## Scrabble

In Scrabble, players pick letter tiles at random from a bag of 100 tiles.
Some letters are more common in English, so there are more of them in the game.

Events are mutually exclusive if they cannot happen at the same time. The chance of one or the other happening can be found by adding each probability.
Eg: chance of an S or a Blank on the first tile: $\frac{4}{100}+\frac{2}{100}=\frac{6}{100}$.

Events are independent if they do not affect one another. The chance of these events all happening can be found by multiplying their probabilities.
Eg: chance of the first tile being an E for two games in a row: $\frac{12}{100} \times \frac{12}{100}=\frac{144}{10000}$.
If events are not independent then a tree diagram can be used to work out what the different probabilities would be before multiplying.
Eg: chance of the first four tiles drawn all being A: $\frac{9}{100} \times \frac{8}{99} \times \frac{7}{98} \times \frac{6}{97}=\frac{3024}{94109400}$.

| Tile | Frequency |
| :---: | ---: |
| Blank | 2 |
| A | 9 |
| B | 2 |
| C | 2 |
| D | 4 |
| E | 12 |
| F | 2 |
| G | 3 |
| H | 2 |
| I | 9 |
| J | 1 |
| K | 1 |
| L | 4 |
| M | 2 |
| N | 6 |
| O | 8 |
| P | 2 |
| Q | 1 |
| R | 6 |
| S | 4 |
| T | 6 |
| U | 4 |
| V | 2 |
| W | 2 |
| X | 1 |
| Y | 2 |
| Z | 1 |
| Total: | 100 |

1. What is the chance of drawing a vowel for the first letter?
2. I play four games of scrabble. What is the chance of getting a vowel for the first letter in all four games?
3. I draw 7 tiles at the start of a game. What is the chance that they are all vowels?
4. Towards the end of the game, there are only 5 tiles left in the bag - a K, two Es, a blank and a T. I pick out two tiles. What is the chance that I get the blank? What is the chance that I don't?
5. I draw three tiles from the bag at the start of a game.

What is the chance that one of them is a Z ?
Hint: First work out the chance of not getting a Z at all.

## Scrabble SOLUTIONS

| Tile | Frequency |
| :---: | ---: |
| Blank | 2 |
| A | 9 |
| B | 2 |
| C | 2 |
| D | 4 |
| E | 12 |
| F | 2 |
| G | 3 |
| H | 2 |
| I | 9 |
| J | 1 |
| K | 1 |
| L | 4 |
| M | 2 |
| N | 6 |
| O | 8 |
| P | 2 |
| Q | 1 |
| R | 6 |
| S | 4 |
| T | 6 |
| U | 4 |
| V | 2 |
| W | 2 |
| X | 1 |
| Y | 2 |
| Z | 100 |
| Total: |  |
|  | 2 |

1. What is the chance of drawing a vowel for the first letter?

$$
\begin{gathered}
P(A \cup E \cup I \cup O \cup U)=P(A)+P(E)+P(I)+P(O)+P(U) \\
=9 \%+12 \%+9 \%+8 \%+4 \%=\mathbf{4 2} \%
\end{gathered}
$$

2. I play four games of scrabble. What is the chance of getting a vowel for the first letter in all four games?

$$
\begin{gathered}
P(\text { vowelfirstfor } 4 \text { games }) \\
=P(\text { vowel }) \times P(\text { vowel }) \times P(\text { vowel }) \times P(\text { vowel }) \\
=0.42 \times 0.42 \times 0.42 \times 0.42=0.03111696 \approx 3.1 \%
\end{gathered}
$$

3. I draw 7 tiles at the start of a game. What is the chance that they are all vowels?

$$
\frac{42}{100} \times \frac{41}{99} \times \frac{40}{98} \times \frac{39}{97} \times \frac{38}{96} \times \frac{37}{95} \times \frac{36}{94} \approx \mathbf{0 . 0 0 1 6 8 5} \text { or } 0.17 \%
$$

4. Towards the end of the game, there are only 5 tiles left in the bag a K, two Es, a blank and a T. I pick out two tiles. What is the chance that I get the blank? What is the chance that I don't?

No blank: $P($ noblank $\cap$ noblank $)=\frac{4}{5} \times \frac{3}{4}=\frac{3}{5}$
Blank: $1-\frac{3}{5}=\frac{2}{5}$
5. I draw three tiles from the bag at the start of a game. What is the chance that one of them is a Z?
Hint: First work out the chance of not getting a $Z$ at all.

$$
\begin{gathered}
P(N o t Z \cap N o t Z \cap N o t Z)=\frac{99}{100} \times \frac{98}{99} \times \frac{97}{98}=\frac{97}{100} \\
P(Z \text { in } f i r s t ~ 3)=1-P(N o t Z \cap N o t Z \cap N o t Z)=1-\frac{97}{100}=\frac{3}{100}=3 \%
\end{gathered}
$$

