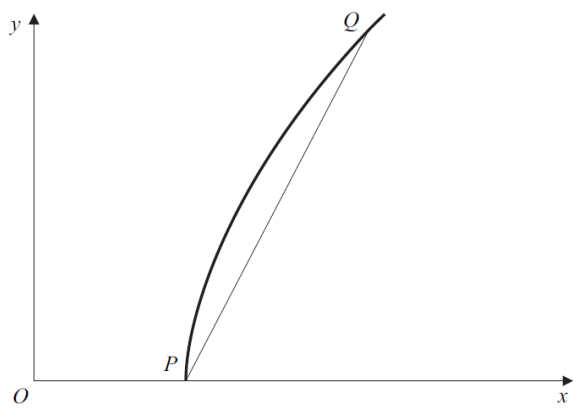
Practice examination questions for parabolas, ellipses and hyperbolas

1. June 2005

8 The diagram shows a part of the curve

$$\frac{x^2}{4} - \frac{y^2}{6} = 1$$

and a chord PQ of the curve, where P lies on the x-axis.



(a) Write down the coordinates of P. *(1 mark)*

(b) The gradient of the chord PQ is 2. Find the coordinates of Q. *(7 marks)*

1. June 2006

7 (a) Describe a geometrical transformation by which the hyperbola

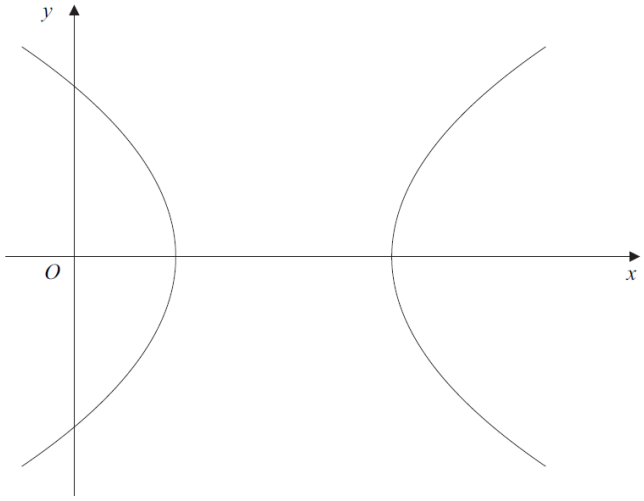
$$x^2 - 4y^2 = 1$$

can be obtained from the hyperbola $x^2 - y^2 = 1$. *(2 marks)*

(b) The diagram shows the hyperbola H with equation

$$x^2 - y^2 - 4x + 3 = 0$$

By completing the square, describe a geometrical transformation by which the hyperbola H can be obtained from the hyperbola $x^2 - y^2 = 1$. *(4 marks)*



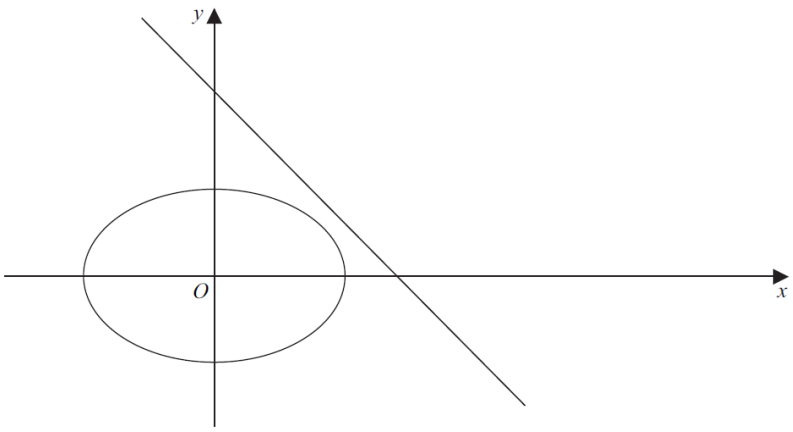
2. June 2007

The diagram shows the curve with equation

$$\frac{x^2}{2} + y^2 = 1$$

and the straight line with equation

$$x + y = 2$$



(a) Write down the exact coordinates of the points where the curve with equation $\frac{x^2}{2} + y^2 = 1$ intersects the coordinate axes. *(2 marks)*

(b) The curve is translated k units in the positive x direction, where k is a constant. Write down, in terms of k, the equation of the curve after this translation. *(2 marks)*

(c) Show that, if the line $x + y = 2$ intersects the **translated** curve, the x-coordinates of the points of intersection must satisfy the equation

$$3x^2 - 2(k+4)x + (k^2 + 6) = 0$$ *(4 marks)*

(d) Hence find the two values of k for which the line $x + y = 2$ is a tangent to the translated curve. Give your answer in the form $p \pm \sqrt{q}$, where p and q are integers. *(4 marks)*

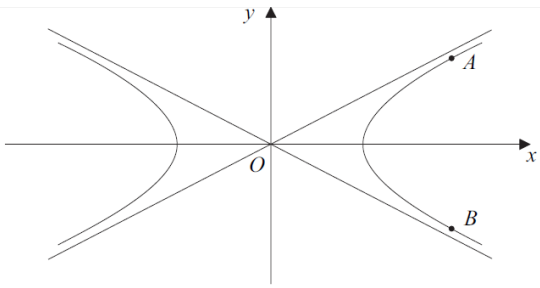
(e) On **Figure 3**, show the translated curves corresponding to these two values of k. *(3 marks)*

3. Jan 2008

5 The diagram shows the hyperbola

$$\frac{x^2}{4} - y^2 = 1$$

and its asymptotes.



