Practice examination questions for RUVAT equations

1.Jan2005

7 The unit vectors **i** and **j** are directed east and north respectively. A yacht moves with a constant acceleration. At time t seconds the position vector of the yacht is **r** metres. When $t = 0$ the velocity of the yacht is $(2\mathbf{i} - \mathbf{j})\,\mathrm{m\,s^{-1}}$, and when $t = 10$ the velocity of the yacht is $(-\mathbf{i} + \mathbf{j})\,\mathrm{m\,s^{-1}}$.

(a) Find the acceleration of the yacht. *(3 marks)*

(b) When $t = 0$ the yacht is 20 metres due east of the origin. Find an expression for **r** in terms of t. *(3 marks)*

(c) (i) Show that when $t = 20$ the yacht is due north of the origin. *(2 marks)*

(ii) Find the speed of the yacht when $t = 20$. *(4 marks)*

2.June2005

7 A particle moves on a smooth horizontal surface with acceleration $(3\mathbf{i} - 5\mathbf{j})\,\mathrm{m\,s^{-2}}$. Initially the velocity of the particle is $4\mathbf{j}\,\mathrm{m\,s^{-1}}$.

(a) Find an expression for the velocity of the particle at time t seconds. *(2 marks)*

(b) Find the time when the particle is travelling in the **i** direction. *(2 marks)*

(c) Show that when $t = 4$ the speed of the particle is $20\,\mathrm{m\,s^{-1}}$. *(4 marks)*

3.Jan 2006

2 A particle P moves with acceleration $(-3\mathbf{i} + 12\mathbf{j})\,\mathrm{m\,s^{-2}}$. Initially the velocity of P is $4\mathbf{i}\,\mathrm{m\,s^{-1}}$.

(a) Find the velocity of P at time t seconds. *(2 marks)*

(b) Find the speed of P when $t = 0.5$. *(3 marks)*

4.June 2006

6 The points A and B have position vectors $(3\mathbf{i} + 2\mathbf{j})$ metres and $(6\mathbf{i} - 4\mathbf{j})$ metres respectively. The vectors **i** and **j** are in a horizontal plane.

(a) A particle moves from A to B with constant velocity $(\mathbf{i} - 2\mathbf{j})\,\mathrm{m\,s^{-1}}$. Calculate the time that the particle takes to move from A to B. *(3 marks)*

(b) The particle then moves from B to a point C with a constant acceleration of $2\mathbf{j}\,\mathrm{m\,s^{-2}}$. It takes 4 seconds to move from B to C.

(i) Find the position vector of C. *(4 marks)*

(ii) Find the distance AC. *(2 marks)*

5.Jan 2007

8 A particle is initially at the origin, where it has velocity $(5\mathbf{i} - 2\mathbf{j})\,\mathrm{m\,s^{-1}}$. It moves with a constant acceleration $\mathbf{a}\,\mathrm{m\,s^{-2}}$ for 10 seconds to the point with position vector $75\mathbf{i}$ metres.

(a) Show that $\mathbf{a} = 0.5\mathbf{i} + 0.4\mathbf{j}$. *(3 marks)*

(b) Find the position vector of the particle 8 seconds after it has left the origin. *(3 marks)*

(c) Find the position vector of the particle when it is travelling parallel to the unit vector $\mathbf{i}$. *(6 marks)*

6.June 2007

8 A boat is initially at the origin, heading due east at $5\,\mathrm{m\,s^{-1}}$. It then experiences a constant acceleration of $(-0.2\mathbf{i} + 0.25\mathbf{j})\,\mathrm{m\,s^{-2}}$. The unit vectors $\mathbf{i}$ and $\mathbf{j}$ are directed east and north respectively.

(a) State the initial velocity of the boat as a vector. *(1 mark)*

(b) Find an expression for the velocity of the boat t seconds after it has started to accelerate. *(2 marks)*

(c) Find the value of t when the boat is travelling due north. *(3 marks)*

(d) Find the bearing of the boat from the origin when the boat is travelling due north. *(6 marks)*

7.Jan 2008

8 A Jet Ski is at the origin and is travelling due north at $5\,\mathrm{m\,s^{-1}}$ when it begins to accelerate uniformly. After accelerating for 40 seconds, it is travelling due east at $4\,\mathrm{m\,s^{-1}}$. The unit vectors $\mathbf{i}$ and $\mathbf{j}$ are directed east and north respectively.

(a) Show that the acceleration of the Jet Ski is $(0.1\,\mathbf{i} - 0.125\,\mathbf{j})\,\mathrm{m\,s^{-2}}$. *(4 marks)*

(b) Find the position vector of the Jet Ski at the end of the 40 second period. *(3 marks)*

(c) The Jet Ski is travelling southeast t seconds after it leaves the origin.

(i) Find t. *(5 marks)*

(ii) Find the velocity of the Jet Ski at this time. *(2 marks)*

8.June 2008

5 The unit vectors $\mathbf{i}$ and $\mathbf{j}$ are directed east and north respectively. A helicopter moves horizontally with a constant acceleration of $(-0.4\mathbf{i} + 0.5\mathbf{j})\,\mathrm{m\,s^{-2}}$. At time $t = 0$, the helicopter is at the origin and has velocity $20\mathbf{i}\,\mathrm{m\,s^{-1}}$.

(a) Write down an expression for the velocity of the helicopter at time t seconds. *(2 marks)*

(b) Find the time when the helicopter is travelling due north. *(3 marks)*

(c) Find an expression for the position vector of the helicopter at time t seconds. *(2 marks)*

(d) When $t = 100$:

(i) show that the helicopter is due north of the origin; *(3 marks)*

(ii) find the speed of the helicopter. *(3 marks)*

9.Jan 2009

6 Two forces, $\mathbf{P} = (6\mathbf{i} - 3\mathbf{j})$ newtons and $\mathbf{Q} = (3\mathbf{i} + 15\mathbf{j})$ newtons, act on a particle. The unit vectors $\mathbf{i}$ and $\mathbf{j}$ are perpendicular.

