Practice examination questions for matrix transformations

1. Jan 2005

5 (a) The transformation T_1 is defined by the matrix

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Describe this transformation geometrically. *(2 marks)*

(b) The transformation T_2 is an anticlockwise rotation about the origin through an angle of 60°.

Find the matrix of the transformation T_2. Use surds in your answer where appropriate. *(3 marks)*

(c) Find the matrix of the transformation obtained by carrying out T_1 followed by T_2. *(3 marks)*

2. June 2005

7 [**Figure 1**, printed on the insert, is provided for use in this question.]

The diagram shows a triangle with vertices $A(1, 1)$, $B(3, 1)$ and $C(3, 2)$.

(a) The triangle DEF is obtained by applying to triangle ABC the transformation T represented by the matrix

$$\begin{bmatrix} 2 & 2 \\ -2 & 2 \end{bmatrix}$$

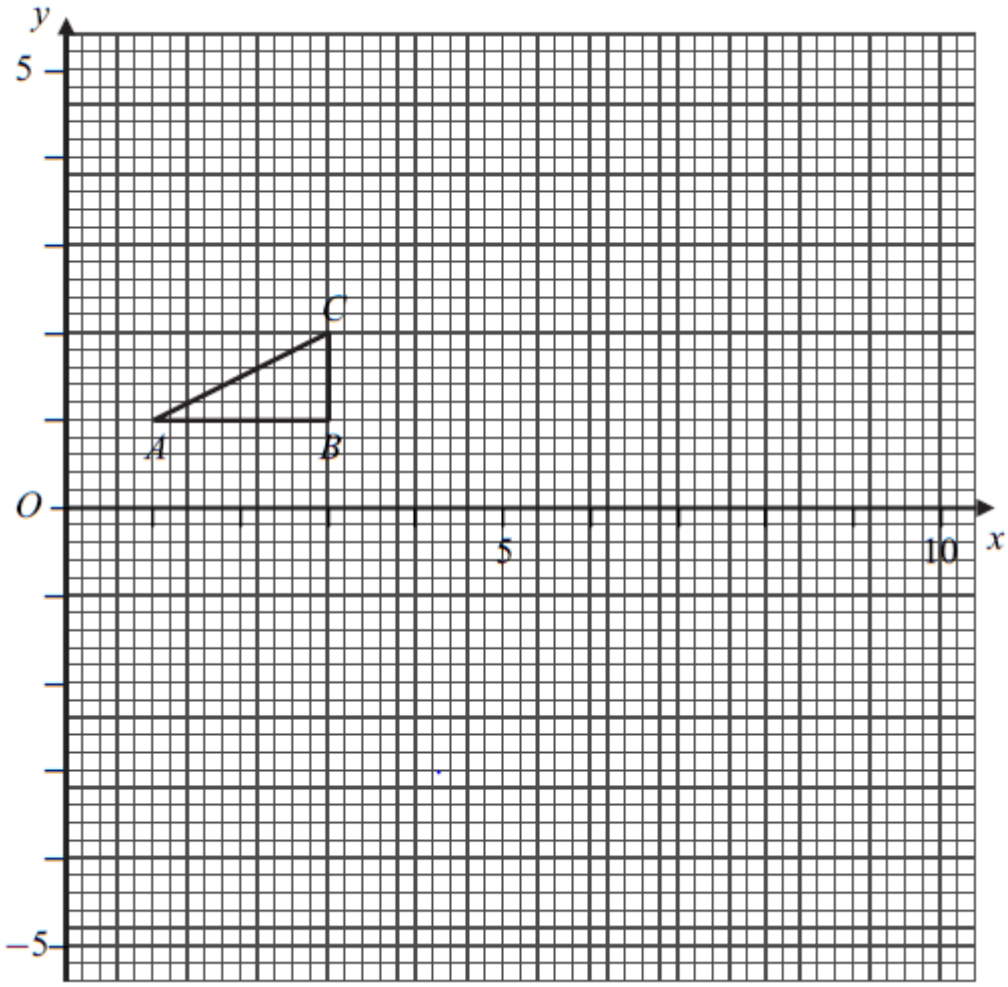
(i) Calculate the coordinates of D, E and F. *(4 marks)*

(ii) Draw the triangle DEF on **Figure 1**. *(2 marks)*

(b) Given that T is a combination of an enlargement and a rotation, find the exact value of:

(i) the scale factor of the enlargement; *(2 marks)*

(ii) the magnitude of the angle of the rotation. *(2 marks)*



3. Jan 2006

7 (a) The transformation T is defined by the matrix **A**, where

$$\mathbf{A} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

(i) Describe the transformation T geometrically. *(2 marks)*

(ii) Calculate the matrix product $\mathbf{A}^2$. *(2 marks)*

(iii) Explain briefly why the transformation T followed by T is the identity transformation. *(1 mark)*

(b) The matrix **B** is defined by

$$\mathbf{B} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

(i) Calculate $\mathbf{B}^2 - \mathbf{A}^2$. *(3 marks)*

(ii) Calculate $(\mathbf{B} + \mathbf{A})(\mathbf{B} - \mathbf{A})$. *(3 marks)*

4. Jan 2006

8 A curve has equation $y^2 = 12x$.

(a) Sketch the curve. *(2 marks)*

(b) (i) The curve is translated by 2 units in the positive y direction. Write down the equation of the curve after this translation. *(2 marks)*

