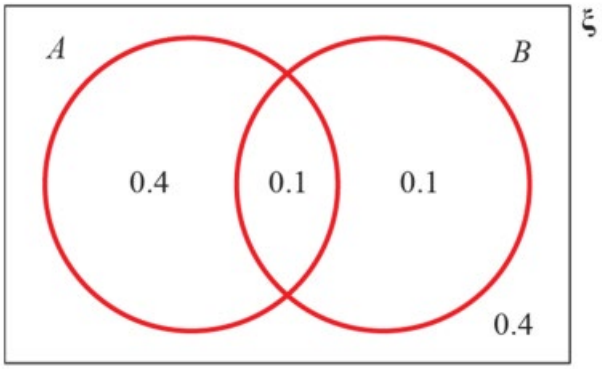
4.

a)



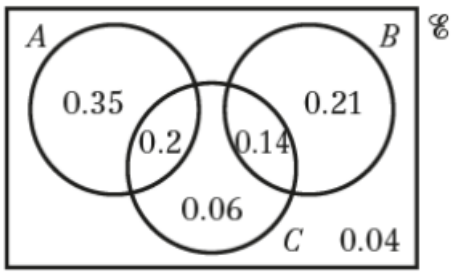
b)
i) $\frac{6}{13} \times \frac{5}{12} = \frac{5}{26}$
ii) 4/11

5.



a) i) 0.4+0.1+0.1 = 0.6
ii) 0.8
iii) 0.4

b)



i) 0.06+0.04 = 0.1
ii) 0.55+0.21 = 0.76
iii) 1

6.

a)

i) $P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{0.2}{0.4} = 0.5$

ii) $P(A'|B) = \frac{P(A' \cap B)}{P(B)} = \frac{0.15}{0.35} = 0.429$

b)

P(match over in two sets) = (0.7 x 0.8) +(0.3 x 0.6) = 0.74

$$P(Anne\ wins|match\ over\ in\ 2\ sets) = \frac{0.7 \times 0.8}{0.74} = 0.757$$

Statistics Skills Check 8: Normal distribution

Record your results here:

Date	***Level (a/b/c)***	***Score (/9)***	***Time (mins)***

1. Recall the condition under which data is modelled using a normal distribution.

a) What assumption is made about the mean, median and mode?	b) What is the total area under the normal distribution curve?	c) What type of data is used with normal distribution?

2. Recall parameters and the empirical rule

a) What are the two parameters of a normal distribution?	b) What is the approximate proportion of data, for a normal distribution, that lies *within one standard deviation* of the mean? What about within two standard deviations?

3. Be able to find probabilities for normal distributions

a) $X \sim N\ (30, 2^2)$ Find i) $P\ (X < 33)$ ii) $P\ (X > 26)$	b) The heights of a large group of women are normally distributed with a mean of 165 cm and a standard deviation of 3.5 cm. A woman is selected at random from this group. Find the probability that she is shorter than 160cm.

4. Be able to find a value a such that $P(x < a) = p$ where p is a given probability

5. Know and use the standard normal distribution

a) State the mean and standard deviation of the standard normal distribution	b) i) Find $P(Z < 2.12)$ ii) Find a such that $P(z > a) = 0.15$	c) The random variable $X \sim N(0.8, 0.05^2)$. For each of the following values of X, write down the corresponding value of the standardised normal distribution i) $x = 0.8$ ii) $x = 0.792$

6. Use the standard normal distribution to find missing mean and/or standard deviation

a) $Y \sim N\ (\mu, 40)$ and $P(Y < 25) = 0.15$ Find the value of μ	b) $X \sim N\ (\mu, \sigma^2)$ Given that $P(X > 15) = 0.2$ and $P(X < 9) = 0.2$ Find the value of μ and the value of σ	c) The masses of the penguins on an island are found to be normally distributed with mean μ and standard deviation σ. Given that 10% of the penguins have a mass less than 18kg and 5% of the penguins have a mass greater than 30kg, find the value of μ and the value of σ.

7 Understand when to approximate a binomial distribution with a normal distribution

a) What two conditions need to be met to approximate a binomial with a normal?	b) How do you find the parameters for the normal distribution from the binomial distribution parameters?	c) What else must you adjust given we are going from discrete to continuous data?

8. Understand how to approximate a binomial distribution with a normal distribution.

a) $X \sim B(100,0.6)$, use a suitable approximation to estimate

$$P(X > 58)$$

b) The probability of a roulette ball landing on red when the wheel is spun is 50/101. On one day in a casino, the wheel is spun 1200 times. Estimate the probability that the ball lands on red in at least half of these spins.

c) A particular breakfast cereal has prizes in 56% of the boxes. A random sample of 100 boxes is taken.
i) Find the exact value of the probability that exactly 55 boxes contain a prize
ii) Find the percentage error when using a normal approximation to calculate the probability that exactly 55 boxes contain prizes.

9. Be able to carry out a hypothesis test testing sample mean

a) $X \sim N(21,1.5^2)$
the sample mean $\bar{x}$ is 21.2 and sample size n is 20
Test at the 5% significance level that the mean is different to 21.

b) The IQ scores of a population are normally distributed with a mean of 100 and a standard deviation of 15. A psychologist wises to test the theory that eating chocolate before sitting and IQ test improves your score. A random sample of 80 people are selected and they are each given an identical bar of chocolate to eat before taking an IQ test. Given the mean score on the test for the sample of 80 people was 102.5.
Test at the 2.5% level whether the psychologist's claim is justified.