(a) Find the resultant of $\mathbf{P}$ and $\mathbf{Q}$. *(2 marks)*

(b) Calculate the magnitude of the resultant of $\mathbf{P}$ and $\mathbf{Q}$. *(2 marks)*

(c) When these two forces act on the particle, it has an acceleration of $(1.5\mathbf{i} + 2\mathbf{j})\,\mathrm{m\,s^{-2}}$. Find the mass of the particle. *(2 marks)*

(d) The particle was initially at rest at the origin.

(i) Find an expression for the position vector of the particle when the forces have been applied to the particle for t seconds. *(2 marks)*

(ii) Find the distance of the particle from the origin when the forces have been applied to the particle for 2 seconds. *(2 marks)*

10.June 2009

7 A particle moves on a smooth horizontal plane. It is initially at the point A, with position vector $(9\mathbf{i} + 7\mathbf{j})$ m, and has velocity $(-2\mathbf{i} + 2\mathbf{j})\,\mathrm{m\,s^{-1}}$. The particle moves with a constant acceleration of $(0.25\mathbf{i} + 0.3\mathbf{j})\,\mathrm{m\,s^{-2}}$ for 20 seconds until it reaches the point B. The unit vectors $\mathbf{i}$ and $\mathbf{j}$ are directed east and north respectively.

(a) Find the velocity of the particle at the point B. *(3 marks)*

(b) Find the velocity of the particle when it is travelling due north. *(4 marks)*

(c) Find the position vector of the point B. *(3 marks)*

(d) Find the average velocity of the particle as it moves from A to B. *(2 marks)*

11.Jan 2010

5 The constant forces $\mathbf{F}_1 = (8\mathbf{i} + 12\mathbf{j})$ newtons and $\mathbf{F}_2 = (4\mathbf{i} - 4\mathbf{j})$ newtons act on a particle. No other forces act on the particle.

(a) Find the resultant force acting on the particle. *(2 marks)*

(b) Given that the mass of the particle is 4 kg, show that the acceleration of the particle is $(3\mathbf{i} + 2\mathbf{j})\,\mathrm{m\,s^{-2}}$. *(2 marks)*

(c) At time t seconds, the velocity of the particle is $\mathbf{v}\,\mathrm{m\,s^{-1}}$.

(i) When $t = 20$, $\mathbf{v} = 40\mathbf{i} + 32\mathbf{j}$.

Show that $\mathbf{v} = -20\mathbf{i} - 8\mathbf{j}$ when $t = 0$. *(3 marks)*

(ii) Write down an expression for $\mathbf{v}$ at time t. *(1 mark)*

(iii) Find the times when the speed of the particle is $8\,\mathrm{m\,s^{-1}}$. *(6 marks)*

12.June 2010

7 A particle, of mass 10 kg, moves on a smooth horizontal surface. A single horizontal force, $(9\mathbf{i} + 12\mathbf{j})$ newtons, acts on the particle.

The unit vectors $\mathbf{i}$ and $\mathbf{j}$ are directed east and north respectively.

(a) Find the acceleration of the particle. *(2 marks)*

(b) At time t seconds, the velocity of the particle is $\mathbf{v}\,\mathrm{m\,s^{-1}}$. When $t = 0$, the velocity of the particle is $(2.2\mathbf{i} + \mathbf{j})\,\mathrm{m\,s^{-1}}$ and the particle is at the origin.

(i) Find the distance between the particle and the origin when $t = 5$. *(4 marks)*

(ii) Express $\mathbf{v}$ in terms of t. *(2 marks)*

(iii) Find t when the particle is travelling north-east. *(3 marks)*

13.Jan 2011

5 A particle moves with constant acceleration $(-0.4\mathbf{i} + 0.2\mathbf{j})\,\mathrm{m\,s^{-2}}$. Initially, it has velocity $(4\mathbf{i} + 0.5\mathbf{j})\,\mathrm{m\,s^{-1}}$. The unit vectors $\mathbf{i}$ and $\mathbf{j}$ are directed east and north respectively.

(a) Find an expression for the velocity of the particle at time t seconds. *(2 marks)*

(b) (i) Find the velocity of the particle when $t = 22.5$. *(2 marks)*

(ii) State the direction in which the particle is travelling at this time. *(1 mark)*

(c) Find the time when the speed of the particle is $5\,\mathrm{m\,s^{-1}}$. *(6 marks)*

14.June 2011

4 Two particles, A and B, are moving on a smooth horizontal surface when they collide. The mass of A is 6 kg and the mass of B is m kg. Before the collision, the velocity of A is $(5\mathbf{i} + 18\mathbf{j})\,\mathrm{m\,s^{-1}}$ and the velocity of B is $(2\mathbf{i} - 5\mathbf{j})\,\mathrm{m\,s^{-1}}$. After the collision, the velocity of A is $8\mathbf{i}\,\mathrm{m\,s^{-1}}$ and the velocity of B is $V\mathbf{j}\,\mathrm{m\,s^{-1}}$.

(a) Find m. *(3 marks)*

(b) Find V. *(3 marks)*

15.June 2011

7 A helicopter is initially hovering above a lighthouse. It then sets off so that its acceleration is $(0.5\mathbf{i} + 0.375\mathbf{j})\,\mathrm{m\,s^{-2}}$. The helicopter does not change its height above sea level as it moves. The unit vectors **i** and **j** are directed east and north respectively.

(a) Find the speed of the helicopter 20 seconds after it leaves its position above the lighthouse. *(4 marks)*

(b) Find the bearing on which the helicopter is travelling, giving your answer to the nearest degree. *(3 marks)*

(c) The helicopter stops accelerating when it is 500 metres from its initial position.

Find the time that it takes for the helicopter to travel from its initial position to the point where it stops accelerating. *(5 marks)*

16.Jan 2012

7 A helicopter is initially at rest on the ground at the origin when it begins to accelerate in a vertical plane. Its acceleration is $(4.2\mathbf{i} + 2.5\mathbf{j})\,\mathrm{m\,s^{-2}}$ for the first 20 seconds of its motion. The unit vectors **i** and **j** are horizontal and vertical respectively.

Assume that the helicopter moves over horizontal ground.

