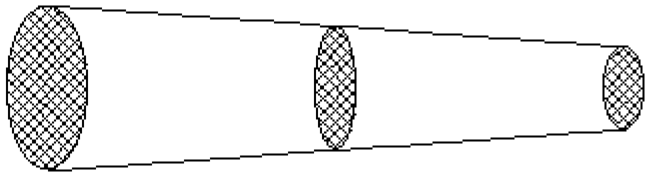
Simpson's Rule – From tree-trunks to spheres

Simpson's Rule for approximating volume:

This rule was quite tricky to develop, but it can be applied – with varying degrees of accuracy – to any three-dimensional solid.

$$V = \frac{h(A_b + 4A_m + A_t)}{6}$$



where: *V = volume*
h = height
A_b = cross-sectional area of the base
A_m = cross-sectional area of the middle
A_t = cross-sectional area of the top

There are a number of shapes where Simpson's rule gives the exact volume.

1) **Show, by substituting into the formula the appropriate values, that Simpson's rule gives the exact volume of a cylinder.**

Hint:
The volume of a cylinder is given by: $V = \pi r^2 h$ where **r** is the radius of the base and **h** is the height.

2) **The volume of a cone is given by:** $V = \frac{1}{3}\pi r^2 h$**. Show that Simpson's rule also works for a cone.**

Hint:
The radius of the circle produced by cutting midway down a cone will be half that of the circle at the base.

3) **Does Simpson's rule work for a sphere? The volume of a sphere is given by :** $V = \frac{4}{3}\pi r^3$**. Make a guess, then attempt to calculate the Simpson's rule approximation to prove or disprove.**

Hint:
The cross-sectional area of the 'end' of a sphere, like the pointed end of the cone, will be 0.