(ii) The **original** curve is reflected in the line $y = x$. Write down the equation of the curve after this reflection. *(1 mark)*

(c) (i) Show that if the straight line $y = x + c$, where c is a constant, intersects the curve $y^2 = 12x$, then the x-coordinates of the points of intersection satisfy the equation

$$x^2 + (2c - 12)x + c^2 = 0$$ *(3 marks)*

(ii) Hence find the value of c for which the straight line is a tangent to the curve. *(2 marks)*

(iii) Using this value of c, find the coordinates of the point where the line touches the curve. *(2 marks)*

(iv) In the case where $c = 4$, determine whether the line intersects the curve or not. *(3 marks)*

5. June 2006

5 The matrix **M** is defined by

$$\mathbf{M} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

(a) Find the matrix:

(i) $\mathbf{M}^2$; *(3 marks)*

(ii) $\mathbf{M}^4$. *(1 mark)*

(b) Describe fully the geometrical transformation represented by **M**. *(2 marks)*

(c) Find the matrix $\mathbf{M}^{2006}$. *(3 marks)*

6. Jan 2007

2 The matrices **A** and **B** are given by

$$\mathbf{A} = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$$

(a) Calculate:

(i) $\mathbf{A} + \mathbf{B}$; *(2 marks)*

(ii) $\mathbf{BA}$. *(3 marks)*

(b) Describe fully the geometrical transformation represented by each of the following matrices:

(i) $\mathbf{A}$; *(2 marks)*

(ii) $\mathbf{B}$; *(2 marks)*

(iii) $\mathbf{BA}$. *(2 marks)*

7. June 2007

1 The matrices $\mathbf{A}$ and $\mathbf{B}$ are given by

$$\mathbf{A} = \begin{bmatrix} 2 & 1 \\ 3 & 8 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

The matrix $\mathbf{M} = \mathbf{A} - 2\mathbf{B}$.

(a) Show that $\mathbf{M} = n\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$, where n is a positive integer. *(2 marks)*

(b) The matrix $\mathbf{M}$ represents a combination of an enlargement of scale factor p and a reflection in a line L. State the value of p and write down the equation of L. *(2 marks)*

(c) Show that

$$\mathbf{M}^2 = q\mathbf{I}$$

where q is an integer and $\mathbf{I}$ is the 2×2 identity matrix. *(2 marks)*

8. Jan 2008

6 The matrix $\mathbf{M}$ is defined by

$$\mathbf{M} = \begin{bmatrix} \sqrt{3} & 3 \\ 3 & -\sqrt{3} \end{bmatrix}$$

(a) (i) Show that

$$\mathbf{M}^2 = p\mathbf{I}$$

where p is an integer and $\mathbf{I}$ is the 2×2 identity matrix. *(3 marks)*

(ii) Show that the matrix $\mathbf{M}$ can be written in the form

$$q\begin{bmatrix} \cos 60^\circ & \sin 60^\circ \\ \sin 60^\circ & -\cos 60^\circ \end{bmatrix}$$

where q is a real number. Give the value of q in surd form. *(3 marks)*

(b) The matrix $\mathbf{M}$ represents a combination of an enlargement and a reflection.

Find:

(i) the scale factor of the enlargement; *(1 mark)*

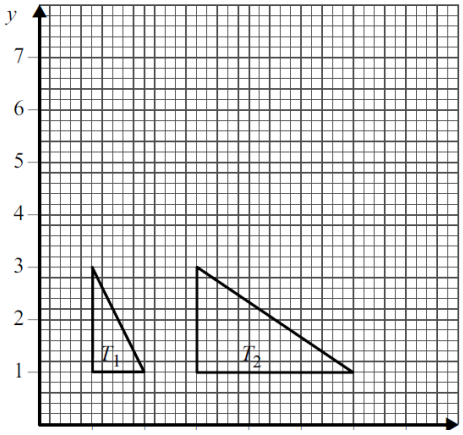
(ii) the equation of the mirror line of the reflection. *(1 mark)*

(c) Describe fully the geometrical transformation represented by $\mathbf{M}^4$. *(2 marks)*

9. June 2008

8 [**Figure 3**, printed on the insert, is provided for use in this question.]

The diagram shows two triangles, T_1 and T_2.



(a) Find the matrix of the stretch which maps T_1 to T_2. *(2 marks)*

(b) The triangle T_2 is reflected in the line $y = x$ to give a third triangle, T_3.

On **Figure 3**, draw the triangle T_3. *(2 marks)*

(c) Find the matrix of the transformation which maps T_1 to T_3. *(3 marks)*

10.Jan 2009

5 The matrices **A** and **B** are defined by

$$\mathbf{A} = \begin{bmatrix} k & k \\ k & -k \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} -k & k \\ k & k \end{bmatrix}$$

where k is a constant.

(a) Find, in terms of k:

(i) $\mathbf{A} + \mathbf{B}$; *(1 mark)*

(ii) $\mathbf{A}^2$. *(2 marks)*

(b) Show that $(\mathbf{A} + \mathbf{B})^2 = \mathbf{A}^2 + \mathbf{B}^2$. *(4 marks)*

(c) It is now given that $k = 1$.