(a) Find the height of the helicopter above the ground at the end of the 20 seconds. *(3 marks)*

(b) Find the velocity of the helicopter at the end of the 20 seconds. *(2 marks)*

(c) Find the speed of the helicopter when it is at a height of 180 metres above the ground. *(7 marks)*

17.June 2012

7 A particle moves with a constant acceleration of $(0.1\mathbf{i} - 0.2\mathbf{j})\,\mathrm{m\,s^{-2}}$. It is initially at the origin where it has velocity $(-\mathbf{i} + 3\mathbf{j})\,\mathrm{m\,s^{-1}}$. The unit vectors **i** and **j** are directed east and north respectively.

(a) Find an expression for the position vector of the particle t seconds after it has left the origin. *(2 marks)*

(b) Find the time that it takes for the particle to reach the point where it is due east of the origin. *(3 marks)*

(c) Find the speed of the particle when it is travelling south-east. *(6 marks)*

18.Jan 2013

7 A particle is initially at the point A, which has position vector $13.6\mathbf{i}$ metres, with respect to an origin O. At the point A, the particle has velocity $(6\mathbf{i} + 2.4\mathbf{j})\,\mathrm{m\,s^{-1}}$, and in its subsequent motion, it has a constant acceleration of $(-0.8\mathbf{i} + 0.1\mathbf{j})\,\mathrm{m\,s^{-2}}$. The unit vectors $\mathbf{i}$ and $\mathbf{j}$ are directed east and north respectively.

(a) Find an expression for the velocity of the particle t seconds after it leaves A. *(2 marks)*

(b) Find an expression for the position vector of the particle, with respect to the origin O, t seconds after it leaves A. *(3 marks)*

(c) Find the distance of the particle from the origin O when it is travelling in a north-westerly direction. *(7 marks)*

Answers

1.Jan 2005

7 (a)	$-\mathbf{i}+\mathbf{j}=2\mathbf{i}-\mathbf{j}+10\mathbf{a}$ $\mathbf{a}=-0.3\mathbf{i}+0.2\mathbf{j}$	M1 A1 A1	3	Use of velocity equation Correct equation Correct **a**
(b)	$\mathbf{r}=(2\mathbf{i}-\mathbf{j})t+\frac{1}{2}(-0.3\mathbf{i}+0.2\mathbf{j})t^2+20\mathbf{i}$ $=\left(2t-0.15t^2+20\right)\mathbf{i}+\left(-t+0.1t^2\right)\mathbf{j}$	M1 A1 A1 ft	3	Use of constant acceleration equation for position Correct **i** component Correct **j** component ft incorrect acceleration
(c) (i)	$\mathbf{r}(20)=(2\times20-0.15\times20^2+20)\mathbf{i}+(-20+0.1\times20^2)\mathbf{j}$ $=0\mathbf{i}+20\mathbf{j}$ so due north of origin	M1 A1	2	Substituting $t=20$ into their expression for **r** Correct conclusion from correct working
(c)(ii)	$\mathbf{v}(20)=2\mathbf{i}-\mathbf{j}+20(-0.3\mathbf{i}+0.2\mathbf{j})$ $=-4\mathbf{i}+3\mathbf{j}$ $\mathbf{v}(20)=\sqrt{4^2+3^2}=5\ \text{ms}^{-1}$	M1 A1 m1 A1ft	4	Finding velocity at $t=20$ Correct velocity Finding magnitude Correct speed ft incorrect acceleration
	Total		**12**	

2.June 2005

7(a)	$\mathbf{v}=4\mathbf{j}+(3\mathbf{i}-5\mathbf{j})t$	M1A1	2	M1: Use of $\mathbf{v}=\mathbf{u}+\mathbf{a}t$ and $\mathbf{u}\neq0$ or integration A1: Correct expression
(b)	$\mathbf{v}=3t\mathbf{i}+(4-5t)\mathbf{j}$ $4-5t=0$ $t=0.8$ s	M1 A1	2	M1: j component of velocity equal to zero A1: Correct t
(c)	$\mathbf{v}=12\mathbf{i}-16\mathbf{j}$ $\mathbf{v}=\sqrt{12^2+16^2}=20$ **AG**	M1 A1 dM1 A1	4	M1: Finding velocity when $t=4$ A1: Correct velocity dM1:Finding the magnitude A1:Correct speed from correct working
	Total		**8**	

3.Jan 2006

2(a)	$\mathbf{v}=4\mathbf{i}+(-3\mathbf{i}+12\mathbf{j})t$	M1 A1	2	use of $\mathbf{v}=\mathbf{u}+\mathbf{a}t$
(b)	$t=0.5,\ \mathbf{v}=2.5\mathbf{i}+6\mathbf{j}$ Speed $=\sqrt{(2.5^2+6^2)}$ Speed $=6.5\,\text{ms}^{-1}$	B1√ M1 A1√	3	√ 2 terms and t subs 2 terms √ 2 terms
	Total		**5**	

4.June 2006

6(a)	$\mathbf{d}=3\mathbf{i}-6\mathbf{j}$ $3\mathbf{i}-6\mathbf{j}=(\mathbf{i}-2\mathbf{j})t$ $t=3$	B1 M1 A1	3	Accept $\pm\mathbf{d}$ or displacements of 3, 6 shown on a diagram Or equivalent method for t Accept ratio of vectors leading directly to ±3 CAO

(b)(i)	$\mathbf{r} = (\mathbf{i} - 2\mathbf{j}) \times 4 + \frac{1}{2} \times 2\mathbf{j} \times 16$	M1 A1		Full method for vector expression giving change in position For correct subs (gives $4\mathbf{i} + 8\mathbf{j}$)
	$+6\mathbf{i} - 4\mathbf{j}$ $= 10\mathbf{i} + 4\mathbf{j}$	M1 A1F	4	FT slip provided obtain vector expression ($\mathbf{u} = 0$ gives $6\mathbf{i} + 12\mathbf{j}$)
(ii)	$A(3,2) \quad C(10,4)$ $\mathbf{d} = 7\mathbf{i} + 2\mathbf{j}$ $\lvert\mathbf{d}\rvert = \sqrt{7^2 + 2^2}$	M1		Attempt to find vector $\overrightarrow{AC}$ or $\overrightarrow{CA}$ (using candidate's C
	$AC = \sqrt{53} = 7.28$	A1F	2	FT **d** provided two non-zero components Accept $\sqrt{53}$
	Total		**9**	