(a) Write down the equations of the two asymptotes. *(2 marks)*

(b) The points on the hyperbola for which $x = 4$ are denoted by A and B.

Find, in surd form, the y-coordinates of A and B. *(2 marks)*

(c) The hyperbola and its asymptotes are translated by two units in the positive y direction.

(c) The hyperbola and its asymptotes are translated by two units in the positive y direction.

Write down:

(i) the y-coordinates of the image points of A and B under this translation; *(1 mark)*

(ii) the equations of the hyperbola and the asymptotes after the translation. *(3 marks)*

4. Jan 2008

9 A curve C has equation

$$y = \frac{2}{x(x-4)}$$

(a) Write down the equations of the three asymptotes of C. *(3 marks)*

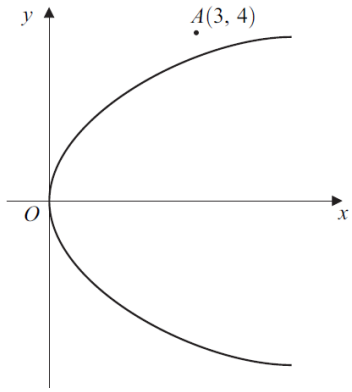
(b) The curve C has one stationary point. By considering an appropriate quadratic equation, find the coordinates of this stationary point.

(No credit will be given for solutions based on differentiation.) *(6 marks)*

(c) Sketch the curve C. *(3 marks)*

5. June 2008

9 The diagram shows the parabola $y^2 = 4x$ and the point A with coordinates $(3, 4)$.



(a) Find an equation of the straight line having gradient m and passing through the point $A(3, 4)$. *(2 marks)*

(b) Show that, if this straight line intersects the parabola, then the y-coordinates of the points of intersection satisfy the equation

$$my^2 - 4y + (16 - 12m) = 0$$ *(3 marks)*

(c) By considering the discriminant of the equation in part (b), find the equations of the two tangents to the parabola which pass through A.

(No credit will be given for solutions based on differentiation.) *(5 marks)*

(d) Find the coordinates of the points at which these tangents touch the parabola. *(4 marks)*

6. Jan 2009

9 A hyperbola H has equation

$$x^2 - \frac{y^2}{2} = 1$$

(a) Find the equations of the two asymptotes of H, giving each answer in the form $y = mx$. *(2 marks)*

(b) Draw a sketch of the two asymptotes of H, using roughly equal scales on the two coordinate axes. Using the same axes, sketch the hyperbola H. *(3 marks)*

(c) (i) Show that, if the line $y = x + c$ intersects H, the x-coordinates of the points of intersection must satisfy the equation

$$x^2 - 2cx - (c^2 + 2) = 0$$ *(4 marks)*

(ii) Hence show that the line $y = x + c$ intersects H in two distinct points, whatever the value of c. *(2 marks)*

(iii) Find, in terms of c, the y-coordinates of these two points. *(3 marks)*

7. June 2009

6 An ellipse E has equation

$$\frac{x^2}{3} + \frac{y^2}{4} = 1$$

(a) Sketch the ellipse E, showing the coordinates of the points of intersection of the ellipse with the coordinate axes. *(3 marks)*

(b) The ellipse E is stretched with scale factor 2 parallel to the y-axis.

Find and simplify the equation of the curve after the stretch. *(3 marks)*

(c) The **original** ellipse, E, is translated by the vector $\begin{bmatrix} a \\ b \end{bmatrix}$. The equation of the translated ellipse is

$$4x^2 + 3y^2 - 8x + 6y = 5$$

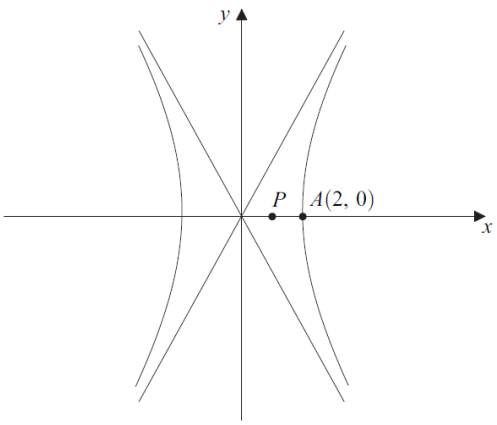
Find the values of a and b. *(5 marks)*

8. Jan 2010

9 The diagram shows the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

and its asymptotes.



The point A on the hyperbola has coordinates $(2, 0)$.

The equations of the asymptotes are $y = 2x$ and $y = -2x$.