(i) Describe the geometrical transformation represented by the matrix $\mathbf{A}^2$. *(2 marks)*

(ii) The matrix **A** represents a combination of an enlargement and a reflection. Find the scale factor of the enlargement and the equation of the mirror line of the reflection. *(3 marks)*

11.June 2009

7 (a) Using surd forms where appropriate, find the matrix which represents:

(i) a rotation about the origin through 30° anticlockwise; *(2 marks)*

(ii) a reflection in the line $y = \frac{1}{\sqrt{3}}x$. *(2 marks)*

(b) The matrix **A**, where

$$\mathbf{A} = \begin{bmatrix} 1 & \sqrt{3} \\ \sqrt{3} & -1 \end{bmatrix}$$

represents a combination of an enlargement and a reflection. Find the scale factor of the enlargement and the equation of the mirror line of the reflection. *(2 marks)*

(c) The transformation represented by **A** is followed by the transformation represented by **B**, where

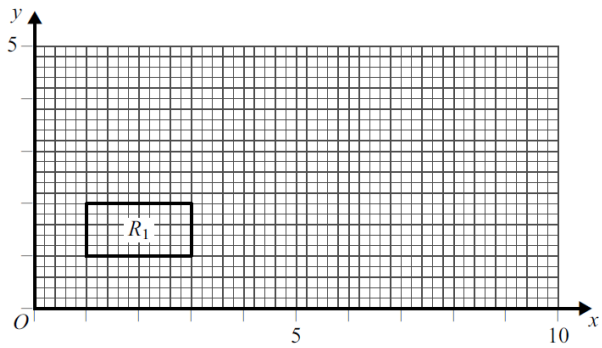
$$\mathbf{B} = \begin{bmatrix} \sqrt{3} & -1 \\ 1 & \sqrt{3} \end{bmatrix}$$

Find the matrix of the combined transformation and give a full geometrical description of this combined transformation. *(5 marks)*

12.Jan 2010

6 [**Figure 1**, printed on the insert, is provided for use in this question.]

The diagram shows a rectangle R_1.



(a) The rectangle R_1 is mapped onto a second rectangle, R_2, by a transformation with matrix $\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$.

(i) Calculate the coordinates of the vertices of the rectangle R_2. *(2 marks)*

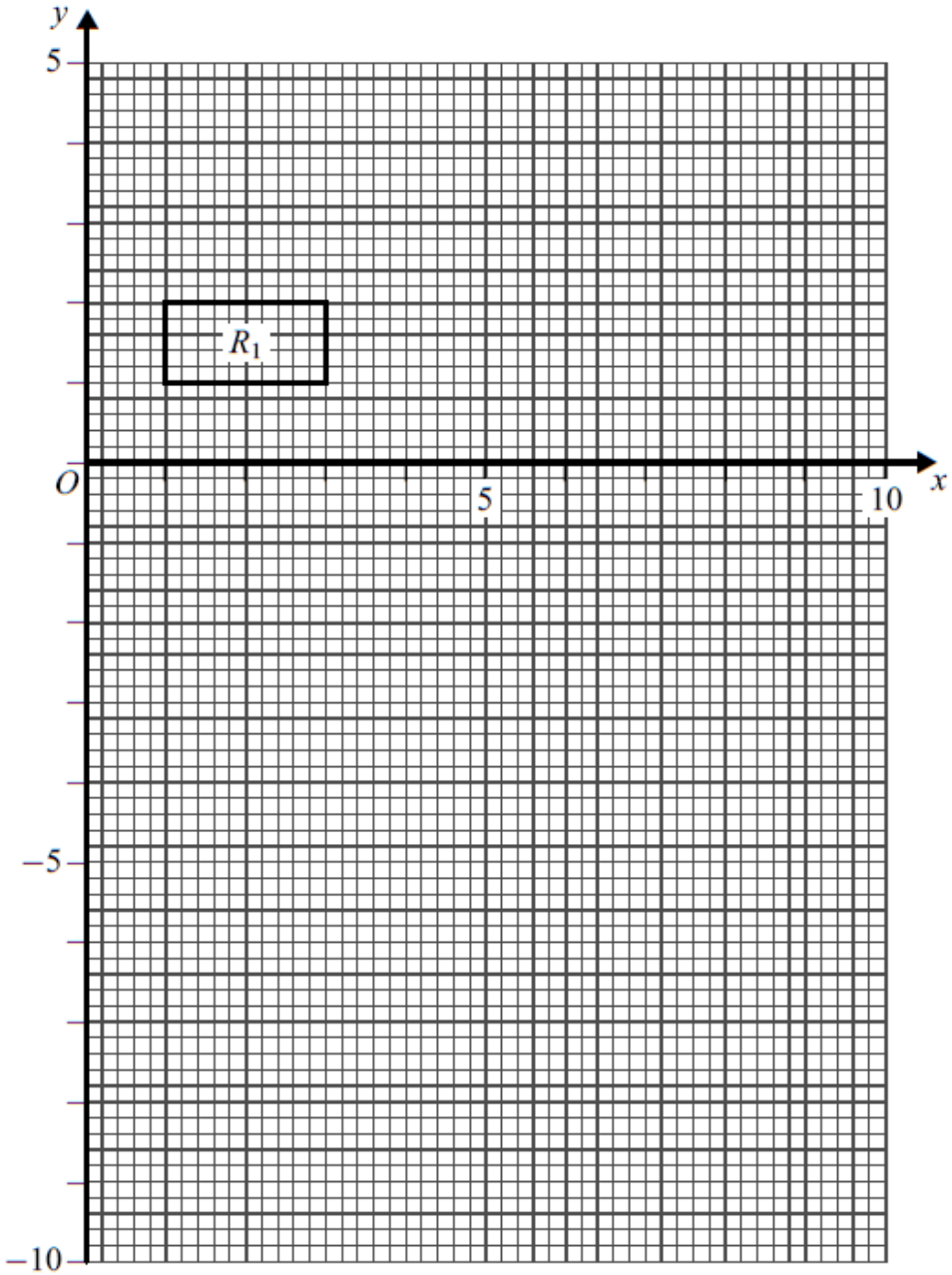
(ii) On **Figure 1**, draw the rectangle R_2. *(1 mark)*