5.Jan 2007

8(a)	$75\mathbf{i} = (5\mathbf{i} - 2\mathbf{j}) \times 10 + \frac{1}{2}\mathbf{a} \times 10^2$	M1 A1		Equation to find **a** from $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ Correct expression
	$\mathbf{a} = \frac{75\mathbf{i} - 50\mathbf{i} + 20\mathbf{j}}{50} = 0.5\mathbf{i} + 0.4\mathbf{j}$	A1	3	**AG** Correct **a** from correct working
(b)	$\mathbf{r} = (5\mathbf{i} - 2\mathbf{j}) \times 8 + \frac{1}{2}(0.5\mathbf{i} + 0.4\mathbf{j}) \times 8^2$	M1 A1		Expression for **r** using $t = 8$ with no extra terms Correct expressions
	$= 56\mathbf{i} - 3.2\mathbf{j}$	A1	3	Correct position vector
(c)	$\mathbf{v} = (5 + 0.5t)\mathbf{i} + (0.4t - 2)\mathbf{j}$ $0.4t - 2 = 0$	M1A1 dM1		Expression for **v**. Correct expression **j** component equal to zero
	$t = \frac{2}{0.4} = 5$	A1		Correct t
	$\mathbf{r} = (5\mathbf{i} - 2\mathbf{j}) \times 5 + \frac{1}{2}(0.5\mathbf{i} + 0.4\mathbf{j}) \times 5^2$ $= 31.25\mathbf{i} - 5\mathbf{j}$	dM1		Expression for **r** using t from **j** component equal to zero
	$= 31.3\mathbf{i} - 5\mathbf{j}$	A1	6	Correct position vector
	Total		**12**	

6.June 2007

8(a)	$\mathbf{u} = 5\mathbf{i}$ or $\begin{bmatrix} 5 \\ 0 \end{bmatrix}$	B1	1	Correct velocity
(b)	$\mathbf{v} = 5\mathbf{i} + (-0.2\mathbf{i} + 0.25\mathbf{j})t$	M1 A1	2	Use of constant acceleration equation, with **u** and **a** not zero Correct velocity M1A0 for using 5**j** or just 5
	OR $\mathbf{v} = \begin{bmatrix} 5 - 0.2t \\ 0.25t \end{bmatrix}$			
(c)	$5 - 0.2t = 0$	M1 A1		Easterly component zero Correct equation
	$t = \frac{5}{0.2} = 25$ seconds	A1	3	Correct t

(d)	$\mathbf{r} = 5\mathbf{i} \times 25 + \frac{1}{2}(-0.2\mathbf{i} + 0.25\mathbf{j}) \times 25^2$ $= 62.5\mathbf{i} + 78.125\mathbf{j}$ $\theta = \tan^{-1}\left(\frac{62.5}{78.125}\right)$ $= 038.7°$ **OR** $\mathbf{r} = \frac{1}{2}(5\mathbf{i} + 6.25\mathbf{j}) \times 25$ $\theta = \tan^{-1}\left(\frac{5}{6.25}\right) = 038.7°$	M1 A1F A1 dM1 A1F A1 (M1) (A1F) (A1) (dM1) (A1F) (A1)	6	Use of constant acceleration equation with t from part (c) Correct expression based on t from part (c) Correct simplification CAO Using tan to find the angle Correct expression based on t from part (c), with correct two values(either way) Correct angle Accept 38.6° or 039°
	Total		**12**	

7.Jan 2008

8(a)	$4\mathbf{i} = 5\mathbf{j} + 40\mathbf{a}$ $\mathbf{a} = \frac{4\mathbf{i} - 5\mathbf{j}}{40} = 0.1\mathbf{i} - 0.125\mathbf{j}$ **AG**	M1 A1 dM1 A1	4	Forming a vector equation based on constant acceleration Correct equation Solving for **a** Correct **a** from correct working For $\frac{4\mathbf{i} - 5\mathbf{j}}{40}$ on its own give M0 Allow verification
(b)	$\mathbf{r} = 5\mathbf{j} \times 40 + \frac{1}{2}(0.1\mathbf{i} - 0.125\mathbf{j}) \times 40^2$ $= 80\mathbf{i} + 100\mathbf{j}$	M1 A1 A1	3	Finding position vector Correct expression Correct simplified result
(c)(i)	$\mathbf{v} = 5\mathbf{j} + (0.1\mathbf{i} - 0.125\mathbf{j})t$ $= 0.1t\mathbf{i} + (5 - 0.125t)\mathbf{j}$ $5 - 0.125t = -0.1t$ $5 = 0.025t$ $t = \frac{5}{0.025} = 200$	M1 A1 dM1 A1 A1	5	Expression for **v** Correct expression for **v** seen or implied Equating components, with or without a minus sign Correct equation Correct time.
(c)(ii)	$\mathbf{v} = 0.1 \times 200\mathbf{i} + (5 - 0.125 \times 200)\mathbf{j}$ $= 20\mathbf{i} - 20\mathbf{j}$	M1 A1F	2	Finding velocity using their time Correct velocity for their time
	Total		**14**	

8.June 2008

5				Use of column vectors is acceptable throughout this question.
(a)	$\mathbf{v} = 20\mathbf{i} + (-0.4\mathbf{i} + 0.5\mathbf{j})t$	M1 A1	2	Use of constant acceleration equation to find expression for **v** Any correct expression.
(b)	$\mathbf{v} = (20 - 0.4t)\mathbf{i} + 0.5t\mathbf{j}$	M1		Simplifying **v**. (May be implied.) (Missing brackets may be condoned if followed by correct working.)