Answers Statistics Skills Check 8: **Answers**

1.

a) Mean = median = mode , it's what gives the bell shaped curve that symmetrical shape!	b) Area under the curve is equal to 1. We assume that all probability events will sum to 1	c) Continuous data i.e. it can take any value (e.g. height)

2.

a) $X \sim N\ (\mu, \sigma^2)$ Parameters are mean μ and variance σ^2	b) Empirical rule is 68 – 95 – 99.7 (68% of the data lies within one st dev of the mean, 95% lies within 2 st dev of the mean and 99.7% lies within 3 st dev of the mean). Note: remembering, approximately, the first one or two of these is helpful, but you can also use Normal CD to find the exact values.

3.

a) Use Normal CD button on your calculator and input the following Lower: a small value e.g. -10^6 Upper: 33 μ: 30 σ: 2 i) 0.9332 Similar to the above for next part but lower is 26 and upper is a big number e.g. 10^6 ii) 0.9772	b) Use Normal CD button on your calculator and input the following Lower: a small value e.g. -10^6 Upper: 160 μ: 165 σ: 3.5 0.0766

4.

a)
Use inverse normal button on your calculator and input the following:
Area: 0.1 σ: 3 μ: 12
i) 8.16
Similar to the above for next part but recall that "area" is always area to the left of the value so input
Area: $1 - 0.65$ σ: 3 μ: 12
ii) 10.84
For next part re-write

$$P(10 \leq X \leq a)$$
$$= P(X \leq a) - P(X \leq 10) = 0.25$$

Then calculate $P(X \leq 10)$ to add to 0.25 before using inverse normal button
iii) 12.02

b)
i) a = 19.12 , b = 18.3
ii) 0.0915

c) $P(X < a) = 0.1$
a = 57.436
$P(X < b) = 0.9$
b = 62.563
b – a = 5.13

5.

a) Standard normal distribution has the following parameters

$$Z \sim N(0,1^2)$$

b)
i) 0.9830
ii) a = 1.0364

c)
Using $Z = \frac{X-\mu}{\sigma}$
i) 0
ii) -0.16

6.

a)

$$P(Y < 25) = 0.15$$
$$P\left(Z < \frac{25-\mu}{\sqrt{40}}\right) = 0.15$$

Use the inverse normal button $z = -1.0364$
So

$$\frac{25-\mu}{\sqrt{40}} = -1.0364$$
$$\mu = 31.554$$

b)
If we have an area of 0.2 greater than 15 and an area of 0.2 less than 9, the mean must lie halfway between 9 and 15 using symmetry.

$$\mu = \frac{1}{2}(9+15) = 12$$
$$P\left(Z > \frac{15-12}{\sigma}\right) = 0.2$$

Use inverse normal button and get 0.8416

$$\frac{15-12}{\sigma} = 0.8416$$
$$\sigma = 3.56$$

c)

$$P(M < 18) = 0.1$$
$$P\left(Z < \frac{18-\mu}{\sigma}\right) = 0.1$$
$$z_1 = -1.28155$$
$$P(M > 30) = 0.05$$
$$P\left(Z > \frac{30-\mu}{\sigma}\right) = 0.05$$
$$z_2 = 1.64485$$

Two simultaneous equations to solve

$$-1.28155\sigma = 18 - \mu$$
$$1.64485\sigma = 30 - \mu$$

Leading to

$$\mu = 23.26, \sigma = 4.101$$

7.

a) n must be sufficiently large and p close to 0.5	b) $X \sim B(n,p)$ Then $Y \sim N\big(np, np(1-p)\big)$	c) Continuity correction – making the area covered by the probability 0.5 bigger, this ensures there are no gaps in the data

8.

a) n is large and p is close to 0.5 so use a normal approximation $Y \sim N(100 \times 0.6, 100 \times 0.6 \times 0.4)$ $Y \sim N(60, 24)$ $P(X > 58) = P(Y > 58.5) = 0.6203$	b) n is large and p is close to 0.5 so use a normal approximation $Y \sim N(1200 \times \frac{50}{101}, 100 \times \frac{50}{101} \times \frac{51}{101})$ $Y \sim N(594.06, 17.32^2)$ $P(X \geq 600) = P(Y > 599.5) = 0.3767$	c) i) binomal PD button on calculator 0.07838 ii) large n and p close to 0.5 → normal $Y \sim N(100 \times 0.56, 100 \times 0.56 \times 0.44)$ $Y \sim N(56, 24.64)$ $P(X = 55) = P(54.5 < Y < 55.5) = 0.07863$ Percentage error = $\frac{0.07838 - 0.07863}{0.07838} \times 100$ -0.31%

9.

a) $H_0: \mu = 21$ $H_1: \mu \neq 21$ Half significance level so 2.5% in each tail Assume $\bar{X} \sim N\left(21, \frac{1.5^2}{20}\right)$ $P(\bar{X} > 21.2) = 0.2755$ 0.2755 > 0.025 so not significant. Accept H_0	b) $H_0: \mu = 100$ $H_1: \mu > 100$ Assume $\bar{X} \sim N\left(100, \frac{15^2}{80}\right)$ $P(\bar{X} > a) = 0.025$ Use inverse normal to get a = 103.287 102.5 < 103.287 so therefore not significant, there is not sufficient evidence to say at the 2.5% significance level that eating chocolate before taking an IQ test improves the result.