Practice examination questions for projectiles

1. Jan 2005

8 A football is placed on a horizontal surface. It is then kicked, so that it has an initial velocity of $12\,\mathrm{m\,s^{-1}}$ at an angle of $40°$ above the horizontal.

(a) State two modelling assumptions that it would be appropriate to make when considering the motion of the football. *(2 marks)*

(b) (i) Find the time that it takes for the ball to reach its maximum height. *(4 marks)*

(ii) Hence show that the maximum height of the ball is 3.04 metres, correct to three significant figures. *(3 marks)*

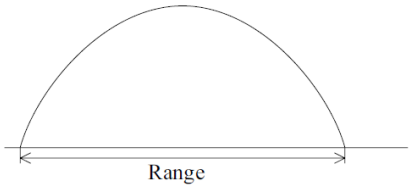
(c) After the ball has reached its maximum height, it hits the bar of a goal at a height of 2.44 metres. Find the horizontal distance of the goal from the point where the ball was kicked. *(7 marks)*

2. June 2005

6 A ball is hit from horizontal ground with velocity $(10\mathbf{i} + 24.5\mathbf{j})\,\mathrm{m\,s^{-1}}$ where the unit vectors $\mathbf{i}$ and $\mathbf{j}$ are horizontal and vertically upwards respectively.

(a) State two assumptions that you should make about the ball in order to make predictions about its motion. *(2 marks)*

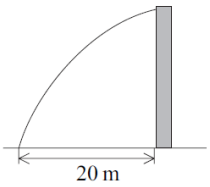
(b) The path of the ball is shown in the diagram.



(i) Show that the time of flight of the ball is 5 seconds. *(4 marks)*

(ii) Find the range of the ball. *(2 marks)*

(c) In fact the ball hits a vertical wall that is 20 metres from the initial position of the ball.

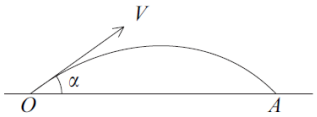


Find the height of the ball when it hits the wall. *(4 marks)*

(d) If a heavier ball were projected in the same way, would your answers to part (b) of this question change? Explain why. *(2 marks)*

3. Jan 2006

5 A golf ball is projected from a point O with initial velocity V at an angle α to the horizontal. The ball first hits the ground at a point A which is at the same horizontal level as O, as shown in the diagram.



It is given that $V\cos\alpha = 6u$ and $V\sin\alpha = 2.5u$.

(a) Show that the time taken for the ball to travel from O to A is $\frac{5u}{g}$. *(4 marks)*

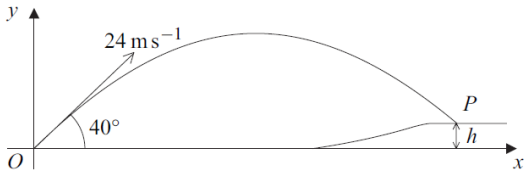
(b) Find, in terms of g and u, the distance OA. *(2 marks)*

(c) Find V, in terms of u. *(2 marks)*

(d) State, in terms of u, the least speed of the ball during its flight from O to A. *(1 mark)*

4. June 2006

7 A golf ball is struck from a point O with velocity $24\,\mathrm{m\,s^{-1}}$ at an angle of $40°$ to the horizontal. The ball first hits the ground at a point P, which is at a height h metres above the level of O.



The horizontal distance between O and P is 57 metres.

(a) Show that the time that the ball takes to travel from O to P is 3.10 seconds, correct to three significant figures. *(3 marks)*

(b) Find the value of h. *(3 marks)*

(c) (i) Find the speed with which the ball hits the ground at P. *(5 marks)*

(ii) Find the angle between the direction of motion and the horizontal as the ball hits the ground at P. *(2 marks)*

5. Jan 2007

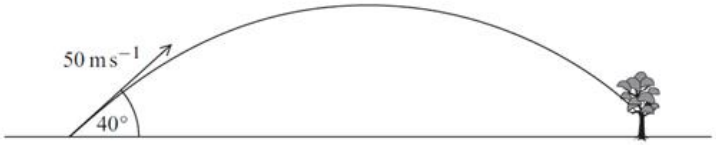
7 A golf ball is struck from a point on horizontal ground so that it has an initial velocity of $50\,\mathrm{m\,s^{-1}}$ at an angle of $40°$ above the horizontal.

Assume that the golf ball is a particle and its weight is the only force that acts on it once it is moving.

(a) Find the maximum height of the golf ball. *(4 marks)*

(b) After it has reached its maximum height, the golf ball descends but hits a tree at a point which is at a height of 6 metres above ground level.

(b) After it has reached its maximum height, the golf ball descends but hits a tree at a point which is at a height of 6 metres above ground level.



Find the time that it takes for the ball to travel from the point where it was struck to the tree. *(6 marks)*

6. June 2007

7 An arrow is fired from a point A with a velocity of $25\,\text{m s}^{-1}$, at an angle of 40° above the horizontal. The arrow hits a target at the point B which is at the same level as the point A, as shown in the diagram.

(a) State **two** assumptions that you should make in order to model the motion of the arrow. *(2 marks)*

(b) Show that the time that it takes for the arrow to travel from A to B is 3.28 seconds, correct to three significant figures. *(4 marks)*

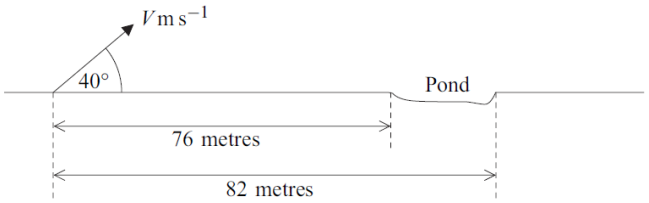
(c) Find the distance between the points A and B. *(2 marks)*

(d) State the magnitude and direction of the velocity of the arrow when it hits the target. *(2 marks)*

(e) Find the minimum speed of the arrow during its flight. *(2 marks)*

7. Jan 2008

7 A golfer hits a ball which is on horizontal ground. The ball initially moves with speed $V\,\text{m s}^{-1}$ at an angle of 40° above the horizontal. There is a pond further along the horizontal ground. The diagram below shows the initial position of the ball and the position of the pond.