	$20-0.4t=0$ $t=\frac{20}{0.4}=50$ seconds	m1 A1	 3	Putting **i** component equal to zero Correct time Candidates who are able to see the correct time without supporting working gain full marks. Condone $\frac{20\mathbf{i}}{0.4\mathbf{i}}=50$
(c)	$\mathbf{r}=20\mathbf{i}\times t+\frac{1}{2}(-0.4\mathbf{i}+0.5\mathbf{j})\times t^2$	M1 A1	 2	Use of constant acceleration equation to find expression for **r** Any correct expression
(d)(i)	$\mathbf{r}=20\mathbf{i}\times 100+\frac{1}{2}(-0.4\mathbf{i}+0.5\mathbf{j})\times 100^2$ $=2000\mathbf{i}-2000\mathbf{i}+2500\mathbf{j}$ $=2500\mathbf{j}$	m1 A1		Substituting $t=100$ into their expression for **r** (dependent on M1 in part (c)) Correct simplified position vector ie $2500\mathbf{j}$
	Therefore due north	A1	3	Conclusion that helicopter is due north provided their position vector is of the form $k\mathbf{j}$, where $k>0$ Note if integration is used there is no need to prove that the constant is zero. Note marks for (d) (i) can be awarded if part c scores zero.
(d)(ii)	$\mathbf{v}=(20-0.4\times 100)\mathbf{i}+0.5\times 100\mathbf{j}$	m1		Substituting $t=100$ into their expression for **v** (dependent on M1 in part (a)) or use of other constant acceleration equation and their position vector (dependent on M1 in part (c))
	$=-20\mathbf{i}+50\mathbf{j}$ $v=\sqrt{20^2+50^2}=53.9$	A1 A1	 3	Correct simplified velocity Correct speed (accept $10\sqrt{29}$) Note marks for (d) (ii) can be awarded if part a scores zero.
	Total		**13**	

9.Jan 2009

6(a)	Resultant $=(6\mathbf{i}-3\mathbf{j})+(3\mathbf{i}+15\mathbf{j})$ $=9\mathbf{i}+12\mathbf{j}$	M1 A1	 2	M1: Summing the two vectors A1: Correct resultant
(b)	Magnitude $=\sqrt{9^2+12^2}$ $=15$ N	M1 A1F	 2	M1: Finding magnitude with an addition sign. A1F: Correct magnitude based on their answer to part (a).
(c)	$1.5m=9$ or $2m=12$ $m=6$ kg $m=6$ kg	M1 A1F	 2	M1: Applying Newton's second law to one or both of the components. A1F: Correct mass, follow through their answer to part (a). Do not award this mark if vector division with 2 components has been used, eg $\frac{9\mathbf{i}+12\mathbf{j}}{1.5\mathbf{i}+2\mathbf{j}}=6$ or $6\mathbf{i}+6\mathbf{j}$ etc without a correct previous statement gives M0A0
(d)(i)	$\mathbf{r}=\frac{1}{2}(1.5\mathbf{i}+2\mathbf{j})t^2$	M1 A1	 2	M1: Using a constant acceleration equation to find the position vector with $\mathbf{u}=0\mathbf{i}+0\mathbf{j}$ A1: Correct position vector.

(d)(ii)	$\mathbf{r}=\frac{1}{2}(1.5\mathbf{i}+2\mathbf{j})\times 2^2=3\mathbf{i}+4\mathbf{j}$ $d=\sqrt{(3)^2+(4)^2}$ $=\sqrt{25}=5$	M1 A1	 2	M1: Finding the position vector when $t=2$. ($\mathbf{r}=(1.5\mathbf{i}+2\mathbf{j})\times 2=3\mathbf{i}+4\mathbf{j}$ scores M0 unless it is clear how the 2 was obtained, possibly by a correct formula in (d) (i)) A1: Correct distance
	Total		**10**	

10.June 2009

7(a)	$\mathbf{v}=(-2\mathbf{i}+2\mathbf{j})+(0.25\mathbf{i}+0.3\mathbf{j})\times 20$ $\mathbf{v}=3\mathbf{i}+8\mathbf{j}$	M1 A1 A1	 3	M1: Finding velocity using $\mathbf{v}=\mathbf{u}+\mathbf{a}t$. A1: Correct expression. A1: Correct velocity in simplest form.
(b)	$-2+0.25t=0$ $t=8$ s $\mathbf{v}=(2+0.3\times 8)\mathbf{j}=4.4\mathbf{j}$	M1A1 A1 A1	 4	M1: One component equal to zero (either **i** or **j** component). A1: Correct equation A1: Correct time A1: Correct velocity
(c)	$\mathbf{r}=(-2\mathbf{i}+2\mathbf{j})\times 20+\frac{1}{2}(0.25\mathbf{i}+0.3\mathbf{j})\times 20^2+(9\mathbf{i}+7\mathbf{j})$ **OR** $\mathbf{r}=\frac{1}{2}\left((-2\mathbf{i}+2\mathbf{j})+(3\mathbf{i}+8\mathbf{j})\right)\times 20+(9\mathbf{i}+7\mathbf{j})$ $\mathbf{r}=19\mathbf{i}+107\mathbf{j}$	M1 A1 A1	 3	M1: Finding position vector using a constant acceleration equation with or without the initial position with $t=20$. A1: Correct expression for position vector including initial position. A1: Correct position vector in simplest form
(d)	$\mathbf{v}_{AVERAGE}=\frac{(19\mathbf{i}+107\mathbf{j})-(9\mathbf{i}+7\mathbf{j})}{20}$ $=\frac{10\mathbf{i}+100\mathbf{j}}{20}$ $=0.5\mathbf{i}+5\mathbf{j}$	M1 A1F	 2	M1: Finding average velocity based on change of position. Subtraction of initial position must be seen or implied. Division by 8 scores M0 A1F: Correct average velocity. Follow through incorrect answers from part (c). Allow $\frac{\mathbf{u}+\mathbf{v}}{2}$
	Total		**12**	

