Round The Bend (1)

When a car travels at speed around a bend, if the road is flat the only force causing circular motion is the friction between the tyres and the road.

Banking a road or track means the normal reaction force acting on the vehicle (which always acts at right angles to the surface) will have a radial component, allowing the vehicle to travel more quickly around the bend.

Where the M69 meets the M6, an interchange comprising an almost circular stretch of road has been constructed. It has radius of curvature 180 metres and is banked at an angle of 4°.

Every banked curve is designed for a specific speed, which is the speed at which you could travel round it without depending on friction at all.

Find the speed this curve is designed for, in mph ($1ms^{-1} \approx 2.237mph$).

The speed limit for this stretch of road is 30mph. Calculate the minimum value for the coefficient of friction for a driver travelling at this speed.

Round The Bend (2)

The Daytona International Speedway tri-oval racetrack has one of the most steeply banked tracks in racing, boasting a 31° slope on a $300m$ radius.

The 'natural speed' of a banked track is the speed at which you could drive around it without friction acting.

Coefficient of friction data

Normal road car, dry road surface:	$\mu \approx 1$
Stock car, dry racing track:	$\mu \approx 1.35$
Formula 1, dry racing track:	$\mu \approx 1.6$

Generally speaking, wet conditions reduce the coefficient of friction by around 30%

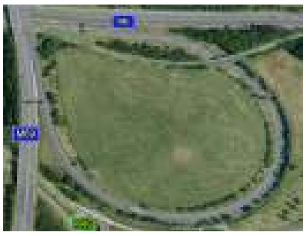
What is the natural speed of this curved piece of track?

What is the maximum speed a stock car could drive around this curve in dry conditions? What is the maximum speed in wet conditions? Use the coefficient of friction data above.

Round The Bend (1) SOLUTIONS

When a car travels at speed around a bend, if the road is flat the only force causing circular motion is the friction between the tyres and the road.

Banking a road or track means the normal reaction force acting on the vehicle (which always acts at right angles to the surface) will have a radial component, allowing the vehicle to travel more quickly around the bend.



Where the M69 meets the M6, an interchange comprising an almost circular stretch of road has been constructed. It has radius of curvature 180 metres and is banked at an angle of 4°.
Every banked curve is designed for a specific speed, which is the speed at which you could travel round it without depending on friction at all. Find the speed this curve is designed for, in mph ($1ms^{-1} \approx 2.237mph$).

With no frictional force, the centripetal force is provided only by the radial component of the normal reaction:

$$Resolving\ vertically: \quad R\cos 4 = mg \quad \Rightarrow \quad R = \frac{mg}{\cos 4}$$

$$Resolving\ radially: \quad R\sin 4 = \frac{mv^2}{r} \quad \Rightarrow \quad \frac{mg}{\cos 4}\sin 4 = \frac{mv^2}{180}$$

$$\Rightarrow \quad 180g\tan 4 = v^2 \quad \Rightarrow \quad v = 11.106\ldots ms^{-1} = \mathbf{24.8mph\ to\ 3\ s.f.}$$

The speed limit for this stretch of road is 30mph. Calculate the minimum value for the coefficient of friction for a driver travelling at this speed.

$$30mph = 13.4ms^{-1}\ to\ 3\ s.f. \quad and\ for\ minimum\ \mu\ assume\ F_r = \mu R$$

$$Resolving\ vertically: \quad R\cos 4 - F_r\sin 4 = mg$$

$$Resolving\ radially: \quad R\sin 4 + F_r\cos 4 = \frac{m(13.4^2)}{180}$$

$$R(\cos 4 - \mu\sin 4) = mg \quad and \quad R(\sin 4 + \mu\cos 4) = 0.99\ldots m$$

$$\frac{(\sin 4 + \mu\cos 4)}{(\cos 4 - \mu\sin 4)} = \frac{0.99\ldots}{g} \quad \Rightarrow \quad \sin 4 + \mu\cos 4 = 0.101\cos 4 - 0.101\mu\sin 4$$

$$\mu(\cos 4 + 0.101\sin 4) = 0.101\cos 4 - \sin 4 \quad \Rightarrow \quad \mu = \frac{0.101\cos 4 - \sin 4}{(\cos 4 + 0.101\sin 4)} = \mathbf{0.032}$$

Round The Bend (2) SOLUTIONS

The Daytona International Speedway tri-oval racetrack has one of the most steeply banked tracks in racing, boasting a 31° slope on a 300m radius.

Coefficient of friction data

Normal road car, dry road surface:	$\mu \approx 1$
Stock car, dry racing track:	$\mu \approx 1.35$
Formula 1, dry racing track:	$\mu \approx 1.6$

Generally speaking, wet conditions reduce the coefficient of friction by around 30%

What is the natural speed of this curved piece of track?

$$\textit{Resolving vertically}: \quad R\cos 31 = mg \quad \Longrightarrow \quad R = \frac{mg}{\cos 31}$$

$$\textit{Resolving radially}: \quad R\sin 31 = \frac{mv^2}{r} \quad \Longrightarrow \quad \frac{mg}{\cos 31}\sin 31 = \frac{mv^2}{300}$$

$$\Longrightarrow \quad 300g\tan 31 = v^2 \quad \Longrightarrow \quad v = 42.03 \ldots ms^{-1} = \mathbf{94.0mph\ to\ 3\ s.f.}$$

What is the maximum speed a stock car could drive around this curve in dry conditions? What is the maximum speed in wet conditions? Use the coefficient of friction data above.

$$\textit{Resolving vertically}: \quad R\cos 31 - F_r\sin 31 = mg$$

$$\textit{Resolving radially}: \quad R\sin 31 + F_r\cos 31 = \frac{mv^2}{r}$$

$$F_r = \mu R \quad \Longrightarrow \quad R(\cos 31 - \mu\sin 31) = mg \quad \textit{and} \quad R(\sin 31 + \mu\cos 31) = \frac{mv^2}{300}$$

$$\Longrightarrow \quad \frac{\sin 31 + \mu\cos 31}{\cos 31 - \mu\sin 31} = \frac{v^2}{300g}$$

$$\textit{In dry conditions } \mu = 1.35 \quad \textit{and} \quad v = 174.2 \ldots ms^{-1} = \mathbf{390mph\ to\ 3\ s.f.}$$

$$\textit{In wet conditions } \mu = 0.945 \quad \textit{and} \quad v = 102.5 \ldots ms^{-1} = \mathbf{231mph\ to\ 3\ s.f.}$$