(a) State **two** assumptions that you should make in order to model the motion of the ball. *(2 marks)*

(b) Show that the horizontal distance, in metres, travelled by the ball when it returns to ground level is

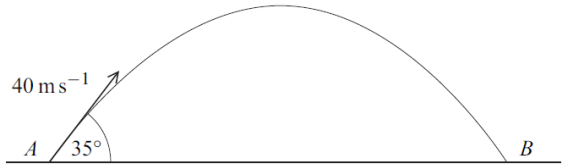
$$\frac{V^2 \sin 40° \cos 40°}{4.9}$$ *(6 marks)*

(c) Find the range of values of V for which the ball lands in the pond. *(4 marks)*

8. June 2008

7 A ball is hit by a bat so that, when it leaves the bat, its velocity is $40\,\mathrm{m\,s^{-1}}$ at an angle of $35°$ above the horizontal. Assume that the ball is a particle and that its weight is the only force that acts on the ball after it has left the bat.

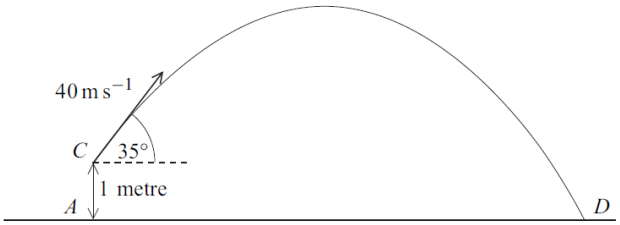
(a) A simple model assumes that the ball is hit from the point A and lands for the first time at the point B, which is at the same level as A, as shown in the diagram.



(i) Show that the time that it takes for the ball to travel from A to B is 4.68 seconds, correct to three significant figures. *(4 marks)*

(ii) Find the horizontal distance from A to B. *(2 marks)*

(b) A revised model assumes that the ball is hit from the point C, which is 1 metre above A. The ball lands at the point D, which is at the same level as A, as shown in the diagram.



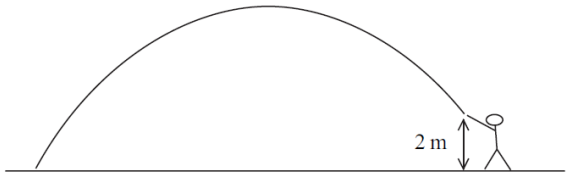
Find the time that it takes for the ball to travel from C to D. *(6 marks)*

9. Jan 2009

8 A cricket ball is hit at ground level on a horizontal surface. It initially moves at $28\,\mathrm{m\,s^{-1}}$ at an angle of $50°$ above the horizontal.

(a) Find the maximum height of the ball during its flight. *(4 marks)*

(b) The ball is caught when it is at a height of 2 metres above ground level, as shown in the diagram.

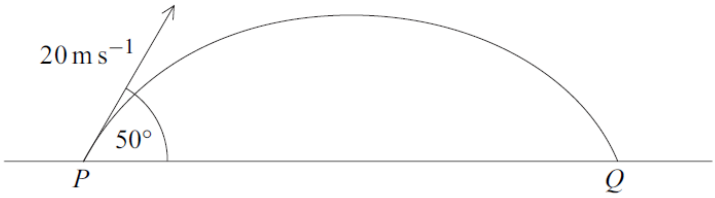


Show that the time that it takes for the ball to travel from the point where it was hit to the point where it was caught is 4.28 seconds, correct to three significant figures. *(5 marks)*

(c) Find the speed of the ball when it is caught. *(5 marks)*

10.June 2009

6 A ball is kicked from the point P on a horizontal surface. It leaves the surface with a velocity of $20\,\mathrm{m\,s^{-1}}$ at an angle of $50°$ above the horizontal and hits the surface for the first time at the point Q. Assume that the ball is a particle that moves only under the influence of gravity.



(a) Show that the time that it takes the ball to travel from P to Q is 3.13 s, correct to three significant figures. *(4 marks)*

(b) Find the distance between the points P and Q. *(2 marks)*

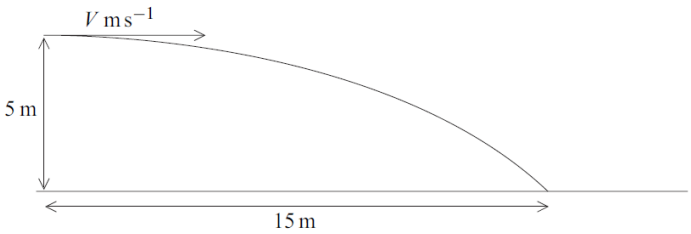
(c) If a heavier ball were projected from P with the same velocity, how would the distance between P and Q, calculated using the same modelling assumptions, compare with your answer to part **(b)**? Give a reason for your answer. *(2 marks)*

(d) Find the maximum height of the ball above the horizontal surface. *(3 marks)*

(e) State the magnitude and direction of the velocity of the ball as it hits the surface. *(2 marks)*

11.Jan 2010

7 A ball is projected horizontally with speed $V\,\mathrm{m\,s^{-1}}$ at a height of 5 metres above horizontal ground. When the ball has travelled a horizontal distance of 15 metres, it hits the ground.



(a) Show that the time it takes for the ball to travel to the point where it hits the ground is 1.01 seconds, correct to three significant figures. *(3 marks)*

(b) Find V. *(2 marks)*

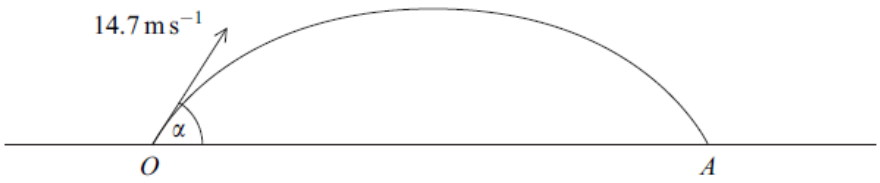
(c) Find the speed of the ball when it hits the ground. *(4 marks)*

(d) Find the angle between the velocity of the ball and the horizontal when the ball hits the ground. Give your answer to the nearest degree. *(3 marks)*

(e) State two assumptions that you have made about the ball while it is moving. *(2 marks)*

12.June 2010

8 A ball is struck so that it leaves a horizontal surface travelling at $14.7\,\mathrm{m\,s^{-1}}$ at an angle α above the horizontal. The path of the ball is shown in the diagram.