11.Jan 2010

5(a)	$(8\mathbf{i}+12\mathbf{j})+(4\mathbf{i}-4\mathbf{j})=12\mathbf{i}+8\mathbf{j}$	M1A1	2	M1: Adding forces to find resultant. A1: Correct resultant force.
(b)	$4\mathbf{a}=12\mathbf{i}+8\mathbf{j}$ or $(\mathbf{a}=)\frac{12\mathbf{i}+8\mathbf{j}}{4}$ $(\mathbf{a}=)3\mathbf{i}+2\mathbf{j}$ **AG**	M1 A1	 2	M1: Use of Newton's second law with 4**a** and their answer to part (a). A1: Correct acceleration from correct equation.
(c)(i)	$40\mathbf{i}+32\mathbf{j}=\mathbf{v}+(3\mathbf{i}+2\mathbf{j})\times 20$ $40\mathbf{i}+32\mathbf{j}=\mathbf{v}+60\mathbf{i}+40\mathbf{j}$ $\mathbf{v}=(40\mathbf{i}+32\mathbf{j})-(60\mathbf{i}+40\mathbf{j})$ **AG** $=-20\mathbf{i}-8\mathbf{j}$	B1 M1 A1	 3	B1: Seeing 60i + 40j or (3i + 2j) × 20 M1: Use of constant acceleration equation with $t=20$ and $\mathbf{a}=3\mathbf{i}\pm 2\mathbf{j}$. A1: Correct velocity from correct working, with one of the intermediate lines of working (or equivalent) shown. Note: Candidates may use **u** instead of **v** in their working. Example: Starting with $\mathbf{v}=40\mathbf{i}+32\mathbf{j}+(3\mathbf{i}+2\mathbf{j})\times 20$ Scores B1M1A0.

				Note on Verification Method: $\mathbf{v} = (-20\mathbf{i} - 8\mathbf{j}) + (3\mathbf{i} + 2\mathbf{j}) \times 20$ B1M1 $= (-20 + 60)\mathbf{i} + (-8 + 40)\mathbf{j}$ $= 40\mathbf{i} + 32\mathbf{j}$ A1 Similarly, verification to confirm acceleration from the two velocities is acceptable.
(c)(ii)	$(\mathbf{v} =)(-20\mathbf{i} - 8\mathbf{j}) + (3\mathbf{i} + 2\mathbf{j})t$	B1	1	B1: Correct velocity vector. Note "**v** = " does not need to be seen.
(c)(iii)	$(\mathbf{v} =)\ (3t - 20)\mathbf{i} + (2t - 8)\mathbf{j}$	M1		M1: Velocity vector seen split into components. Condone omission of **i** and **j** Note: This can be implied by later working, such as the second line of this solution.
	$(3t - 20)^2 + (2t - 8)^2 = 8^2$	dM1 A1		dM1: Equation based on speed of 8. A1: Correct unsimplified equation.
	$13t^2 - 152t + 400 = 0$	A1		A1: Simplified quadratic equation
	$t = \dfrac{152 \pm \sqrt{152^2 - 4 \times 13 \times 400}}{2 \times 13}$	dM1		dM1: Solving quadratic equation, to obtain two solutions.
	$t = 4$ or $t = 7.69$	A1	6	A1: Both correct solutions. Accept AWRT 7.7 or 7.6 or $\dfrac{100}{13}$. Note: Using calculator to solve quadratic is acceptable.
	Total		**14**	

12.June 2010

7(a)	$10\,\mathbf{a} = 9\mathbf{i} + 12\,\mathbf{j}$ $\mathbf{a} = (0.9\mathbf{i} + 1.2\,\mathbf{j})\ \text{ms}^{-2}$	M1 A1	2	M1: Application of Newton's second Law with $m = 10$ in vector form. A1: Correct acceleration. If acceleration incorrect follow their value through for the rest of this question.
(b)(i)	$\mathbf{r}(5) =$ $(2.2\mathbf{i} + 1\mathbf{j}) \times 5 + \frac{1}{2}(0.9\mathbf{i} + 1.2\,\mathbf{j}) \times 5^2$ $= 22.25\mathbf{i} + 20\,\mathbf{j}$	M1 A1F		M1: Use of constant acceleration to find position vector at $t = 5$, with $\mathbf{u} \neq 0\mathbf{i} + 0\,\mathbf{j}$. A1F: Correct position vector, for candidate's acceleration which must be a vector. Allow 22.3**i** + 20**j**.
	$d = \sqrt{22.25^2 + 20^2} = 29.9$ metres	dM1 A1F	4	dM1: Calculation of distance from position vector. Must see + sign. A1F: Correct distance, for their acceleration. Accept 30 from 22.3**i** + 20**j**.
(ii)	$\mathbf{v} = (2.2\mathbf{i} + 1\mathbf{j}) + (0.9\mathbf{i} + 1.2\,\mathbf{j})t$	M1 A1F	2	M1: Use of constant acceleration equation to find an expression for **v**, with $\mathbf{u} \neq 0\mathbf{i} + 0\,\mathbf{j}$. A1F: Correct **v** for their acceleration.
(iii)	$\mathbf{v} = (2.2 + 0.9t)\mathbf{i} + (1 + 1.2t)\,\mathbf{j}$ $2.2 + 0.9t = 1 + 1.2t$ $1.2 = 0.3t$ $t = 4$	M1 A1F A1F	3	M1: Equation involving both **i** and **j** components of their velocity. Could have incorrect signs, for example $2.2 + 0.9t = -(1 + 1.2t)$. A1F: Correct equation. A1F: Correct time, for their acceleration.
	Total		**11**	

13.Jan 2011

5(a)	$\mathbf{v} = (4\mathbf{i} + 0.5\mathbf{j}) + (-0.4\mathbf{i} + 0.2\mathbf{j})t$	M1A1	2	M1: Use of constant acceleration equation to find **v** with $\mathbf{u} \neq 0\mathbf{i} + 0\mathbf{j}$ A1: Correct **v**. (Could be done as a column vector.)
(b)(i)	$\mathbf{v} = (4\mathbf{i} + 0.5\mathbf{j}) + (-0.4\mathbf{i} + 0.2\mathbf{j}) \times 22.5$ $= -5\mathbf{i} + 5\mathbf{j}$	M1 A1	2	M1: Substitution of 22.5 into their expression for the velocity, even if no marks awarded in part (a). A1: Correct velocity CAO (Could be done using column vectors.)
(b)(ii)	North-west	B1	1	B1: Correct statement of direction. Accept 315°. Must follow from correct answer to (b)(i).
(c)	$(\mathbf{v} =)(4 - 0.4t)\mathbf{i} + (0.5 + 0.2t)\mathbf{j}$ $5^2 = (4 - 0.4t)^2 + (0.5 + 0.2t)^2$ $0.2t^2 - 3t - 8.75 = 0$ $t^2 - 15t - 43.75 = 0$ $t = 17.5$ or $t = -2.5$ $t = 17.5$	B1 M1A1 A1 dM1 A1	6	B1: Grouping **i** and **j** components at some point in the solution. (Could be done as column vectors.) Allow $5 = (4 - 0.4t)\mathbf{i} + (0.5 + 0.2t)\mathbf{j}$ M1: Seeing both components of their velocity squared and added A1: Correct equation. (Condone including **i** and **j**.) For example: $5 = (4 - 0.4t)\mathbf{i}^2 + (0.5 + 0.2t)\mathbf{j}^2$ scores B1M1A0 $5^2 = (4 - 0.4t)\mathbf{i}^2 + (0.5 + 0.2t)\mathbf{j}^2$ scores B1M1A1 A1: Any correct simplified quadratic equation, with exactly three terms. dM1: Solving the quadratic equation. (Allow one substitution error in correctly quoted formula) Candidates with an incorrect quadratic equation must show method to get dM1. A1:Correct positive solution stated.
	Total		**11**	