(b) The rectangle R_2 is rotated through 90° clockwise about the origin to give a third rectangle, R_3.

(i) On **Figure 1**, draw the rectangle R_3. *(2 marks)*

(ii) Write down the matrix of the rotation which maps R_2 onto R_3. *(1 mark)*

(c) Find the matrix of the transformation which maps R_1 onto R_3. *(2 marks)*


y
5
R_1
O
5
10
x
−5
−10

13.June 2010

6 The matrices **A** and **B** are defined by

$$\mathbf{A} = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

Describe fully the geometrical transformation represented by each of the following matrices:

(a) **A**; *(2 marks)*

(b) **B**; *(2 marks)*

(c) $\mathbf{A}^2$; *(2 marks)*

(d) $\mathbf{B}^2$; *(2 marks)*

(e) **AB**. *(3 marks)*

14.Jan 2011

3 (a) Write down the 2×2 matrix corresponding to each of the following transformations:

(i) a rotation about the origin through 90° clockwise; *(1 mark)*

(ii) a rotation about the origin through 180°. *(1 mark)*

(b) The matrices **A** and **B** are defined by

$$\mathbf{A} = \begin{bmatrix} 2 & 4 \\ -1 & -3 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} -2 & 1 \\ -4 & 3 \end{bmatrix}$$

(i) Calculate the matrix **AB**. *(2 marks)*

(ii) Show that $(\mathbf{A} + \mathbf{B})^2 = k\mathbf{I}$, where **I** is the identity matrix, for some integer k. *(3 marks)*

(c) Describe the single geometrical transformation, or combination of two geometrical transformations, represented by each of the following matrices:

(i) $\mathbf{A} + \mathbf{B}$; *(2 marks)*

(ii) $(\mathbf{A} + \mathbf{B})^2$; *(2 marks)*

(iii) $(\mathbf{A} + \mathbf{B})^4$. *(2 marks)*

15. June 2011

7 The matrix **A** is defined by

$$\mathbf{A} = \begin{bmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{bmatrix}$$

(a) (i) Calculate the matrix $\mathbf{A}^2$. *(2 marks)*

(ii) Show that $\mathbf{A}^3 = k\mathbf{I}$, where k is an integer and $\mathbf{I}$ is the 2×2 identity matrix. *(2 marks)*

(b) Describe the single geometrical transformation, or combination of two geometrical transformations, corresponding to each of the matrices:

(i) $\mathbf{A}^3$; *(2 marks)*

(ii) $\mathbf{A}$. *(3 marks)*

16. Jan 2012

8 The diagram below shows a rectangle R_1 which has vertices (0, 0), (3, 0), (3, 2) and (0, 2).

(a) On the diagram, draw:

(i) the image R_2 of R_1 under a rotation through 90° clockwise about the origin; *(1 mark)*

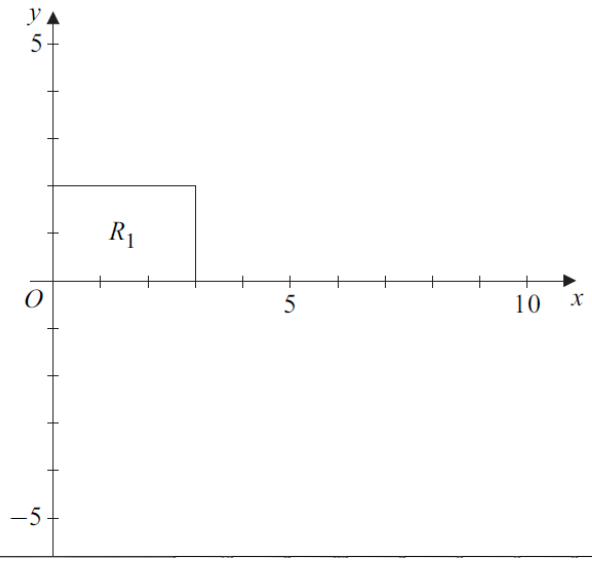
(ii) the image R_3 of R_2 under the transformation which has matrix

$$\begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$$ *(3 marks)*

(b) Find the matrix of:

(i) the rotation which maps R_1 onto R_2; *(1 mark)*

(ii) the combined transformation which maps R_1 onto R_3. *(3 marks)*

QUESTION PART REFERENCE	
(a)	

17. June 2012

6 (a) Using surd forms, find the matrix of a rotation about the origin through 135° anticlockwise. *(2 marks)*

(b) The matrix **M** is defined by $\mathbf{M} = \begin{bmatrix} -1 & -1 \\ 1 & -1 \end{bmatrix}$.