(a) Show that the ball takes $\dfrac{3\sin\alpha}{2}$ seconds to reach its maximum height. *(3 marks)*

(b) The ball reaches a maximum height of 7 metres.

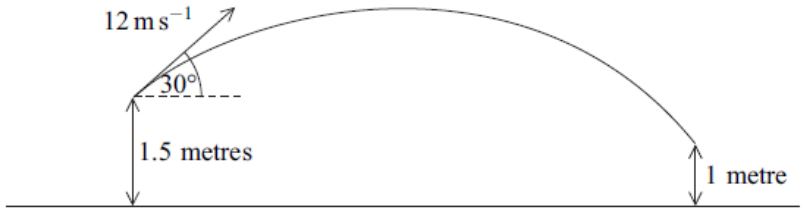
(i) Find α. *(5 marks)*

(ii) Find the range, OA. *(3 marks)*

(c) State two assumptions that you needed to make in order to answer the earlier parts of this question. *(2 marks)*

13.Jan 2011

7 An arrow is fired from a point at a height of 1.5 metres above horizontal ground. It has an initial velocity of $12\,\mathrm{m\,s^{-1}}$ at an angle of 30° above the horizontal. The arrow hits a target at a height of 1 metre above horizontal ground. The path of the arrow is shown in the diagram.



Model the arrow as a particle.

(a) Show that the time taken for the arrow to travel to the target is 1.30 seconds, correct to three significant figures. *(5 marks)*

(b) Find the horizontal distance between the point where the arrow is fired and the target. *(2 marks)*

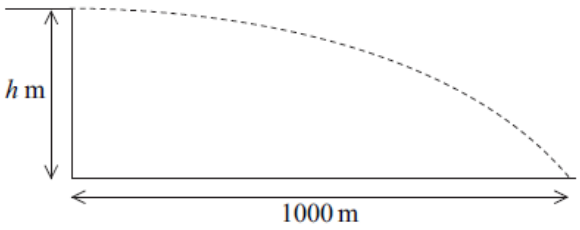
(c) Find the speed of the arrow when it hits the target. *(4 marks)*

(d) Find the angle between the velocity of the arrow and the horizontal when the arrow hits the target. *(2 marks)*

(e) State one assumption that you have made about the forces acting on the arrow. *(1 mark)*

14. June 2011

6 A bullet is fired horizontally from the top of a vertical cliff, at a height of h metres above the sea. It hits the sea 4 seconds after being fired, at a distance of 1000 metres from the base of the cliff, as shown in the diagram.



(a) Find the initial speed of the bullet. *(2 marks)*

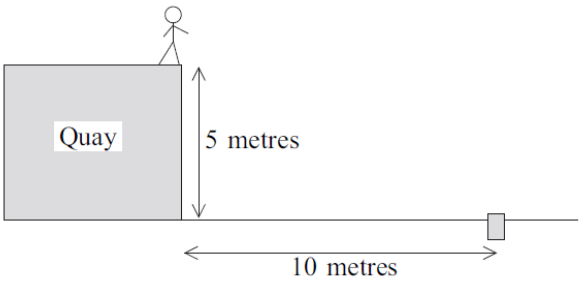
(b) Find h. *(2 marks)*

(c) Find the speed of the bullet when it hits the sea. *(4 marks)*

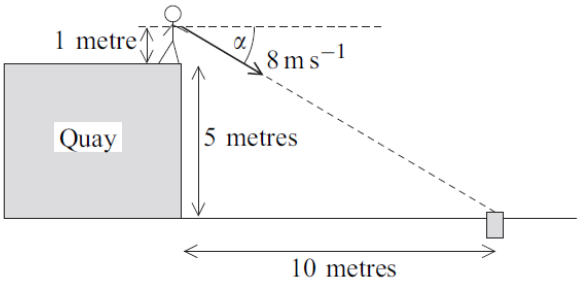
(d) Find the angle between the velocity of the bullet and the horizontal when it hits the sea. *(3 marks)*

15. Jan 2012

8 A girl stands at the edge of a quay and sees a tin can floating in the water. The water level is 5 metres below the top of the quay and the can is at a horizontal distance of 10 metres from the quay, as shown in the diagram.



The girl decides to throw a stone at the can. She throws the stone from a height of 1 metre above the top of the quay. The initial velocity of the stone is $8\,\mathrm{m\,s^{-1}}$ at an angle α below the horizontal, so that the initial velocity of the stone is directed at the can, as shown in the diagram.



Assume that the stone is a particle and that it experiences no air resistance as it moves.

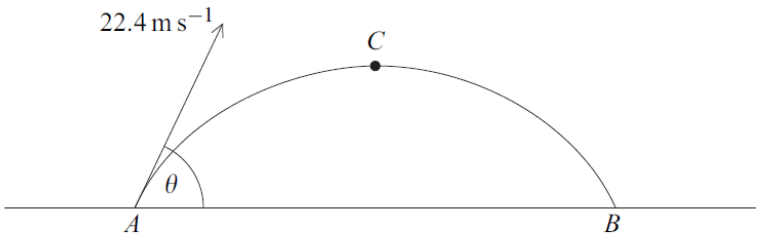
(a) Find α. *(2 marks)*

(b) Find the time that it takes for the stone to reach the level of the water. *(6 marks)*

(c) Find the distance between the stone and the can when the stone hits the water. *(4 marks)*

16. June 2012

8 A particle is launched from the point A on a horizontal surface, with a velocity of $22.4\,\mathrm{m\,s^{-1}}$ at an angle θ above the horizontal, as shown in the diagram.



After 2 seconds, the particle reaches the point C, where it is at its maximum height above the surface.