(a) Show that $a = 2$ and $b = 4$. *(4 marks)*

(b) The point P has coordinates $(1, 0)$. A straight line passes through P and has gradient m. Show that, if this line intersects the hyperbola, the x-coordinates of the points of intersection satisfy the equation

$$(m^2 - 4)x^2 - 2m^2x + (m^2 + 16) = 0$$ *(4 marks)*

(c) Show that this equation has equal roots if $3m^2 = 16$. *(3 marks)*

(d) There are two tangents to the hyperbola which pass through P. Find the coordinates of the points at which these tangents touch the hyperbola.

(No credit will be given for solutions based on differentiation.) *(5 marks)*

9. June 2010

9 A parabola P has equation $y^2 = x - 2$.

(a) (i) Sketch the parabola P. *(2 marks)*

(ii) On your sketch, draw the two tangents to P which pass through the point $(-2, 0)$. *(2 marks)*

(b) (i) Show that, if the line $y = m(x + 2)$ intersects P, then the x-coordinates of the points of intersection must satisfy the equation

$$m^2x^2 + (4m^2 - 1)x + (4m^2 + 2) = 0$$ *(3 marks)*

(ii) Show that, if this equation has equal roots, then

$$16m^2 = 1$$ *(3 marks)*

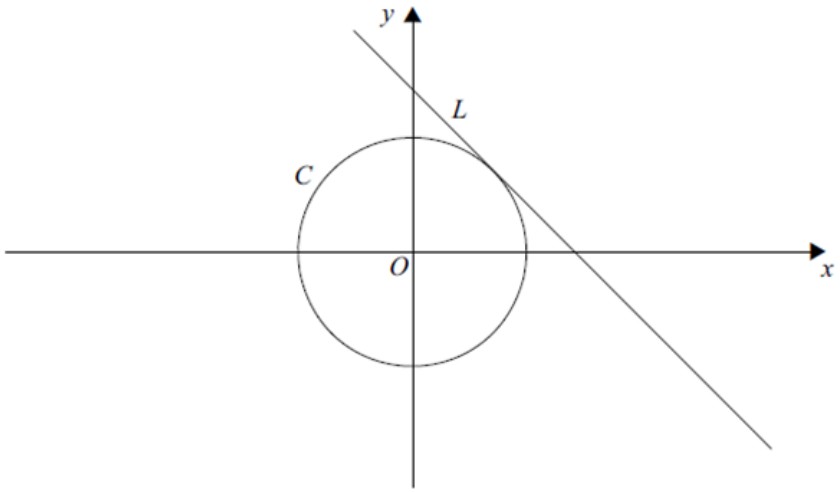
(iii) Hence find the coordinates of the points at which the tangents to P from the point $(-2, 0)$ touch the parabola P. *(3 marks)*

10.Jan 2011

6 The diagram shows a circle C and a line L, which is the tangent to C at the point $(1, 1)$. The equations of C and L are

$$x^2 + y^2 = 2 \quad \text{and} \quad x + y = 2$$

respectively.



The circle C is now transformed by a stretch with scale factor 2 parallel to the x-axis. The image of C under this stretch is an ellipse E.

(a) **On the diagram below**, sketch the ellipse E, indicating the coordinates of the points where it intersects the coordinate axes. *(4 marks)*

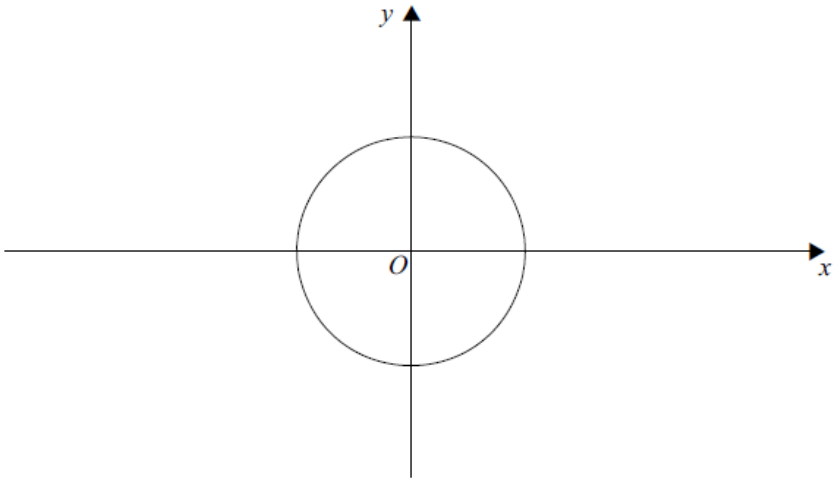
(b) Find equations of:

(i) the ellipse E; *(2 marks)*

(ii) the tangent to E at the point $(2, 1)$. *(2 marks)*

QUESTION PART REFERENCE

(a)



12. June 2011

9 The diagram shows a parabola P which has equation $y = \frac{1}{8}x^2$, and another parabola Q which is the image of P under a reflection in the line $y = x$.

The parabolas P and Q intersect at the origin and again at a point A.

The line L is a tangent to both P and Q.