(i) Given that **M** represents an enlargement followed by a rotation, find the scale factor of the enlargement and the angle of the rotation. *(3 marks)*

(ii) The matrix $\mathbf{M}^2$ also represents an enlargement followed by a rotation. State the scale factor of the enlargement and the angle of the rotation. *(2 marks)*

(iii) Show that $\mathbf{M}^4 = k\mathbf{I}$, where k is an integer and **I** is the 2×2 identity matrix. *(2 marks)*

(iv) Deduce that $\mathbf{M}^{2012} = -2^n\mathbf{I}$ for some positive integer n. *(2 marks)*

18. Jan 2013

6 (a) The matrix **X** is defined by $\begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$.

(i) Given that $\mathbf{X}^2 = \begin{bmatrix} m & 2 \\ 3 & 6 \end{bmatrix}$, find the value of m. *(1 mark)*

(ii) Show that $\mathbf{X}^3 - 7\mathbf{X} = n\mathbf{I}$, where n is an integer and **I** is the 2×2 identity matrix. *(4 marks)*

(b) It is given that $\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$.

(i) Describe the geometrical transformation represented by $\mathbf{A}$. *(1 mark)*

(ii) The matrix $\mathbf{B}$ represents an anticlockwise rotation through 45° about the origin.

Show that $\mathbf{B} = k\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$, where k is a surd. *(2 marks)*

(iii) Find the image of the point $P(-1, 2)$ under an anticlockwise rotation through 45° about the origin, followed by the transformation represented by $\mathbf{A}$. *(4 marks)*

Answers

1. Jan 2005

5(a)	Transformation is a reflection in $y = x$	B2	2	
(b)	Matrix is $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$	M1 A2,1	3	M1 for matrix for a rotation; A1 for correct trig expressions
(c)	Attempt to multiply the matrices ... in the correct order	M1 m1		
	Matrix is $\begin{bmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$	A1√	3	Wrong answer to (b)
	Total		**8**	

2. June 2005

7(a)(i)	$D(4, 0)$ $E(8, -4)$, $F(10, -2)$	M1A1 A1A1	4	M1 if at least one point correct
(ii)	Correct sketch	m1A1F	2	Ft one error
(b)(i)	Scale factor is $2\sqrt{2}$	M1A1	2	NMS 2/2; 1/2 for AWRT 2.8
(ii)	Angle 45°	M1A1	2	NMS 2/2; condone 315°;1/2 for AWRT 44-46° OE
	Total		**10**	

3. Jan 2006

7(a)(i)	Reflection ... ... in $y = -x$	M1 A1	2	OE
(ii)	$\mathbf{A}^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	M1A1	2	M1A0 for three correct entries
(iii)	$\mathbf{A}^2 = \mathbf{I}$ or geometrical reasoning	E1	1	
(b)(i)	$\mathbf{B}^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$	M1A1		M1A0 for three correct entries
	$\mathbf{B}^2 - \mathbf{A}^2 = \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}$	A1√	3	ft errors, dependent on both M marks
(ii)	$(\mathbf{B} + \mathbf{A})(\mathbf{B} - \mathbf{A}) = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$	B1		
	$\ldots = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$	M1 A1√	3	ft one error; M1A0 for three correct (ft) entries
	Total		**11**	

4. Jan 2006

8(a)	Good attempt at sketch	M1		
	Correct at origin	A1	2	
(b)(i)	y replaced by $y - 2$	B1		
	Equation is $(y-2)^2 = 12x$	B1√	2	ft $y + 2$ for $y - 2$
(ii)	Equation is $x^2 = 12y$	B1	1	
(c)(i)	$(x+c)^2 = x^2 + 2cx + c^2$	B1		
	$\ldots = 12x$	M1		
	Hence result	A1	3	convincingly shown (AG)
(ii)	Tangent if $(2c-12)^2 - 4c^2 = 0$	M1		
	ie if $-48c + 144 = 0$ so $c = 3$	A1	2	
(iii)	$x^2 - 6x + 9 = 0$	M1		
	$x = 3, y = 6$	A1	2	
(iv)	$c = 4 \Rightarrow$ discriminant $= -48 < 0$	M1A1		OE
	So line does not intersect curve	A1	3	
	Total		**15**	

5. June 2006

5(a)(i)	$\mathbf{M}^2 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$	M1 A2,1	3	M1 if 2 entries correct M1A1 if 3 entries correct
(ii)	$\mathbf{M}^4 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$	B1√	1	ft error in $\mathbf{M}^2$ provided no surds in $\mathbf{M}^2$
(b)	Rotation (about the origin)	M1		
	... through 45° clockwise	A1	2	
(c)	Awareness of $\mathbf{M}^8 = \mathbf{I}$	M1		OE; NMS 2/3
	$\mathbf{M}^{2006} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$	m1 A1√	3	complete valid method ft error in $\mathbf{M}^2$ as above
	Total		**9**	