(a) Show that $\sin\theta = 0.875$. *(3 marks)*

(b) Find the height of the point C above the horizontal surface. *(3 marks)*

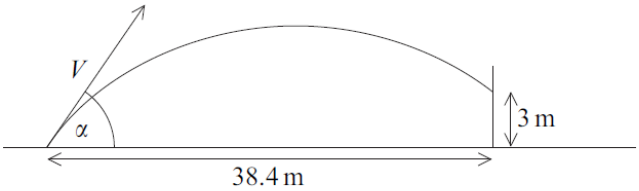
(c) The particle returns to the surface at the point B. Find the distance between A and B. *(3 marks)*

(d) Find the length of time during which the height of the particle above the surface is greater than 5 metres. *(5 marks)*

(e) Find the minimum speed of the particle. *(2 marks)*

17. Jan 2013

8 A golf ball is hit from a point on a horizontal surface, so that it has an initial velocity $V\,\mathrm{m\,s^{-1}}$ at an angle α above the horizontal. The ball travels through the air and after 2.4 seconds hits a vertical wall at a height of 3 metres. The wall is at a horizontal distance of 38.4 metres from the point where the ball was hit. The path of the ball is shown in the diagram.



Assume that the weight of the ball is the only force that acts on it as it travels through the air.

(a) Find the horizontal component of the velocity of the ball. *(2 marks)*

(b) Find V. *(5 marks)*

(c) Find α. *(3 marks)*

Answers

1. Jan 2005

8(a)	Ball is a particle No air resistance	B1 B1	 2	One appropriate assumption Second appropriate assumption
(b)(i)	$0 = 12\sin 40° - 9.8t$ $t = \dfrac{12\sin 40°}{9.8} = 0.787$ s	M1 A1 M1 A1	 4	Equation to find time at maximum height Correct equation Solving for t Correct time
(ii)	$h = 12\sin 40° \times 0.7871 - 4.9 \times 0.7871^2$ $= 3.04$ m	M1 A1 A1	 3	Substituting time from previous into expression for height Correct expression AG; correct height from correct working
(c)	$2.44 = 12\sin 40° t - 4.9t^2$ $4.9t^2 - 12\sin 40° t + 2.44 = 0$ $t = 0.4385$ or 1.136 $s = 12\cos 40° \times 1.136 = 10.4$ m	M1 A1 A1 m1 A1 M1 A1	 7	Equation for time to get to the bar, based on height being 2.44 Correct LHS Correct RHS Solving quadratic Correct time / times Substituting their larger time into an expression for the horizontal displacement Correct distance
	Total		**16**	

2. June 2005

6(a)	Ball is a particle/no spin No air resistance/Only gravity or weight	B1 B1	 2	B1: One assumption B1: Second assumption
(b)(i)	$24.5t - 4.9t^2 = 0$ $\left(t = 0 \text{ or } t = \dfrac{24.5}{4.9} = 5 \text{ s}\right)$ **AG**	M1 A1 dM1 A1	 4	M1: Equation for vertical motion with height zero A1:Correct equation dM1: Solving for t A1: Correct time from correct working
(b)(ii)	$R = 10 \times 5 = 50$ m	M1 A1	 2	M1: Use of horizontal component of velocity to find the range A1: Correct range
(c)	$20 = 10t$ $t = 2$ $h = 24.5 \times 2 - 4.9 \times 2^2 = 29.4$ m	M1 A1 dM1 A1	 4	M1: Horizontal equation A1: Time to reach wall dM1: Vertical equation for height with $u = 24.5$ and a negative acceleration A1: Correct height
(d)	No change as acceleration and initial velocity do not change with the mass	B1 B1	 2	B1: No change B1: Explanation
	Total		**14**	

3. Jan 2006

5(a)	$s = ut + \frac{1}{2}at^2$ $0 = 2\frac{1}{2}ut - \frac{1}{2}gt^2$ $0 = t\left(2\frac{1}{2}u - \frac{1}{2}gt\right)$ $t = \dfrac{5u}{g}$	 M1 A1 m1 A1	 4	 full method required for time (equation of motion, or standard result) (if $g = 9.8$ used, lose last A1)

(b)	$OA = 6u \times \frac{5u}{g}$	M1		
	$= \frac{30u^2}{g}$	A1	2	cao
(c)	$\text{speed}^2 = (6u)^2 + \left(2\frac{1}{2}u\right)^2$	M1		
	$\text{speed} = 6\frac{1}{2}u$	A1	2	cao
(d)	Least speed, at top, $= 6u$	B1	1	
	Total		**9**	

4. June 2006

7(a)	$57 = 24\cos 40° \times t$	M1		Component attempted and acceleration $= 0$
		A1		All correct
	$t = 3.10$ sec	A1	3	CAO
(b)	$h = 24\sin 40° \times 3.1 - \frac{1}{2} \times 9.8 \times 3.1^2$	M1		Component attempted & acceleration = 9.8
		A1		All correct
	$h = 0.734$ m	A1F	3	FT one slip e.g. +9.8 used Accept 2 s.f. answer, AWRT $0.71 - 0.74$
(c)(i)	horizontal, $u = 24\cos 40° = 18.39$ ms^{-1}	B1		Seen anywhere in (c) accept 18.4
	vertical, $v = 24\sin 40° - 9.8 \times 3.1$	M1		Component attempted & acceleration = 9.8
	$v = -14.95$ ms^{-1}	A1		(Accept -15.0)
	$V = \sqrt{(18.39)^2 + (-14.95)^2}$	M1		Use of candidate's u and new v (when $t = 3.1$)
	$V = 23.7$ms^{-1}	A1F	5	FT use of candidate's u and v and new v when $t = 3.1$
(ii)	$\tan\theta = \frac{14.95}{18.39}$	M1		Use of candidate's u and v Accept inverted ratio
	$\left.\begin{array}{l}\theta = 39.1° \text{ or } 39.2° \\ \text{Also } 140.8° \text{ or } 140.9°\end{array}\right\}$ accept $\pm$	A1F	2	FT use of candidates u and v and V
	Total		**13**	

5. Jan 2007

7(a)	$0^2 = (50\sin 40°)^2 + 2 \times (-9.8)h$	M1A1		Equation for h with $v = 0$ and a component of velocity. Correct equation
	$h = \frac{(50\sin 40°)^2}{2 \times 9.8} = 52.7$	dM1 A1		Solving for h Correct h
	Alt			
	$0 = 50\sin 40° - 9.8t$	(M1)		Equation for t with $v = 0$ and a component of velocity
	$t = \frac{50\sin 40°}{9.8} = 3.280$	(A1)		Correct t
	$h = 50\sin 40° \times 3.280 - \frac{1}{2} \times 9.8 \times 3.280^2$	(dM1)		Expression for h with a component of velocity
	$= 52.7$ ALLOW 52.6	(A1)	4	Correct h