(a) (i) Find the coordinates of the point A. *(2 marks)*

(ii) Write down an equation for Q. *(1 mark)*

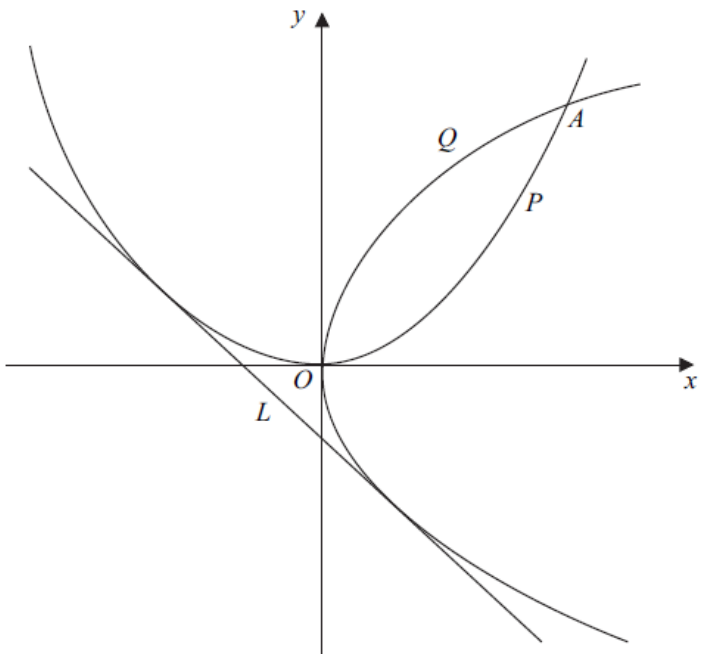
(iii) Give a reason why the gradient of L must be -1. *(1 mark)*

(b) (i) Given that the line $y = -x + c$ intersects the parabola P at two distinct points, show that

$$c > -2$$ *(3 marks)*

(ii) Find the coordinates of the points at which the line L touches the parabolas P and Q.

(No credit will be given for solutions based on differentiation.) *(4 marks)*



13. Jan 2012

7 A hyperbola H has equation

$$\frac{x^2}{9} - y^2 = 1$$

(a) Find the equations of the asymptotes of H. *(1 mark)*

(b) The asymptotes of H are shown in the diagram opposite. On the same diagram, sketch the hyperbola H. Indicate on your sketch the coordinates of the points of intersection of H with the coordinate axes. *(3 marks)*

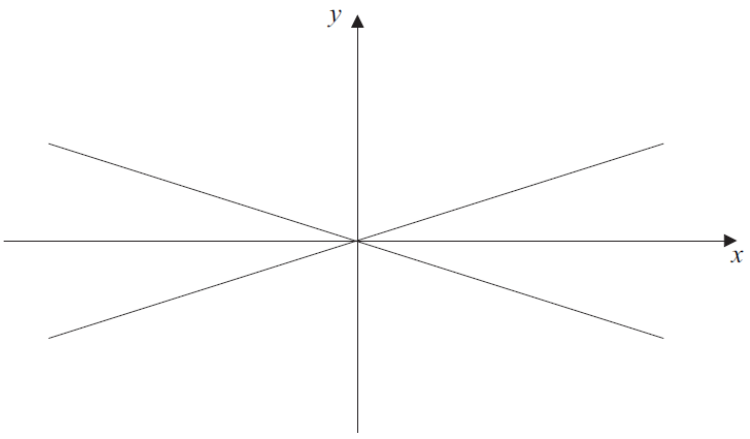
(c) The hyperbola H is now translated by the vector $\begin{bmatrix} -3 \\ 0 \end{bmatrix}$.

(i) Write down the equation of the translated curve. *(2 marks)*

(ii) Calculate the coordinates of the two points of intersection of the translated curve with the line $y = x$. *(4 marks)*

(d) From your answers to part **(c)(ii)**, **deduce** the coordinates of the points of intersection of the original hyperbola H with the line $y = x - 3$. *(2 marks)*

(b)



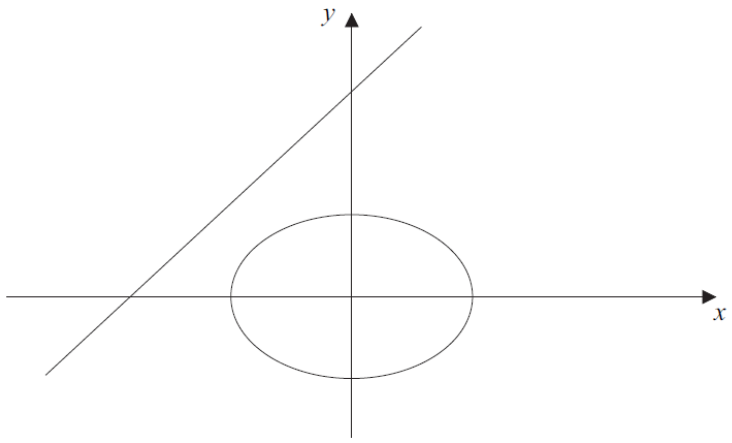
14. June 2012

8 The diagram shows the ellipse E with equation

$$\frac{x^2}{5} + \frac{y^2}{4} = 1$$

and the straight line L with equation

$$y = x + 4$$



(a) Write down the coordinates of the points where the ellipse E intersects the coordinate axes. *(2 marks)*