6. Jan 2007

2(a)(i)	$\mathbf{A} + \mathbf{B} = \begin{bmatrix} \sqrt{3} & 0 \\ 1 & 0 \end{bmatrix}$	M1A1	2	M1A0 if 3 entries correct; Condone $\frac{2\sqrt{3}}{2}$ for $\sqrt{3}$
(ii)	$\mathbf{BA} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$	B3,2,1	3	Deduct one for each error; **SC** B2,1 for **AB**
(b)(i)	Rotation 30° anticlockwise (abt O)	M1A1	2	M1 for rotation
(ii)	Reflection in $y = (\tan 15°)x$	M1A1	2	M1 for reflection
(iii)	Reflection in x-axis	B2F	2	1/2 for reflection in y-axis ft (M1A1) only for the **SC**
	Alt: Answer to (i) followed by answer to (ii)	M1A1F	(2)	M1A0 if in wrong order or if order not made clear
	Total		**11**	

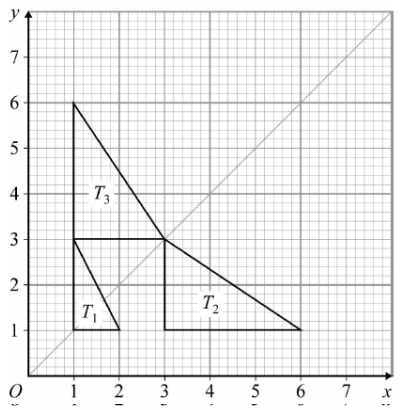
7. June 2007

1(a)	$\mathbf{M} = \begin{bmatrix} 0 & -3 \\ -3 & 0 \end{bmatrix}$	B2,1	2	B1 if subtracted the wrong way round
(b)	$p = 3$	B1F		ft after B1 in (a)
	L is $y = -x$	B1	2	Allow $p = -3$, L is $y = x$
(c)	$\mathbf{M}^2 = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$	B1F		Or by geometrical reasoning; ft as before
	$\ldots = 9\mathbf{I}$	B1F	2	ft as before
	Total		**6**	

8. Jan 2008

6(a)(i)	$\mathbf{M}^2 = \begin{bmatrix} 12 & 0 \\ 0 & 12 \end{bmatrix}$	M1A1		M1 if zeroes appear in the right places
	$= 12\mathbf{I}$	A1F	3	ft provided of right form
(ii)	$q \cos 60° = \frac{1}{2} q = \sqrt{3} \Rightarrow q = 2\sqrt{3}$	M1A1		OE **SC** $q = 2\sqrt{3}$ NMS 1/3
	Other entries verified	E1	3	surd for sin 60° needed
(b)(i)	SF $= q = 2\sqrt{3}$	B1F	1	ft wrong value for q
(ii)	Equation is $y = x \tan 30°$	B1	1	
(c)	$\mathbf{M}^4 = 144\mathbf{I}$	B1F		PI; ft wrong value in (a)(i)
	$\mathbf{M}^4$ gives enlargement SF 144	B1F	2	ft if c's $\mathbf{M}^4 = k\mathbf{I}$
	Total		**10**	

9. June 2008

8(a)	Matrix is $\begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$	M1A1	2	M1 if zeros in correct positions; allow NMS
(b)				
	Third triangle shown correctly	M1A1	2	M1A0 if one point wrong

10.Jan 2009

5(a)(i)	$\mathbf{A}+\mathbf{B}=\begin{bmatrix}0 & 2k\\ 2k & 0\end{bmatrix}$	B1	1	
(ii)	$\mathbf{A}^2=\begin{bmatrix}2k^2 & 0\\ 0 & 2k^2\end{bmatrix}$	B2,1	2	B1 if three entries correct
(b)	$(\mathbf{A}+\mathbf{B})^2 = =\begin{bmatrix}4k^2 & 0\\ 0 & 4k^2\end{bmatrix}$	B2,1		B1 if three entries correct
	$\mathbf{B}^2=\mathbf{A}^2$, hence result	B1B1	4	
(c)(i)	$\mathbf{A}^2$ is an enlargement (centre O) with SF 2	M1 A1	2	Condone $2k^2$
(ii)	Scale factor is now $\sqrt{2}$ Mirror line is $y=x\tan 22\frac{1}{2}°$	B1 M1A1	3	Condone $\sqrt{2}k$
	Total		**12**	

11.June 2009

7(a)(i)	Matrix is $\begin{bmatrix}\sqrt{3}/2 & -1/2\\ 1/2 & \sqrt{3}/2\end{bmatrix}$	M1A1	2	M1 for $\begin{bmatrix}\cos 30° & \sin 30°\\ -\sin 30° & \cos 30°\end{bmatrix}$ (PI)
(ii)	Matrix is $\begin{bmatrix}1/2 & \sqrt{3}/2\\ \sqrt{3}/2 & -1/2\end{bmatrix}$	M1A1	2	M1 for $\begin{bmatrix}\cos 60° & \sin 60°\\ \sin 60° & -\cos 60°\end{bmatrix}$ (PI)
(b)	SF 2, line $y=\frac{1}{\sqrt{3}}x$	B1B1	2	OE
(c)	Attempt at **BA** or **AB**	M1		
	$\mathbf{BA}=\begin{bmatrix}0 & 4\\ 4 & 0\end{bmatrix}$	m1A1		m1 if zeros in correct positions
	Enlargement SF 4	B1F		ft use of **AB** (answer still 4) or after $\mathbf{BA}=\begin{bmatrix}0 & k\\ k & 0\end{bmatrix}$
	... and reflection in line $y=x$	B1F	5	ft only from $\mathbf{BA}=\begin{bmatrix}0 & k\\ k & 0\end{bmatrix}$
	Total		**11**	