(b)	$6 = 50\sin 40°t - 4.9t^2$	M1A1		Forming a quadratic in t. Correct terms with any signs
	$0 = 4.9t^2 - 50\sin 40°t + 6$	A1		Correct equation
	$t = \dfrac{50\sin 40° \pm \sqrt{(50\sin 40°)^2 - 4\times 4.9\times 6}}{2\times 4.9}$	dM1		Solving quadratic
	$= 0.192$ or 6.37			
	$t = 6.37$	A2	6	Correct solution selected
	Alt			
	$46.7 = 4.9t_1^2$	(M1)		Finding two times
		(dM1)		Equation for time to go down
	$t_1 = 3.087$	(A1)		Correct time
	$t_2 = 3.280$	(A1)		Time to go up
	$t = 3.087 + 3.280 = 6.37$	(A2)		Correct total
	Total		**10**	

6. June 2007

7(a)	A particle or no spin	B1		First assumption
	No air resistance or no wind or only gravity acting	B1	2	Second assumption
				If more than 2 assumptions given, subtract one mark for each incorrect additional assumption
(b)	$0 = 25\sin 40°t - 4.9t^2$	M1		Equation for time of flight
		A1		Correct equation
	$0 = t(25\sin 40° - 4.9t)$	dM1		Solving for t
	$t = 0$ or $t = \dfrac{25\sin 40°}{4.9}$			
	Time of flight = 3.28 s	A1	4	AG Correct final answer from correct working (Verification method M1A1M1A0)
(c)	$s = 3.28\times 25\cos 40° = 62.8$ m	M1		Finding range
		A1	2	Correct range
(d)	25 ms^{-1} at 40° **below** the horizontal	B1		Speed
		B1	2	Direction
(e)	$v_{min} = 25\cos 40° = 19.2$ ms^{-1}	M1		Horizontal component of velocity
		A1	2	Correct speed
	OR			Accept 19.1 ms^{-1}
	$v_{min} = \dfrac{62.807}{3.2795} = 19.2$ ms^{-1}			
	Total		**12**	

7. Jan 2008

7(a)	It is a particle /No air resistance / lift forces act on the ball.	B1		Particle
		B1	2	Other acceptable assumption Deduct one mark for each additional incorrect assumption.
(b)	$V\sin 40°t - \frac{1}{2}\times 9.8t^2 = 0$	M1		Vertical equation to find t.
		A1		Correct equation (Equals zero may be implied)
	$t = \dfrac{V\sin 40°}{4.9}$	dM1		Solving for t
		A1		Correct t
	$s = V\cos 40°\times\dfrac{V\sin 40°}{4.9}$ **AG**			
	$= \dfrac{V^2\cos 40°\sin 40°}{4.9}$	M1		Finding range with their t
		A1	6	Correct range from correct working SC Quoting the formula for the range 2 marks.

(c)	$76 < \frac{V^2 \cos 40° \sin 40°}{4.9} < 82$ $\sqrt{\frac{76 \times 4.9}{\cos 40° \sin 40°}} < V < \sqrt{\frac{82 \times 4.9}{\cos 40° \sin 40°}}$ $27.5 < V < 28.6$	M1 A1 A1 A1	4	An equation to find one value of V. Correct value for V Other value of V correct Correct range of values Accept 27.5 – 28.6 but not 28.6-27.5 For using values close to 76 and 82 deduct one mark.
	Total		**12**	

8. June 2008

7(a)(i)	$0 = 40\sin 35° t - 4.9t^2$ $t = \frac{40\sin 35°}{4.9} = 4.68$ s **AG**	M1 A1 m1 A1	4	Equation to find time of flight with 40, sin/cos35° and –4.9 or $-\frac{g}{2}$ Correct equation Solving for t Correct time from correct working Note: candidates must have a method for the complete time of flight before any marks can be awarded. Condone the use of a formula for the time of flight.
(b)	$-1 = 40\sin 35° t - 4.9t^2$ $4.9t^2 - 40\sin 35° t - 1 = 0$ $t = \frac{40\sin 35° \pm \sqrt{(40\sin 35°)^2 - 4 \times 4.9 \times (-1)}}{2 \times 4.9}$ $t = 4.73$ or $t = -0.0432$ $t = 4.73$	M1 A1 A1 m1 A1 A1	6	Equation to find time of flight with a ±1, 40, sin/cos35° and –4.9 or $-\frac{g}{2}$ Correct terms Correct signs Solving quadratic equation Accept AWRT 4.73 or 4.72 Rejection of negative solution indicated (Only 4.73 or 4.72 given award 5/6 marks)
	Alternative methods based on finding two times. For example, $t = 4.682 + 0.044 = 4.73$ $t = 2.341 + 2.384 = 4.73$	(M1) (A1) (m1) (A1) (A1) (A1)		Addition of two times Use of AWRT 4.68 or AWRT 2.34 Calculation of time for 'second' part Correct expression for time for 'second' part Correct time (Allow AWRT 0.04 or AWRT 2.38) Correct total time Accept 4.72
	Total		**12**	

9. Jan 2009

8				If candidates have already used $g = 9.81$ do not penalise again on this question.
(a)	$0^2 = (28\sin 50°)^2 + 2 \times (-9.8)s$	M1 A1		M1: Equation to find the max height, with $v = 0$, $u = 28\sin 50°$ or $u = 28\cos 50°$ and –9.8 or $-g$. A1: Correct equation