(b) The ellipse E is translated by the vector $\begin{bmatrix} p \\ 0 \end{bmatrix}$, where p is a constant. Write down the equation of the translated ellipse. *(2 marks)*

(c) Show that, if the translated ellipse intersects the line L, the x-coordinates of the points of intersection must satisfy the equation

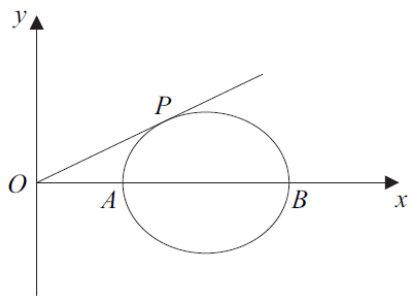
$$9x^2 - (8p - 40)x + (4p^2 + 60) = 0$$ *(3 marks)*

(d) Given that the line L is a tangent to the translated ellipse, find the coordinates of the two possible points of contact.

(No credit will be given for solutions based on differentiation.) *(8 marks)*

15. Jan 2013

9 An ellipse is shown below.



(a) Find the x-coordinates of A and B. *(2 marks)*

(b) The line $y = mx$ $(m > 0)$ is a tangent to the ellipse, with point of contact P.

(i) Show that the x-coordinate of P satisfies the equation

$$(1 + 4m^2)x^2 - 8x + 12 = 0$$ *(3 marks)*

(ii) Hence find the exact value of m. *(4 marks)*

(iii) Find the coordinates of P. *(4 marks)*

Answers

1. June 2005

8(a)	P is (2, 0)	B1	1	
(b)	PQ is $y = 2(x - 2)$	M1A1F		ft wrong value for x_P
	Elimination of y (or of x)	m1A1F		ft numerical error
	$(x - 2)(5x - 22) = 0$	A1		
	Q is (4.4, 4.8)	A1A1	7	
	Total		**8**	

2. June 2006

7(a)	Stretch parallel to y axis ...	B1		
	... scale-factor $\frac{1}{2}$ parallel to y axis	B1	2	
(b)	$(x - 2)^2 - y^2 = 1$	M1A1		
	Translation in x direction ...	A1		
	... 2 units in positive x direction	A1	4	
	Total		**6**	

3. June 2007

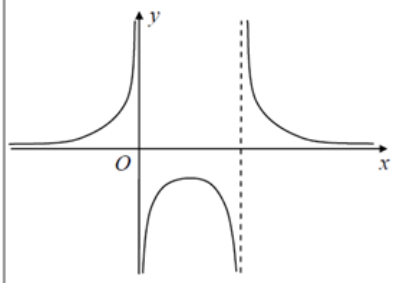
8(a)	$\int\left(x^{\frac{1}{3}} + x^{-\frac{1}{3}}\right) dx = \frac{3}{4}x^{\frac{4}{3}} + \frac{3}{2}x^{\frac{2}{3}} (+ c)$	M1A1		M1 for adding 1 to index at least once
	$\int_0^1 \ldots = \left(\frac{3}{4} + \frac{3}{2}\right) - 0 = \frac{9}{4}$	m1A1	4	Condone no mention of limiting process; m1 if "− 0" stated or implied
(b)	Second term is $x^{-\frac{4}{3}}$	B1		
	Integral of this is $-3x^{-\frac{1}{3}}$	M1A1		M1 for correct index
	$x^{-\frac{1}{3}} \to \infty$ as $x \to 0$, so no value	E1	4	
	Total		**8**	

4. Jan 2008

5(a)	Asymptotes are $y = \pm\frac{1}{2}x$	M1A1	2	OE; M1 for $y = \pm mx$
(b)	$x = 4$ substituted into equation	M1		
	$y^2 = 3$ so $y = \pm\sqrt{3}$	A1	2	Allow NMS
(c)(i)	y-coords are $2 \pm \sqrt{3}$	B1F	1	ft wrong answer to (b)
(ii)	Hyperbola is $\frac{x^2}{4} - (y - 2)^2 = 1$	M1A1		M1A0 if $y + 2$ used
	Asymptotes are $y = 2 \pm \frac{1}{2}x$	B1F	3	ft wrong gradients in (a)
	Total		**8**	

5. Jan 2008

9(a)	Asymptotes $x = 0$, $x = 4$, $y = 0$	B1 × 3	3	
(b)	$y = k \Rightarrow 2 = kx(x - 4)$	M1		
	... $\Rightarrow 0 = kx^2 - 4kx - 2$	A1		
	Discriminant $= (4k)^2 + 8k$	m1		
	At SP $y = -\frac{1}{2}$	A1		not just $k = -\frac{1}{2}$

	$\ldots \Rightarrow 0 = -\frac{1}{2}x^2 + 2x - 2$	m1		
	So $x = 2$	A1	6	
(c)		B1		Curve with three branches approaching vertical asymptotes correctly
		B1		Outer branches correct
		B1	3	Middle branch correct
	Total		**12**	

