Practice examination questions for series

1. June 2005

3 (a) Use the formulae

$$\sum_{r=1}^{n} r^2 = \tfrac{1}{6}n(n+1)(2n+1)$$

and

$$\sum_{r=1}^{n} r^3 = \tfrac{1}{4}n^2(n+1)^2$$

to show that

$$\sum_{r=1}^{n} r^2(r-1) = \tfrac{1}{12}n(n^2-1)(3n+2)$$ *(4 marks)*

(b) Use the result from part (a) to find the value of

$$\sum_{r=4}^{11} r^2(r-1)$$ *(3 marks)*

2. June 2006

3 Show that

$$\sum_{r=1}^{n} (r^2 - r) = kn(n+1)(n-1)$$

where k is a rational number. *(4 marks)*

3. Jan 2007

6 (a) (i) Expand $(2r-1)^2$. *(1 mark)*

(ii) Hence show that

$$\sum_{r=1}^{n} (2r-1)^2 = \tfrac{1}{3}n(4n^2-1)$$ *(5 marks)*

(b) Hence find the sum of the squares of the odd numbers between 100 and 200. *(4 marks)*

4. Jan 2008

4 (a) Find

$$\sum_{r=1}^{n} (r^3 - 6r)$$

expressing your answer in the form

$$kn(n+1)(n+p)(n+q)$$

where k is a fraction and p and q are integers. *(5 marks)*

(b) It is given that

$$S = \sum_{r=1}^{1000} (r^3 - 6r)$$

Without calculating the value of S, show that S is a multiple of 2008. *(2 marks)*

5. Jan 2009

4 It is given that

$$S_n = \sum_{r=1}^{n}(3r^2 - 3r + 1)$$

(a) Use the formulae for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r$ to show that $S_n = n^3$. *(5 marks)*

(b) Hence show that $\sum_{r=n+1}^{2n} (3r^2 - 3r + 1) = kn^3$ for some integer k. *(2 marks)*

6. Jan 2010

8 (a) Show that

$$\sum_{r=1}^{n} r^3 + \sum_{r=1}^{n} r$$

can be expressed in the form

$$kn(n+1)(an^2 + bn + c)$$

where k is a rational number and a, b and c are integers. *(4 marks)*

(b) Show that there is exactly one positive integer n for which

$$\sum_{r=1}^{n} r^3 + \sum_{r=1}^{n} r = 8\sum_{r=1}^{n} r^2$$ *(5 marks)*

7.Jan 2011

8 (a) The equation

$$x^3 + 2x^2 + x - 100\,000 = 0$$

has one real root. Taking $x_1 = 50$ as a first approximation to this root, use the Newton-Raphson method to find a second approximation, x_2, to the root. *(3 marks)*

(b) (i) Given that $S_n = \sum_{r=1}^{n} r(3r+1)$, use the formulae for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r$ to show that

$$S_n = n(n+1)^2$$ *(5 marks)*

(ii) The lowest integer n for which $S_n > 100\,000$ is denoted by N.

Show that

$$N^3 + 2N^2 + N - 100\,000 > 0$$ *(1 mark)*

(c) Find the value of N, justifying your answer. *(3 marks)*

8. Jan 2012

4 (a) Use the formulae for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r^3$ to show that

$$\sum_{r=1}^{n} r^2(4r-3) = kn(n+1)(2n^2-1)$$

where k is a constant. *(5 marks)*

(b) Hence evaluate

$$\sum_{r=20}^{40} r^2(4r-3)$$ *(2 marks)*

9. Jan 2013

8 (a) Show that

$$\sum_{r=1}^{n} 2r(2r^2-3r-1) = n(n+p)(n+q)^2$$

where p and q are integers to be found. *(6 marks)*

(b) Hence find the value of

$$\sum_{r=11}^{20} 2r(2r^2-3r-1)$$ *(2 marks)*

Answer

1. June 2005

3(a)	$\Sigma r^2(r-1) = \Sigma r^3 - \Sigma r^2$ Good progress with expansion Factors n and $n+1$ found $\ldots = \frac{1}{12}n(n^2-1)(3n+2)$	M1 m1 A1 A1	 4	With attempt to use the given formulae or use of common factors Allow verification here Convincingly shown (AG)
(b)	Use of f(11) – f(3) in above f(11) = 3850 f(3) = 22 (so answer is 3828)	M1 A1 A1	 3	M1 for f(11) – f(4) PI by correct answer ditto
	Total		**7**	