12.Jan 2010

6(a)(i)	Coords (3, 2), (9, 2), (9, 4), (3, 4)	M1A1	2	M1 for multn of x by 3 or y by 2 (PI)
(ii)	R_2 shown correctly on insert	B1	1	
(b)(i)	R_3 shown correctly on insert	B2,1F	2	B1 for rectangle with 2 vertices correct; ft if c's R_2 is a rectangle in 1st quad
(ii)	Matrix of rotation is $\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}$	B1	1	

(c)	Multiplication of matrices	M1		(either way) or other complete method
	Required matrix is $\begin{bmatrix} 0 & 2 \\ -3 & 0 \end{bmatrix}$	A1	2	
	Total		**8**	

13.June 2010

6(a)	Rotation 45° (anticlockwise)(about O)	M1A1	2	M1 for 'rotation'
(b)	Reflection in $y = x \tan 22.5°$	M1A1	2	M1 for 'reflection'
(c)	Rotation 90° (anticlockwise)(about O)	M1A1F	2	M1 for 'rotation' or correct matrix; ft wrong angle in (a)
(d)	Identity transformation	B2,1F	2	ft wrong mirror line in (b); B1 for $\mathbf{B}^2 = \mathbf{I}$
(e)	$\mathbf{AB} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	M1A1		allow M1 if two entries correct
	Reflection in $y = x$	A1	3	
	Total		**11**	

14.Jan 2011

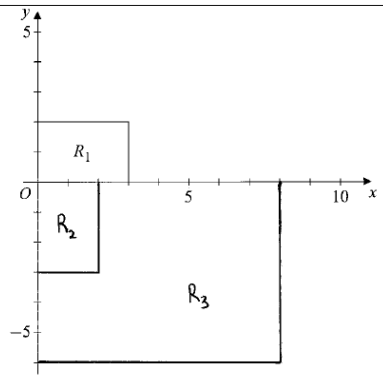
3(a)(i)	$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$	B1	1	
(ii)	$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$	B1	1	
(b)(i)	$\mathbf{AB} = \begin{bmatrix} -20 & 14 \\ 14 & -10 \end{bmatrix}$	M1A1	2	M1A0 if 3 entries correct
(ii)	$\mathbf{A} + \mathbf{B} = \begin{bmatrix} 0 & 5 \\ -5 & 0 \end{bmatrix}$	B1		
	$(\mathbf{A} + \mathbf{B})^2 = \begin{bmatrix} -25 & 0 \\ 0 & -25 \end{bmatrix}$	B1		
	$\ldots = -25\mathbf{I}$	B1F	3	ft if c's $(\mathbf{A} + \mathbf{B})^2$ is of the form $k\mathbf{I}$
(c)(i)	Rot'n 90° clockwise, enlargem't SF 5	B2, 1	2	OE
(ii)	Rotation 180°, enlargement SF 25	B2, 1F	2	Accept 'enlargement SF −25'; ft wrong value of k
(iii)	Enlargement SF 625	B2, 1F	2	B1 for pure enlargement; ft ditto
	Total		**13**	

15.June 2011

7(a)(i)	$\mathbf{A}^2 = \begin{bmatrix} -2 & 2\sqrt{3} \\ -2\sqrt{3} & -2 \end{bmatrix}$	M1A1	2	M1 if at least two entries correct
(ii)	$\mathbf{A}^3 = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$	M1		if at least two entries correct
	……..= $8\mathbf{I}$	A1	2	
(b)(i)	$\mathbf{A}^3$ gives enlargement with SF 8 (centre the origin)	M1A1F	2	M1 for enlargement (only); ft wrong value for k

(ii)	Enlargement and rotation Enlargement scale factor 2 Rotation through 120° (antic'wise)	M1 A1 A1	 3	Some detail needed
	Total		**9**	

16.Jan 2012

8(a)(i)		B1	1	Rectangle with vertices (0, 0), (0, −3), (2, −3), (2, 0)
(ii)		M1		Rectangle with vertices **either** whose x-coords are c's (a)(i) x-coord vertices multiplied by 4 **or** whose y-coords are c's (a)(i) y-coord vertices multiplied by 2
		A2,1	3	A2 if rectangle with vertices (0, 0), (0, −6), (8, −6), (8, 0) (A1 if either the four x-coords are correct or the four y-coords are correct)
(b)(i)	Matrix is $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$	B1	1	
(ii)	$\begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} =$	M1		Attempt to multiply $\begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$ with c's (b)(i) matrix in **either** order.
		m1		Multiplication in **correct** order with at least two of the four ft multiplications carried out correctly.
	$\begin{bmatrix} 0 & 4 \\ -2 & 0 \end{bmatrix}$	A1	3	For $\begin{bmatrix} 0 & 4 \\ -2 & 0 \end{bmatrix}$ NMS $\begin{bmatrix} 0 & 4 \\ -2 & 0 \end{bmatrix}$ scores B3 $\begin{bmatrix} 0 & 2 \\ -4 & 0 \end{bmatrix}$ scores B1
	Total		**8**	