14.June 2011

4 (a)	$6(5\mathbf{i} + 18\mathbf{j}) + m(2\mathbf{i} - 5\mathbf{j}) = 6(8\mathbf{i}) + m(V\mathbf{j})$ $6 \times 5 + 2m = 6 \times 8$ $30 + 2m = 48$ $m = \frac{48 - 30}{2} = 9$	M1 A1 A1	3	M1: Conservation of momentum, with addition of terms, as either 4 term vector equation (seen either in part (a) or part (b) **OR** three term equation for **i** component. Allow one error, for example switching masses. A1: Correct equation for **i** components. A1: Correct m.
(b)	$6 \times 18 - 5 \times 9 = 9V$ $108 - 45 = 9V$ $V = \frac{108 - 45}{9} = 7$	M1A1F A1F	3	M1: Conservation of momentum for **j** component with correct signs. Allow one error, for example switching masses. Note: omitting any mass scores M0. A1F: Correct equation. Allow m instead of 9 at this stage.

				A1F: Correct velocity. Condone 7**j** FT candidate's mass from part (a). Only award FT marks if mass positive. Note $V=\frac{108}{m}-5$
	Total		**6**	

15.June 2011

7(a)	$\mathbf{v}=(0.5\mathbf{i}+0.375\mathbf{j})\times 20(=10\mathbf{i}+7.5\mathbf{j})$ $v=\sqrt{10^2+7.5^2}=12.5\text{ ms}^{-1}$	M1A1 dM1A1	4	M1: Calculating velocity with $\mathbf{u}=0\mathbf{i}+0\mathbf{j}$ and $t=20$. A1: Correct expression for velocity. dM1: Calculating speed. A1: Correct speed.
(b)	$\tan\theta=\frac{0.5}{0.375}$ or $\frac{10}{7.5}$ $\left(\text{or } \tan\theta=\frac{0.375}{0.5} \text{ or } \frac{7.5}{10}\right)$ $\theta=053°$ **OR** $\cos\theta=\frac{7.5}{12.5}$ or $\frac{0.375}{0.625}$ $\left(\text{or } \cos\theta=\frac{10}{12.5}\right)$ $\theta=053°$ **OR** $\sin\theta=\frac{10}{12.5}$ or $\frac{0.5}{0.625}$ $\left(\text{or } \sin\theta=\frac{7.5}{12.5}\right)$ $\theta=053°$	M1A1F A1 (M1A1F) (A1) (M1A1F) (A1)	3	M1: Using trig to find angle. Can be inverted as shown in brackets. A1F: Correct trig expression with any correct equivalent fraction. A1: Correct angle to the nearest degree. Accept 53°. Note: For 37° award M1A0A0 But for 90 – 37 = 53° award M1A1A1. For 127°, award M1A1A0 Note: 53.1° as final answer scores M1A1A0 Condone finding angle from
(c)	$(\mathbf{r}=)\frac{1}{2}(0.5\mathbf{i}+0.375\mathbf{j})t^2\left(=0.25t^2\mathbf{i}+0.1875t^2\mathbf{j}\right)$ $500^2=\left(0.25t^2\right)^2+\left(0.1875t^2\right)^2$ $t=\sqrt[4]{\frac{500^2}{0.25^2+0.1875^2}}=40$ seconds **OR** $a=0.625$ $500=\frac{1}{2}0.625t^2$ $t=40$ **OR**	M1A1 dM1A1 A1 (M1A1) (dM1A1) (A1)	5	M1: Finding an expression for position vector in terms of t. A1: Correct position vector. dM1: Using distance to form an equation for t. A1: Correct equation. A1: Correct time. M1: Finding magnitude of acceleration. A1: Correct acceleration dM1: Using distance to form an equation for t. A1: Correct equation. A1: Correct time.
	$400=\frac{1}{2}\times 0.5t^2$ or $300=\frac{1}{2}\times 0.375t^2$ $t^2=1600$ $t=40$	(M1A1) (A1) (dM1) (A1)		M1: Working with one component. A1: Correct distance (300 or 400) A1: Correct equation. dM1: Solving for t. A1: Correct t. Note: 500÷12.5=40 is not acceptable and scores 0
	Total		**12**	

16.Jan 2012

7(a)	$h=\frac{1}{2}\times 2.5\times 20^2$ $=500\text{ m}$	M1 A1A1	3	M1: Expression for height or position vector at $t=20$. A1: Correct expression for height or position vector with correct **j** component (…**i** + 500 **j**) A1: Correct height stated. Condone 500 **j**.