2. June 2006

3	$\Sigma(r^2-r) = \Sigma r^2 - \Sigma r$ At least one linear factor found $\Sigma(r^2-r) = \frac{1}{6}n(n+1)(2n+1-3)$ $\ldots = \frac{1}{3}n(n+1)(n-1)$	M1 m1 m1 A1	 4	 OE
	Total		**4**	

3. Jan 2007

6(a)(i)	$(2r-1)^2 = 4r^2 - 4r + 1$	B1	1	
(ii)	$\sum(2r-1)^2 = 4\sum r^2 - 4\sum r + \sum 1$ $\ldots = \frac{4}{3}n^3 - \frac{4}{3}n + \sum 1$ $\sum 1 = n$ Result convincingly shown	M1 m1A1 B1 A1	 5	 **AG**
(b)	Sum = f (100) – f (50) $\ldots = 1\,166\,650$	M1A1 A2	 4	M1 for 100 ± 1 and 50 ± 1 **SC** f(100) – f(51) = 1 156 449: 3/4
	Total		**10**	

4. Jan 2008

4(a)	Use of formula for $\sum r^3$ or $\sum r$ n is a factor of the expression So is $(n+1)$ $S_n = \frac{1}{4}n(n+1)(n^2+n-12)$ $\ldots = \frac{1}{4}n(n+1)(n+4)(n-3)$	M1 m1 m1 A1 A1F	 5	 clearly shown ditto ft wrong value for k
(b)	$n = 1000$ substituted into expression Conclusion convincingly shown Need $\frac{1000}{4}$ is even, hence conclusion	m1 A1	 2	The factor 1004, or 1000 + 4, seen not '2008 × 124749625' OE
	Total		**7**	

5. Jan 2009

4(a)	$S_n = 3\Sigma r^2 - 3\Sigma r + \Sigma 1$	M1		
	Correct expressions substituted	m1		At least for first two terms
	Correct expansions	A1		
	$\Sigma 1 = n$	B1		
	Answer convincingly obtained	A1	5	AG
(b)	$S_{2n} - S_n$ attempted	M1		Condone $S_{2n} - S_{n+1}$ here
	Answer $7n^3$	A1	2	
	Total		**7**	

6. Jan 2010

8(a)	$\Sigma r^3 + \Sigma r = \frac{1}{4}n^2(n+1)^2 + \frac{1}{2}n(n+1)$	M1		at least one term correct
	Factor n clearly shown	m1		or $n + 1$ clearly shown to be a factor
	$\ldots = \frac{1}{4}n(n+1)(n^2+n+2)$	A1A1	4	OE; A1 for $\frac{1}{4}$, A1 for quadratic
(b)	Valid equation formed	M1		
	Factors n, $n + 1$ removed	m1		
	$3n^2 - 29n - 10 = 0$	A1		OE
	Valid factorisation or solution	m1		of the correct quadratic
	$n = 10$ is the only pos int solution	A1	5	SC 1/2 for $n = 10$ after correct quad
	Total		**9**	

7.Jan 2011

8(a)	$x_2 = 50 - \dfrac{50^3 + 2(50^2) + 50 - 100\,000}{3(50^2) + 4(50) + 1}$	B1 B1		For numerator (PI by value 30050) For denominator (PI by value 7701)
	$x_2 \approx 46.1$	B1	3	Allow AWRT 46.1
8(b)(i)	$\Sigma r(3r+1) = 3\Sigma r^2 + \Sigma r$	M1		
	$\ldots = 3\left(\frac{1}{6}n\right)(n+1)(2n+1) + \frac{1}{2}n(n+1)$	m1		correct formulae substituted
	$\ldots = \frac{1}{2}n(n+1)(2n+1+1)$	m1m1		m1 for each factor (n and $n + 1$)
	$\ldots = n(n+1)^2$ convincingly shown	A1	5	AG
(ii)	Correct expansion of $n(n+1)^2$	B1	1	and conclusion drawn (AG)
(c)	Attempt at value of S_{46}	M1		
	Attempt at value of S_{45}	m1		
	$S_{45} < 100000 < S_{46}$, so $N = 46$	A1	3	
	Alternative method			
	Root of equation in (a) is 45.8			Allow AWRT 45.7 or 45.8
	So lowest integer value is 46	(B3)		
	Total		**12**	