6. June 2008

9(a)	Equation is $y - 4 = m(x - 3)$	M1A1	2	OE; M1A0 if one small error
(b)	Elimination of x	M1		
	$4y - 16 = m(y^2 - 12)$	A1		OE (no fractions)
	Hence result	A1	3	convincingly shown (AG)
(c)	Discriminant equated to zero	M1		
	$(3m - 1)(m - 1) = 0$	m1A1		OE; m1 for attempt at solving
	Tangents $y = x + 1,\ y = \frac{1}{3}x + 3$	A1A1	5	OE
(d)	$m = 1 \Rightarrow y^2 - 4y + 4 = 0$	M1		OE; $m = 1$ needed for this
	so point of contact is (1, 2)	A1		
	$m = \frac{1}{3} \Rightarrow \frac{1}{3}y^2 - 4y + 12 = 0$	M1		OE; $m = \frac{1}{3}$ needed for this
	so point of contact is (9, 6)	A1	4	
	Total		**14**	

7. Jan 2009

9(a)	Asymptotes are $y = \pm\sqrt{2}x$	M1A1	2	M1A0 if correct but not in required form
(b)	Asymptotes correct on sketch	B1F		With gradients steeper than 1; ft from $y = \pm mx$ with $m > 1$
	Two branches in roughly correct positions	B1		
	Approaching asymptotes correctly	B1	3	Asymptotes $y = \pm mx$ needed here
(c)(i)	Elimination of y	M1		
	Clearing denominator correctly	M1		
	$x^2 - 2cx - (c^2 + 2) = 0$	m1A1	4	Convincingly found (AG)
(ii)	Discriminant $= 8c^2 + 8$	B1		Accept unsimplified
	$\ldots > 0$ for all c, hence result	E1	2	OE
(iii)	Solving gives $x = c \pm \sqrt{2(c^2 + 1)}$	M1A1		
	$y = x + c = 2c \pm \sqrt{2(c^2 + 1)}$	A1	3	Accept $y = c + \frac{2c \pm \sqrt{8c^2 + 8}}{2}$
	Total		**14**	

8. June 2009

6(a)	Ellipse with centre of origin $(\pm\sqrt{3}, 0)$ and (0 ± 2) shown on diagram	B1 B2,1	3	Allow unequal scales on axes Condone AWRT 1.7 for $\sqrt{3}$; B1 for incomplete attempt
(b)	y replaced by $\frac{1}{2}y$ Equation is now $\frac{x^2}{3}+\frac{y^2}{16}=1$	M1A1 A1	3	M1A0 for $2y$ instead of $\frac{1}{2}y$
(c)	Attempt at completing the square $4(x-1)^2+3(y+1)^2 \ldots$ [Alt: replace x by $x-a$ and y by $y-b$ $4x^2-8ax+3y^2-6by \ldots$] $a=1$ and $b=-1$	M1 A1A1 (M1) (m1A1) A1A1	5	M1 if one replacement correct Condone errors in constant terms
	Total		**11**	

9. Jan 2010

9(a)	$x=2, y=0 \Rightarrow \frac{4}{a^2}-0=1$ so $a=2$ Asymps $\Rightarrow \pm\frac{b}{a}=\pm 2$ so $b=2a=4$	E2,1 E2,1	4	E1 for verif'n or incomplete proof ditto
(b)	Line is $y-0=m(x-1)$ Elimination of y $4x^2-m^2(x^2-2x+1)=16$ So $(m^2-4)x^2-2m^2x+(m^2+16)=0$	B1 M1 A1 A1	4	OE OE (no fractions) convincingly shown (AG)
(c)	Discriminant equated to zero $4m^4-4m^4-64m^2+16m^2+256=0$ $-3m^2+16=0$, hence result	M1 A1 A1	3	 OE convincingly shown (AG)
(c)	Discriminant equated to zero $4m^4-4m^4-64m^2+16m^2+256=0$ $-3m^2+16=0$, hence result	M1 A1 A1	3	 OE convincingly shown (AG)
(d)	$m^2=\frac{16}{3} \Rightarrow \frac{4}{3}x^2-\frac{32}{3}x+\frac{64}{3}=0$ $x^2-8x+16=0$, so $x=4$ Method for y-coordinates $y=\pm 4\sqrt{3}$	M1 m1A1 m1 A1	5	 using $m=\pm\frac{4}{\sqrt{3}}$ or from equation of hyperbola; dep't on previous m1
	Total		**16**	

10.June 2010

9(a)(i)	Parabola drawn passing through (2, 0)	M1 A1	2	with x-axis as line of symmetry
(ii)	Two tangents passing through (−2, 0)	B1B1	2	to c's parabola
(b)(i)	Elimination of y Correct expansion of $(x+2)^2$ Result	M1 B1 A1	3	 convincingly shown (AG)

(ii)	Correct discriminant	B1		
	$16m^4 - 8m^2 + 1 = 16m^4 + 8m^2$	M1		OE
	Result	A1	3	convincingly shown (AG)
(iii)	$\frac{1}{16}x^2 - \frac{3}{4}x + \frac{9}{4} = 0$	M1		OE
	$x = 6, y = \pm 2$	A1,A1	3	
	Total		**13**	