(b)	$\mathbf{v}(20) = (4.2\mathbf{i} + 2.5\mathbf{j}) \times 20$ $= 84\mathbf{i} + 50\mathbf{j}$	M1 A1	2	M1: Using $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ to find the velocity at $t = 20$ with $\mathbf{u} = 0\mathbf{i} + 0\mathbf{j}$. A1: Correct velocity.
(c)	$1.25t^2 = 180$ $t = \sqrt{\frac{180}{1.25}} = 12$ s $\mathbf{v} = (4.2\mathbf{i} + 2.5\mathbf{j}) \times 12$ $= 50.4\mathbf{i} + 30\mathbf{j}$ $v = \sqrt{50.4^2 + 30^2} = 58.7 \text{ ms}^{-1}$ OR $a = \sqrt{4.2^2 + 2.5^2} = 4.89$ $v = 4.89 \times 12 = 58.7 \text{ ms}^{-1}$	M1A1 A1 dM1 A1 dM1A1 (dM1A1) (dM1A1)	7	M1: Equation based on height of 180 to find t. A1: Correct equation. A1: Correct t. dM1: Using $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ to find the velocity at their time with $\mathbf{u} = 0\mathbf{i} + 0\mathbf{j}$. A1: Correct velocity. dM1: Finding speed from their velocity. A1: Correct speed. Accept 58.6 or AWRT 58.7. dM1: finding magnitude of acceleration. A1: correct magnitude. dM1: acceleration × 12. A1: correct speed.
	OR $x = 4.2 \times \frac{180}{2.5} = 302.4$ $v_x = \sqrt{2 \times 4.2 \times 302.4} = 50.4$ $v_y = \sqrt{2 \times 2.5 \times 180} = 30$ $v = \sqrt{50.4^2 + 30^2} = 58.7$	(M1A1) (dM1A1) (A1) (dM1A1)		M1: finding horizontal displacement when height is 180. Must see 4.2, 2.5 and 180. May be implied by seeing 302.4. A1: Seeing 302.4 dM1: Finding both components of velocity. A1: Seeing 50.4. A1: Seeing 30. dM1: Finding the speed. A1: Final answer of 58.7
7(c) cont	OR $v_y = \sqrt{2 \times 2.5 \times 180} = 30$ $30 = 0 + 2.5t$ $t = \frac{30}{2.5} = 12$ $v_x = 0 + 4.2 \times 12 = 50.4$ $v = \sqrt{50.4^2 + 30^2} = 58.7 \text{ ms}^{-1}$	(M1A1) (A1) (dM1A1) (dM1A1)		M1: Equations based on height of 180 to find v and then t. A1: Correct equation for t. A1: Correct t. dM1: Using $v = u + at$ to find the **j** component of velocity at their time with $u = 0$. A1: Correct velocity. dM1: Finding speed from their velocity. A1: Correct speed. Accept 58.6 or AWRT 58.7.
	OR $\tan^{-1}\left(\frac{2.5}{4.2}\right) = 30.76°$ $v_y = \sqrt{2 \times 2.5 \times 180} = 30$ $v = \frac{30}{\sin 30.76} = 58.7 \text{ m s}^{-1}$	(M1A1) (A1) (dM1A1) (dM1A1)		M1: Finding angle using acceleration components. A1: Correct expression for acceleration components A1: Correct angle. dM1: Finding v_y at height of 180 A1: Correct speed of 30. dM1: Using trig to get v. A1: Correct speed. Accept 58.6 or AWRT 58.7.
	Total		**12**	

17.June 2012

7(a)	$\mathbf{r} = (-\mathbf{i} + 3\mathbf{j})t + \frac{1}{2}(0.1\mathbf{i} - 0.2\mathbf{j})t^2$	M1A1	2	M1: Using constant acceleration equation to get **r**. A1: Correct expression for **r**. Allow equivalent column vector answer.
(b)	$3t - 0.1t^2 = 0$ $t(3 - 0.1t) = 0$ $t = 0$ or $t = 30$ $t = 30$ seconds	M1A1 A1	 3	M1: Putting their **j** component equal to zero to form a quadratic equation. A1: Correct equation. A1: For 30 seconds. No need to see $t = 0$.
(c)	$\mathbf{v} = (0.1t - 1)\mathbf{i} + (3 - 0.2t)\mathbf{j}$ $0.1t - 1 = -(3 - 0.2t)$ $2 = 0.1t$ $t = 20$ $\mathbf{v} = \mathbf{i} - \mathbf{j}$ $v = \sqrt{2} = 1.41$ ms^{-1}	B1 M1A1 A1 dM1 A1	 6	B1: Correct expression for the velocity in terms of t. Can be implied by subsequent working in terms of t. M1: For $0.1t - 1 = \pm(3 - 0.2t)$. May be with their components if velocity stated incorrectly. A1: Correct equation. A1: t =20 dM1: finding velocity and speed at their time A1: Correct speed.
				Special cases If the equation in t in line 2 is not seen: then seeing t=20 and **v**=**i** - **j** and v =1.41 award 4 out of 6 or then seeing t=20 and **v**=**i** - **j** award 2 out of 6
	Total		**11**	

18.Jan 2013

7(a)	$\mathbf{v} = (6\mathbf{i} + 2.4\mathbf{j}) + (-0.8\mathbf{i} + 0.1\mathbf{j})t$	M1A1	2	M1: Using constant acceleration equation to get **v**. A1: Correct expression for the velocity. Allow equivalent column vector answer.
(b)	$\mathbf{r} = (6\mathbf{i} + 2.4\mathbf{j})t + \frac{1}{2}(-0.8\mathbf{i} + 0.1\mathbf{j})t^2 + 13.6\mathbf{i}$ $\left(= (6t - 0.4t^2 + 13.6)\mathbf{i} + (2.4t + 0.05t^2)\mathbf{j}\right)$	M1A1 A1	3	M1: Use of $\mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ or other constant acceleration equation. A1: Position vector with or without 13.6**i**. A1: Correct position vector.
(c)	$\mathbf{v} = (6 - 0.8t)\mathbf{i} + (2.4 + 0.1t)\mathbf{j}$ $6 - 0.8t = -(2.4 + 0.1t)$ $8.4 = 0.7t$ $t = \frac{8.4}{0.7} = 12$ s $\mathbf{r} = 28\mathbf{i} + 36\mathbf{j}$ $d = \sqrt{28^2 + 36^2} = 45.6$ m	B1 M1A1 A1 dM1A1 A1	 7	B1: Velocity simplified into **i** and **j** components. Could be implied. M1: $6 - 0.8t = \pm(2.4 + 0.1t)$ A1: Correct equation. A1: Correct t. dM1: Finding position vector using their time. A1: Correct position vector. A1: Correct distance. Accept AWRT 45.6
				Do not penalise the use of other methods, such as trial and improvement, to find the time.
	Total		**12**	