Exponentially Increasing Like Rabbits

Along with the convicts and entrepreneurs, the 18^{th} century settlers of Australia brought rabbits.

In 1859, 24 rabbits were released into the wild from Geelong, Victoria so they could be hunted for sport.

Farmers couldn't shoot them fast enough, and with very few natural predators the rabbit population exploded, and by 1916 it had reached a million.

1.
The rate of increase of the population can be modelled as proportional to the population. Use the information above to formulate a relationship between P and t of the form $P = Ae^{kt}$ where A and k are constants to be found.

2.
By 1950 the country was overrun. Use your formula to calculate an estimate for the number of rabbits in 1950, giving your answer to 2 significant figures.

3.
Shortly afterwards, when large-scale culling and the famous 1000 mile rabbit-proof fence had proved ineffectual, and the population was now estimated at 600 million, the Myxomatosis virus was introduced. This disease devastated the rabbit population, which declined to 100 million in just 2 years. Assuming the decline in population is also proportional to the population, calculate how long it would take, at this rate, for the number of rabbits to drop as low as 1000.

4.
In actual fact, the 100 million remaining rabbits were largely resistant to the virus, and between 1952 and 1991 the population rose to 250 million. Formulate an equation for this new rate of growth and use it to estimate the year in which the rabbit population will once again exceed 600 million.

Exponentially Increasing Like Rabbits - SOLUTIONS

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The rate of increase of the population can be modelled as proportional to the population. Use the information above to formulate a relationship between P and t of the form $P = Ae^{kt}$ where A and k are constants to be found.

$$A = 24 \ (initial\ population\ at\ time\ t = 0)$$

$$1000000 = 24e^{57k} \quad \Rightarrow \quad k = \frac{1}{57}\ln\frac{125000}{3} \approx 0.1866\ to\ 4\ s.f.$$

$$\boldsymbol{P = 24e^{\left(\frac{1}{57}\ln\frac{125000}{3}\right)t}}$$

2.
By 1950 the country was overrun. Use your formula to calculate an estimate for the number of rabbits in 1950, giving your answer to 2 significant figures.

$$P = 24e^{\left(\frac{1}{57}\ln\frac{125000}{3}\right)\times 91} \approx 569722629 = \mathbf{570000000\ to\ 2\ s.f.}$$

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$$A = 600000000 \quad and \quad 100000000 = 600000000e^{2k} \quad \Rightarrow \quad k = \frac{1}{2}\ln\frac{1}{6}$$

$$P = 600000000e^{\left(\frac{1}{2}\ln\frac{1}{6}\right)t}$$

$$1000 = 600000000e^{\left(\frac{1}{2}\ln\frac{1}{6}\right)t} \quad \Rightarrow \quad t = \frac{\ln\frac{1}{600000}}{\frac{1}{2}\ln\frac{1}{6}} \approx 14.85 = \mathbf{15\ years\ to\ the\ nearest\ year}$$

4.
In actual fact, the 100 million remaining rabbits were largely resistant to the virus, and between 1952 and 1991 the population rose to 250 million. Formulate an equation for this new rate of growth and use it to estimate the year in which the rabbit population will once again exceed 600 million.

$$A = 100000000 \quad and \quad 250000000 = 100000000e^{39k} \quad \Rightarrow \quad k = \frac{1}{39}\ln 2.5$$

$$P = 100000000e^{\left(\frac{1}{39}\ln 2.5\right)t}$$

$$600000000 = 100000000e^{\left(\frac{1}{39}\ln 2.5\right)t} \quad \Rightarrow \quad t = \frac{\ln 6}{\frac{1}{39}\ln 2.5} \approx 76.26 \quad \Rightarrow \quad \boldsymbol{In\ the\ year\ 2028}$$