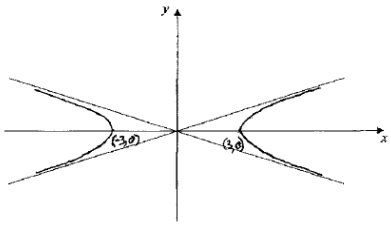
11.Jan 2011

6(a)	Sketch of ellipse	M1		centred at origin
	Correct relationship to circle	A1		
	Coords $(\pm 2\sqrt{2}, 0), (0, \pm\sqrt{2})$	B2,1	4	Accept $\sqrt{8}$ for $2\sqrt{2}$; B1 for any 2 of $x = \pm 2\sqrt{2}$, $y = \pm\sqrt{2}$ allow B1 if all correct except for use of decimals (at least one DP)
(b)(i)	Replacing x by $\frac{x}{2}$	M1		or by $2x$
	E is $(\frac{x}{2})^2 + y^2 = 2$	A1	2	OE
(ii)	Tangent is $\frac{x}{2} + y = 2$	M1A1	2	M1 for complete valid method
	Total		**8**	

12.June 2011

9(a)(i)	Elimination to give $x = \frac{1}{8}x^2$	M1		OE
	A is (8, 8)	A1	2	NMS 2/2
(ii)	Equation of Q is $x = \frac{1}{8}y^2$	B1	1	OE; condone $y = \sqrt{8x}$
(iii)	Points of contact are images in $y = x$	E1	1	
(b)(i)	Eliminating y to give $-x + c = \frac{1}{8}x^2$ (ie $x^2 + 8x - 8c = 0$)	M1		
	Distinct roots if $\Delta > 0$	E1		stated or implied
	$\Delta = 64 + 32c$, so $c > -2$	A1	3	convincingly shown (AG)
(ii)	For tangent $c = -2$, so $x^2 + 8x + 16 = 0$	M1		OE
	... and $x = -4$, $y = 2$	A1		
	Reflection in $y = x$	M1		or other complete method
	$x = 2$, $y = -4$	A1F	4	ft wrong answer for first point; allow NMS 2/2
	Total		**11**	

13.Jan 2012

7(a)	$y = \pm\frac{1}{3}x$	B1	1	ACF Need both
(b)		B1 B1 B1	3	2-branch curve with branches in correct regions above and below x-axis Curve approaching asymptotes Coords (±3, 0), as **only** points of intersection with coordinate axes, indicated. Condone −3 and +3 marked on x-axis at points of intersection as (±3, 0) indicated.
(c)(i)	$\frac{(x+3)^2}{9} - y^2 = 1$	M1 A1	2	Replacing x by either $x + 3$ or $x - 3$ ACF
(ii)	$\frac{(x+3)^2}{9} - x^2 = 1$	M1		Substitution into c's (c)(i) eqn of $y = x$ to eliminate y or of $x = y$ to eliminate x

(c)(i)	$\frac{(x+3)^2}{9} - y^2 = 1$	M1 A1	2	Replacing x by either $x + 3$ or $x - 3$ ACF
(ii)	$\frac{(x+3)^2}{9} - x^2 = 1$	M1		Substitution into c's (c)(i) eqn of $y = x$ to eliminate y or of $x = y$ to eliminate x
	$x^2 + 6x + 9 = 9(x^2 + 1)$	A1F		Correct expansion of $(x \pm 3)^2$ equated to $9(x^2 + 1)$ OE ft; [OE in y]
	$8x^2 - 6x = 0 \quad (8x^2 = 6x)$	A1F		Ft on error $(x - 3)$ for $(x + 3)$ in (c)(i) which gives $8x^2 + 6x = 0 \quad (8x^2 = -6x)$ [OE in y]
	Points are (0, 0), $\left(\frac{3}{4}, \frac{3}{4}\right)$	A1	4	Both. ACF
(d)		M1		Adding 3 to c's (c)(ii) **two** x-coords keeping y-coordinates unchanged.
	Points are (3, 0), $\left(3\frac{3}{4}, \frac{3}{4}\right)$	A1F	2	Ft on c's (c)(ii) coordinates for the **two** points If not deduced then M0A0
	Total		**12**	

14.June 2012

8(a)	$\left(\pm\sqrt{5}\ ,\ 0\right),\quad (0\ ,\ \pm 2)$	B2,1	2	If not B2, award B1 if either at least two of these 4 correct pts or if '$x = \pm\sqrt{5}$ and $y = \pm 2$'
8(b)	$\frac{(x-p)^2}{5} + \frac{y^2}{4} = 1$	M1 A1	2	Replacing x by either $x + p$ or $x - p$ and keeping y unchanged or as $y \pm 0$ ACF
8(c)	$\frac{(x-p)^2}{5} + \frac{(x+4)^2}{4} = 1$	M1		Substitution into c's (**b**) eqn of $y = x+4$ to eliminate y
	$4(x-p)^2 + 5(x+4)^2 = 4 \times 5$ $4(x^2 - 2px + p^2) + 5(x^2 + 8x + 16) = 20$	m1		Denominators 5 and 4 cleared in a correct manner and at least either a correct expansion of $(x \pm p)^2$ or a correct expansion of $(x + 4)^2$
	$4x^2 - 8px + 4p^2 + 5x^2 + 40x + 80 = 20$			CSO No errors in any line of working.
	$9x^2 - (8p - 40)x + 4p^2 + 60 = 0$	A1	3	AG. Must see brackets correctly removed and all terms involving x, p correctly rearranged to same side before the printed