	$s=\frac{(28\sin 50^\circ)^2}{2\times 9.8}=23.5$ m **OR** $0=28\sin 50^\circ-9.8t$ $t=\frac{28\sin 50^\circ}{9.8}=2.1887$ $s=28\sin 50^\circ\times 2.1887-4.9\times 2.1887^2=23.5$	dM1 A1 (M1) (A1) (dM1) (A1)	4	dM1: Solving for the height A1: Correct height. Awrt 23.5 Note: If using a memorised formula, either 4 marks if final answer correct, 3 marks if substituted correctly but evaluated incorrectly, otherwise zero. M1: Equation to find time to the max height, with $v=0$, $u=28\sin 50^\circ$ or $u=28\cos 50^\circ$ and -9.8 or $-g$. A1: Correct time dM1: Finding the height with their time and $u=28\sin 50^\circ$ or $u=28\cos 50^\circ$ and -4.9 or $-g/2$ A1: Correct height. Awrt 23.5

10.June 2009

6(a)	$20\sin 50^\circ t-4.9t^2=0$ $t=\frac{20\sin 50^\circ}{4.9}$ or $3.126...=3.13$ s AG	M1A1 dM1 A1		M1: Equation to find time, with $y=0$, $u=20\sin 50^\circ$ or $u=20\cos 50^\circ$ and ± 9.8 or $\pm g$. A1: Correct equation dM1: Solving for t. A1: Correct time from correct working. Must see division by 4.9 oe or more than 3sf Verification methods can only gain first 2 marks Special case $t=\frac{15.3}{4.9}=3.12$ or 3.13 scores
	OR $0=20\sin 50^\circ-9.8t$ $t=\frac{20\sin 50^\circ}{9.8}=1.563$ $T=2\times 1.563$ $=3.13$	(M2) (A2)	4	M1A1dM1A0 M2: doubling time to max height (could use cos instead of sin) but must use ± 9.8 or $\pm g$. A2: Correct time from correct working. Don't penalise use of $g=9.81$ if already done earlier on script. Would obtain time as 3.12 seconds. Note: If using a memorised formula either 4 marks if final answer correct, 3 marks if substituted correctly, otherwise zero.
(b)	$PQ=20\cos 50^\circ\times 3.127=40.2$ m	M1 A1	2	M1: Calculation of range, could use sin instead of cos. A1: Correct range Accept 40.1
(c)	No change because a greater mass would not change the acceleration. OR Mass is not used in the equations.	B1 B1	2	B1: No change B1: Explanation following a correct statement.

11.Jan 2010

7(a)	$5=\frac{1}{2}\times 9.8t^2$	M1 A1		M1: Equation based on vertical motion with no velocity component, with ± 5 and ± 9.8 A1: Correct equation

	$t=\sqrt{\frac{5}{4.9}}=1.01\text{ s}$ **AG**	A1	3	A1: Correct time from correct working. Must see square root or $t^2=1.02$ OE Note: If g = 9.81 is used for the first time deduct one mark. Should still get 1.01 seconds.
(b)	$15=V\times\sqrt{\frac{5}{4.9}}$ $V=15\sqrt{\frac{4.9}{5}}=14.8$	M1 A1	2	M1: Using distance = speed×time OE A1: Correct speed. Accept AWRT 14.8 or 14.9. Note: If g = 9.81 is used for the first time deduct one mark. Should get 14.9 ms^{-1} from g = 9.81.
(c)	$v_V=\pm 9.8\times\sqrt{\frac{5}{4.9}}\,(=\pm 9.899)$ or $v_V=\sqrt{2\times 9.8\times 5}=9.899$ $v=\sqrt{9.899^2+14.8^2}=17.8\text{ ms}^{-1}$	M1A1 dM1 A1F	4	M1: Calculating vertical component of velocity. A1: Correct value. Accept 9.9 or similar dM1: Finding magnitude (with addition not subtraction of squares inside the square root). A1: Correct speed. Accept AWRT 17.8 or AWRT 17.9. Note: If g = 9.81 is used for the first time deduct one mark. Should get 17.9 ms^{-1} from g = 9.81
(d)	$\tan\alpha=\frac{9.899}{14.8}$ or $\frac{14.8}{9.899}$ $\alpha=34°$ $\sin\alpha=\frac{9.899}{17.8}$ or $\frac{14.8}{17.8}$ $\alpha=34°$ $\cos\alpha=\frac{14.8}{17.8}$ or $\frac{9.899}{17.8}$ $\alpha=34°$	M1 A1F A1F (M1) (A1F) (A1F) (M1) (A1F) (A1F)	3	M1: Use of one of trig equations shown. A1F: Anything which rounds to 34° or 56° A1F: 34° CAO (33° scores M1A1A0) Only follow through if all method marks in (b) and (c) have been awarded (except the dM if tan used) .
(e)	Particle Experiences no air resistance **or** no wind **or** only gravity **or** no other forces acting **or** no spin.	B1 B1	2	B1: Particle assumption B1: Other assumption. Ignore any other assumptions.
	Total		**14**	

12.June 2010

8(a)	$14.7\sin\alpha-9.8t\,(=0)$ $t=\frac{14.7\sin\alpha}{9.8}=\frac{3\sin\alpha}{2}$ **AG** **OR** $14.7\sin\alpha T-4.9T^2\ (=0)$ $T=\frac{14.7\sin\alpha}{4.9}=3\sin\alpha$ $t=\frac{3\sin\alpha}{2}$	M1A1 A1 (M1) (A1) (A1)	3	M1: Equation for vertical velocity being zero at highest point. Must have $\sin\alpha$ with ± 9.8. A1: Correct equation. A1: Correct result from correct working. All marks awarded for last line, from correct working.

(b)(i)	$7 = 14.7\sin\alpha\left(\frac{3\sin\alpha}{2}\right) - 4.9\left(\frac{3\sin\alpha}{2}\right)^2$	M1 A1		M1: Expression including vertical displacement at height 7, using expression from part (a) and with $\pm g$ or equivalent. A1: Correct expression.
	$7 = 11.025\sin^2\alpha$	dM1		dM1: Simplified expression with $\sin^2\alpha$.
	$\alpha = \sin^{-1}\left(\sqrt{\frac{7}{11.025}}\right) = 52.8°$	dM1 A1	5	dM1: Finding an angle. Must have previous dM1 mark. A1: Correct angle. Accept 52.7°, 52.9°.
	OR			
	$0^2 = (14.7\sin\alpha)^2 + 2\times(-9.8)\times 7$ $\sin^2\alpha = \frac{2\times 9.8\times 7}{14.7^2}$ $\alpha = 52.8°$	(M1) (A1) (dM1) (dM1) (A1)		
(ii)	$OA = 14.7\cos 52.8° \times 3\sin 52.8°$	B1M1		B1: Use of $3\sin\alpha$ with their α. M1: Finding horizontal displacement. including $14.7\cos\alpha$ with $3\sin\alpha$ or $\frac{3\sin\alpha}{2}$.
	$OA = 21.2$ m	A1	3	A1: Correct distance. Accept 21.3 m.
(c)	Ball is a particle/No spin. No air resistance/No wind/Constant acceleration of 9.8/Only force is weight.	B1 B1	2	B1: Particle assumption. B1: Air resistance assumption.
	Total		**13**	