17.June 2012

6(a)	$\begin{bmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$	M1 A1	2	If A1 not scored, award M1A0 for all correct entries expressed in trig form eg $\begin{bmatrix} \cos 135 & -\sin 135 \\ \sin 135 & \cos 135 \end{bmatrix}$
(b)(i)	$\mathbf{M}=\begin{bmatrix} -1 & -1 \\ 1 & -1 \end{bmatrix} = \sqrt{2} \times \begin{bmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$	M1		Or better PI by cand. having both a correct scale

	$\lfloor \sqrt{2} \quad \sqrt{2} \rfloor$ $= \left(= \begin{bmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{2} \end{bmatrix} \right)$	M1		PI by cand. having both a correct scale factor of enlargement and a correct corresponding angle of rotation
	Scale factor of enlargement is $\sqrt{2}$	A1		SF = $\sqrt{2}$ OE surd form
	Angle of rotation is 135 (degrees anticlockwise)	A1	3	Angle = 135 OE eg −225 If M0 give B1 for SF= $\sqrt{2}$ OE surd and B1 for angle = 135 OE
(b)(ii)	For $\mathbf{M}^2$, SF of enlargement = 2	B1F		OE If incorrect, ft on [c's SF in (b)(i)]^2
	Angle of rotation is 270 (degrees anticlockwise)	B1F	2	OE, eg − 90(degrees), eg 90 (degrees) clockwise If incorrect, ft on 2×c's angle in (b)(i) (neither B1F B1 nor B1 B1F is possible)
(iii)	$\mathbf{M}^2 = \begin{bmatrix} -1 & -1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} -1 & -1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$ $\mathbf{M}^4 = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix}$	M1		For complete method (matrix calculation or geometrical reasoning) Matrix for $\mathbf{M}^2$ could be seen earlier (M0 if >1 independent error in matrix multiplication) Geometrically SF = 4, rotation angle= 540 OE scores M1 and completion scores A1
	$\mathbf{M}^4 = -4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = -4\mathbf{I}$	A1	2	Either of these two forms convincingly shown
(iv)	$\mathbf{M}^{2012} = (\mathbf{M}^4)^{503} = (-4\mathbf{I})^{503} =$ $-(2^2)^{503}\mathbf{I} = -2^{1006}\mathbf{I}$	E1		OE Fully explained, algebraically from $(-4\mathbf{I})^{503}$, or geometrically
	$\mathbf{M}^{2012} = -2^{1006}\mathbf{I}$	B1	2	$M^{2012} = -2^{1006}\mathbf{I}$ (n = 1006) (B0 if FIW)
	(Geometrically: $\mathbf{M}^{2012}$ represents an enlargement with SF 2^{1006} followed by a rotation of angle 2012×135° ie 754.5 revolutions, being equivalent to rotation of 180° ie matrix is $-\mathbf{I}$ so $\mathbf{M}^{2012} = -2^{1006}\mathbf{I}$)			
	Total		**11**	

18.Jan 2013

6(a)(i)	$\mathbf{X}^2 = \begin{bmatrix} 7 & 2 \\ 3 & 6 \end{bmatrix}$; (m =) 7	B1	1	(m =)7 or 7 as top left element of $\mathbf{X}^2$
(ii)	$\mathbf{X}^3 = \begin{bmatrix} 13 & 14 \\ 21 & 6 \end{bmatrix}$;	M1		At least 2 elements correct
	$7\mathbf{X} = \begin{bmatrix} 7 & 14 \\ 21 & 0 \end{bmatrix}$	B1		PI
	$\mathbf{X}^3 - 7\mathbf{X} = \begin{bmatrix} 13-7 & 14-14 \\ 21-21 & 6-0 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$	A1F		Ft on c's m value
	$= 6\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 6\,\mathbf{I}$	A1	4	CSO Accept either form but at least one must be shown explicitly
(b)(i)	Reflection in the x-axis	B1	1	OE

(ii)	$\mathbf{B} = \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ \\ \sin 45^\circ & \cos 45^\circ \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$	M1		Either OE. For M mark, accept dec. equiv. (at least 3sf) for $\frac{1}{\sqrt{2}}$
	$= \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$	A1	2	NMS SC1 for $k = \frac{1}{\sqrt{2}}$ or better.
(iii)	$\mathbf{AB}\begin{bmatrix} -1 \\ 2 \end{bmatrix} = k\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} -1 \\ 2 \end{bmatrix}$	M1		Attempt to find $\mathbf{AB}\begin{bmatrix} -1 \\ 2 \end{bmatrix}$
	$= k\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} -3 \\ 1 \end{bmatrix}$ $\left\{\text{or } k\begin{bmatrix} 1 & -1 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} -1 \\ 2 \end{bmatrix}\right\}$	A1		Either $\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$ or $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & -1 \end{bmatrix}$
	$= k\begin{bmatrix} -3 \\ -1 \end{bmatrix}$	m1		Completing the matrix mult. to reach a 2×1 matrix
	(Image of P is the point) $\left(-\frac{3}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$	A1	4	**CSO** **SC** Wrong order, works with $\mathbf{BA}\begin{bmatrix} -1 \\ 2 \end{bmatrix}$, mark out of a max of M1A0 m1A0
	Total		**12**	