8. Jan 2012

4(a)	$\Sigma r^2(4r-3) = 4\Sigma r^3 - 3\Sigma r^2 \ldots$	M1		Splitting up the sum into two separate sums. PI by next line.
	$= 4\left(\frac{1}{4}\right)n^2(n+1)^2 - 3\left(\frac{1}{6}\right)n(n+1)(2n+1)$	m1		Substitution of the two summations from FB
	$= n(n+1)\left[n(n+1) - \frac{1}{2}(2n+1)\right]$	m1 A1		Taking out common factors n and $n + 1$. Remaining expression eg our […] in ACF not just simplified to AG

	Sum $= \frac{1}{2}n(n+1)(2n^2-1)$	A1	5	Be convinced as form of answer is given, penalise any jumps or backward steps
(b)	$\sum_{r=20}^{40} r^2(4r-3)$ $= \sum_{r=1}^{40} r^2(4r-3) - \sum_{r=1}^{19} r^2(4r-3)$	M1		Attempt to take S(19) from S(40) using part (a)
	$= 20(41)(3199) - 9.5(20)(721)$ $= 2623180 - 136990$			
	$\sum_{r=20}^{40} r^2(4r-3) = 2486190$	A1	2	2486190 ; Since 'Hence' NMS 0/2. **SC** $\sum_{r=1}^{40}\ldots\ldots - \sum_{r=1}^{20}\ldots\ldots$ clearly attempted and evaluated to 2455390 scores B1
	Total		**7**	

9. Jan 2013

8(a)	$\sum_{r=1}^{n} 2r(2r^2-3r-1) = \sum_{r=1}^{n} 4r^3 - \sum_{r=1}^{n} 6r^2 - \sum_{r=1}^{n} 2r$ $= 4\sum_{r=1}^{n} r^3 - 6\sum_{r=1}^{n} r^2 - 2\sum_{r=1}^{n} r$	M1		Splitting up the sum into three separate sums. PI by m1 line below.
	$= 4\times\frac{1}{4}n^2(n+1)^2 - 6\times\frac{1}{6}n(n+1)(2n+1) - 2\times\frac{1}{2}n(n+1)$	m1		Substitution of the three summations from FB into $a\sum_{r=1}^{n} r^3 + b\sum_{r=1}^{n} r^2 + c\sum_{r=1}^{n} r$
	$= n^2(n+1)^2 - n(n+1)(2n+1) - n(n+1)$	A1		PI by later expressions
	$= n(n+1)[n(n+1) - (2n+1) - 1]$	m1		Taking out factor $n(n+1)$ from correct expressions
	$= n(n+1)[n^2 - n - 2]$	A1		
	$= n(n+1)(n+1)(n-2)$ $(= n(n-2)(n+1)^2$ $(p=-2, q=1))$	A1	6	
(b)	$\sum_{r=11}^{20} 2r(2r^2-3r-1)$ $= \sum_{r=1}^{20} 2r(2r^2-3r-1) - \sum_{r=1}^{10} 2r(2r^2-3r-1)$	M1		$\sum_{r=1}^{20}\ldots - \sum_{r=1}^{10}\ldots$ PI by next line (ft c's p&q)
	$= 20(20+p)(20+q)^2 - 10(10+p)(10+q)^2$			
	$= 20\times18\times21^2 - 10\times8\times11^2 = 158760 - 9680 = 149080$	A1	2	NMS 0/2 A0 if not showing use of fully factorised form.
	Total		**8**	