13.Jan 2011

7(a)	$12\sin 30° t - 4.9t^2 = -0.5$ $4.9t^2 - 12\sin 30° t - 0.5 = 0$ $t = 1.30281...or - 0.078323...$ $t = 1.30$ seconds (to 3sf) AG	M1A1A1 dM1 A1	5	M1: Three term equation for vertical motion, with $\pm g$, ± 0.5 (or ± 1 and ± 1.5) and $12\sin 30° t$ or $12\cos 30° t$. A1: Correct terms. (one must be equivalent to ± 0.5) A1: Correct signs. dM1: Solving the quadratic to find t. Must see use of quadratic equation formula or can be implied by seeing 1.303 or 1.302 or similar. A1: Correct time from correct working. Must see more than 3 significant figures in candidate's working before the final answer or two correct solutions to the quadratic (eg 1.3 and –0.08). Accept 1.3
	OR time up = 0.6122 time down = 0.6122+0.0783=0.6905 total time = 0.6122+0.6905 = 1.30 (to 3sf)	(M1A1 dM1A1A1)		M1:Adding time up to time down having used a quadratic. A1: 0.6122 dM1: Finding time down with a quadratic A1: 0.6905 A1: Correct answer Accept 1.3
	OR $-6.767 = 12\sin 30° - gt$ $t = \frac{12\sin 30° + 6.767}{g} = 1.30281 = 1.30$ (to 3sf)	(M1A1A1) (dM1A1)		M1:Forms an equation to find t having found v first A1: Correct terms A1: Correct signs dM1: Solving for t

(b)	$12\cos 30° \times 1.303 = 13.5$ m	M1A1	2	A1: Correct time from correct working. Must see more than 3 significant figures in candidate's working before the final answer. Accept 1.3 M1: Finding horizontal displacement using 1.30 (or better) and $12\cos 30°$. Do not allow $12\sin 30°$. A1: Correct distance. AWRT 13.5.

14.June 2011

6 (a)	$1000 = V \times 4$ $V = 250$ ms^{-1}	M1 A1	2	M1: Equation for horizontal motion to find V. Must not contain g. Could contain cos0° or equivalent. A1: Correct V.
(b)	$(h =)\frac{1}{2} \times 9.8 \times 4^2$ $= 78.4$ metres to 3sf	M1 A1	2	M1: Vertical equation to find height with $u = 0$ and $a = \pm 9.8$. A1: Correct height. Accept –78.4
(c)	$(v_y =) 9.8 \times 4 = 39.2$ ms^{-1} or $(v_y =)\sqrt{2 \times 9.8 \times 78.4} = 39.2$ ms^{-1} $(v =)\sqrt{250^2 + 39.2^2} = 253$ ms^{-1}	M1A1 dM1A1	4	M1: Calculation of vertical component of velocity with $u = 0$ and $a = \pm 9.8$. A1: Correct vertical component. dM1: Calculation of speed. A1: Correct speed.
(d)	$\tan\alpha = \frac{39.2}{250}$ $\left(\text{or } \tan\alpha = \frac{250}{39.2}\right)$ $\alpha = 8.91°$ **OR** $\sin\alpha = \frac{39.2}{253}$ $\left(\text{or } \sin\alpha = \frac{250}{253}\right)$ $\alpha = 8.91°$	M1A1F A1 (M1A1F) (A1)	3	M1: Using tan to find angle with opposite and adjacent sides. Can be inverted as shown in brackets. A1F: Correct trig expression. A1: Correct angle. M1: Using sin to find angle with hypotenuse and one other side. Can be changed as shown in brackets. A1F: Correct trig expression. A1: Correct angle.
	OR $\cos\alpha = \frac{250}{253(.055)}$ $\left(\text{or } \cos\alpha = \frac{39.2}{253}\right)$ $\alpha = 8.91°$	(M1A1F) (A1)		M1: Using cos to find angle with hypotenuse and one other side. Can be changed as shown in brackets. A1F: Correct trig expression. A1: Correct angle. Accept 8.83° from this method. Note: Accept 8.98° from 253.1 Accept negative angles Note: FT value of V from (a) and speed from (c) if needed. Do not FT 39.2 from (c) in place of 253. Note: Accept energy methods if used correctly in part (c).
	Total		**11**	

15. Jan 2012

8(a)	$\tan\alpha = \frac{6}{10}$ $\alpha = 31.0°$	M1 A1	2	M1: Using tan with 10 and 5 or 6, OR sin or cos with $\sqrt{136}$ and 6 or 10, OR sin or cos with $\sqrt{125}$ and 5 or 10.

				A1: Correct angle. Accept 30.9° or AWRT 31°
(b)	$8\sin\alpha t + 4.9t^2 = 6$ $4.9t^2 + 4.116t - 6 = 0$ $t = 0.76359$ or $t = -1.60$ s $t = 0.764$	M1 A1FA1F A1 dM1 A1	6	M1: equation for the vertical motion containing ±6 or ±5, $\pm 4.9t^2$ and $\pm 8\sin\alpha$ or $\pm 8\cos\alpha$, where α has a value related to their answer to part (a) (May be a negative angle). A1F: Correct terms. A1F: Correct signs and terms Follow through angle from part (a). A1: Correct equation rearranged equal to zero, but may be implied by subsequent working. dM1: Attempting to solve their quadratic equation. Only award method mark if method seen or correct answers obtained or –0.764 with +1.60. A1: Correct solution obtained. Accept 0.763 or AWRT 0.764.
	$v = \sqrt{(8\sin 31.0°)^2 + 2\times 9.8\times 6} = 11.60$ $11.60 = 8\sin 31° + 9.8t$ $t = \dfrac{11.60 - 8\sin 31°}{9.8} = 0.763$	(M1) (A1FA1F) (dM1) (A1) (A1)		M1: Use a constant acceleration equation $v^2 = u^2 + 2as$ to find v. A1F: Correct equation. A1F: Correct v. dM1: Use of $v = u + at$ to find t A1: Correct equation. A1: Correct t (0.763)
(c)	$d = 10 - 8\cos\alpha \times 0.764$ $= 10 - 5.238$ $= 4.76$ m	M1dM1 A1 A1	4	M1: Finding a horizontal distance using 8cosα or 8sinα multiplied by their time from part (b). dM1: For subtracting their distance from 10. A1: Seeing AWRT 5.24 or 5.23 from 0.763. A1: Correct final answer. Accept AWRT 4.76. Accept 4.77 from use of 0.763.
	Total		**12**	