				answer is stated. Must have '= 0' although brackets around $4p^2 + 60$ may be omitted
(d)	Discriminant is $(8p - 40)^2 - 4\,(9)\,(4p^2 + 60)$ For tangency	B1		OE Must be isolated, not just within the quadratic formula
	$(8p - 40)^2 - 4\,(9)\,(4p^2 + 60) = 0$	M1		OE Equating c's discriminant to zero before obtaining any values for p
	$p^2 + 8p + 7 = 0$	A1		ACF with like terms collected
	$\{(p + 1)\,(p + 7) = 0 \Rightarrow\}\ p = -1,\ p = -7$ (*)	B1		Correct values $-1, -7$ for p
	$\underline{p = -1}$: $9x^2 + 48x + 64\ (= 0)$ $\underline{p = -7}$: $9x^2 + 96x + 256\ (= 0)$	M1		Substitutes at least one of c's **two** values for p either into the given quadratic in (c) OE or into $\dfrac{8p - 40}{18}$
	$\underline{p = -1}$: $9x^2 + 48x + 64\ (= 0) \Rightarrow x = -\dfrac{8}{3}$	A1		$x = -\dfrac{8}{3}$ OE as only root from the quadratic or from $\dfrac{8p - 40}{18}$. Apply FIW if (*) is B0
	$\underline{p = -7}$: $9x^2 + 96x + 256\ (=0) \Rightarrow x = -\dfrac{16}{3}$	A1		$x = -\dfrac{16}{3}$ OE as only root from the quadratic or from $\dfrac{8p - 40}{18}$. Apply FIW if (*) is B0
	$x = -\dfrac{8}{3},\ y = \dfrac{4}{3};\quad x = -\dfrac{16}{3},\ y = -\dfrac{4}{3}$ $\left(-\dfrac{8}{3}, \dfrac{4}{3}\right)\quad \left(-\dfrac{16}{3}, -\dfrac{4}{3}\right)$	A1	8	CSO Previous 7 marks must have been awarded and coordinates of both points need to be correct and exact but accept in either format
	Total		**15**	

15.Jan 2013

9(a)	$y = 0,\ \dfrac{(x-4)^2}{4} = 1;\quad (x-4)^2 = 4$	M1		OE Sub y=0 in eqn of ellipse and either eliminate fraction **or** take sq root, condoning missing ±, ie $\dfrac{(x-4)}{2} = (\pm)1$
	$\Rightarrow x = 2, 6\ \ (x_A = 2,\ x_B = 6)$	A1	2	Both 2 and 6 NMS Mark as B2 or B0
(b)(i)	$\dfrac{(x-4)^2}{4} + (mx)^2 = 1 \quad \Rightarrow$	M1		Substitute y=mx to eliminate y
	$(x-4)^2 + 4(mx)^2 = 4 \Rightarrow x^2 - 8x + 16 + 4(mx)^2 = 4$	A1		Eliminate fractions correctly and expand $(x - 4)^2$ correctly
	$\Rightarrow x^2 - 8x + 16 + 4m^2x^2 - 4 = 0$ $\Rightarrow (1+4m^2)\,x^2 - 8x + 12 = 0$	A1	3	CSO AG
(ii)	Discriminant $b^2 - 4ac\ \ \{(-8)^2 - 4(1+4m^2)(12)\}$	M1		b^2–$4ac$ in terms of m condone one sign or copying error OE
	For tangency, $(-8)^2 - 4(1+4m^2)(12) = 0$	A1		A correct equation with m^2 being the only unknown at any stage.
	$192m^2 - 16\ \ (=0)$	A1		OE eg $12m^2$–$1(=0)$ OE PI by a correct value for m condoning wrong sign
	(m>0 so) $m = \dfrac{1}{\sqrt{12}}$	A1	4	ACF of an **exact** value for m eg $\dfrac{1}{2\sqrt{3}}$, $\dfrac{\sqrt{3}}{6}$. Dep on prev 3 mrks

(iii)	$(1+4\times\left\{\frac{1}{\sqrt{12}}\right\}^2)x^2-8x+12\ (=0)$ $\frac{4}{3}x^2-8x+12=0$; $4x^2-24x+36=0$ $x^2-6x+9=0$ $x=\frac{-(-8)\pm\sqrt{0}}{\frac{8}{3}}$; $(x-3)^2\ (=0)$ $x=3$ Coordinates of P are $\left(3,\ \frac{3}{\sqrt{12}}\right)$	M1 m1 A1 A1	4	Subst value for m in LHS of eqn (b)(i); ft on c's value of m. Valid method to solve a **correct** quadratic equation; as far as either correct subst into quadratic formula with b^2-4ac evaluated to 0 or correct factorisation or correct value of x after $\frac{4}{3}x^2-8x+12=0$ or better seen.; OE, correct use of $-b/2a$ Must see earlier justification Correct coordinates with the y-coord in any correct exact form eg $\frac{\sqrt{3}}{2}$. NMS SC 1 for $\left(3,\ \frac{3}{\sqrt{12}}\right)$
	Total		**13**	