16. June 2012

8(a)	$22.4\sin\theta - 2\times 9.8 = 0$ $\sin\theta = \dfrac{19.6}{22.4} = \dfrac{7}{8} = 0.875$ **AG** Or $0 = 22.4\sin\theta \times 4 - \dfrac{1}{2}\times 9.8\times 4^2$ $\sin\theta = \dfrac{4.9\times 16}{22.4\times 4} = 0.875$	M1A1 A1 (M1A1) (A1)	3	M1: Use of $v = u + at$ vertically with $u = 22.4\sin\theta$, $v = 0$, $t = 2$ and $a = \pm 9.8$. A1: Correct equation. (May be in terms of g or contain 9.81.. A1: Must see either $22.4\sin\theta = 19.6$ or $\dfrac{19.6}{22.4}$. M1: Use of $s = ut + \dfrac{1}{2}at^2$ with $u = 22.4\sin\theta$, $s = 0$, $t = 4$ and $a = \pm 9.8$. A1: Correct equation. A1: must see $89.6\sin\theta = 78.4$ or $\dfrac{78.4}{89.6}$ OE

8(b)	$h_{MAX} = 22.4\times\sin\theta\times 2 - \dfrac{1}{2}\times 9.8\times 2^2$ $= 19.6$ m	M1A1 A1	3	M1: Using a constant acceleration equation to find height, with $t = 2$, u=22.4 sin θ or 19.6 and $a = \pm 9.8$.

	Or $0^2 = (22.4 \times \sin\theta)^2 + 2 \times (-9.8) h_{MAX}$ $h_{MAX} = 19.6$ m	(M1A1) (A1)		A1: Correct equation. A1: Correct height. AWRT 19.6 Note using $g = 9.81$ gives 19.6, also accept 19.5. Note: other constant acceleration equations will lead to the same result
(c)	$\cos\theta = \frac{\sqrt{15}}{8} = 0.4841$ or $\theta = 61.04°$ $AB = 22.4 \times \frac{\sqrt{15}}{8} \times 4 = 43.4$ m	B1 M1A1F	3	B1: Correct value for $\cos\theta$ (accept 0.484) or θ (accept 61.0° or 61° or 1.06 or 1.065 or 1.07 radians). Can be implied. M1: Calculation for range with value for $\cos\theta$ and with $t = 4$. A1F: Correct distance. Follow through incorrect θ. Accept AWRT 43.4 or 43.3 or 43.2. Do not accept 43.
(d)	$22.4 \times (\sin\theta)t - 4.9t^2 = 5$ $4.9t^2 - 19.6t + 5 = 0$ $t = 0.274$ or $t = 3.726$ Time $= 3.726 - 0.274 = 3.45$ seconds	M1 A1 dM1 A1 A1	5	M1: Use of $s = ut + \frac{1}{2}at^2$ with correct terms, but not necessarily signs. A1: Correct equation. dM1: Solving their quadratic. A1: At least one correct solution. Allow 0.27 or 0.28 and 3.72 or 3.73 A1: Correct difference. Accept 3.46. Note: there are other methods which will lead to the correct time: M1dM1A1 for a constant acceleration equation that gives a time or times from which the final answer can be obtained A1 Correct time or times A1 Correct final answer
(e)	$v_{MIN} = 22.4 \times \cos\theta$ $= 10.8$ ms^{-1} Or $v_{min} = \frac{43.4}{4} = 10.9$ to 3sf	M1 A1 (M1) (A1)	2	M1: Finding horizontal component with candidate's value for $\cos\theta$. Do not award if combined with a non-zero vertical component. A1: Correct speed. Accept 10.9 or 10.85. M1: range divided by time of flight A1: Correct speed. Accept 10.9 or 10.85.
	Total		**16**	

17. Jan 2013

8(a)	$(V_H =)\frac{38.4}{2.4} = 16$ m s^{-1}	M1A1	2	M1: Horizontal range divided by time. A1: Correct speed.
(b)	$3 = V_V \times 2.4 - \frac{1}{2} \times 9.8 \times 2.4^2$ $V_V = \frac{3 + 28.224}{2.4} = 13.01$ $V = \sqrt{13.01^2 + 16^2} = 20.6$ m s^{-1}	M1A1 A1 dM1A1	5	M1: Equation to find the vertical component, with $s = \pm 3$, $t = 2.4$ and $a = \pm g$ or ± 9.8 or ± 9.81. A1: Correct equation with g or 9.8 or ± 9.81. A1: Correct vertical component. Accept AWRT 13. dM1: Finding speed using their answer from part (a) and their vertical component. A1:Correct final speed. Accept AWRT 20.6.

(c)	$\tan\alpha = \frac{13.01}{16}$ or $\sin\alpha = \frac{13.01}{20.6}$ or $\cos\alpha = \frac{16}{20.6}$ $\alpha = 39.1°$	M1A1F A1F	3	M1: Trig equation to find the angle with: cos with 13 or 16 in the numerator and 20.6 in denominator sin with 13 or 16 in the numerator and 20.6 in denominator tan with 13 and 16 in any position A1F: Correct equation. A1F: Correct angle. Accept AWRT 39° Follow through incorrect answers to part (a) and (b), provided their speed from (b) is the resultant of two components.
	Total		**10**	