Practice examination questions for complex numbers

1. Jan 2005

3 It is given that $z = x + iy$, where x and y are real numbers.

(a) Write down, in terms of x and y, an expression for z^*, the complex conjugate of z. *(1 mark)*

(b) Find, in terms of x and y, the real and imaginary parts of

$$2z - iz^*$$ *(2 marks)*

(c) Find the complex number z such that

$$2z - iz^* = 3i$$ *(3 marks)*

2. June 2005

6 The equation

$$x^2 - 4x + 13 = 0$$

has roots α and β.

(a) (i) Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(ii) Deduce that $\alpha^2 + \beta^2 = -10$. *(2 marks)*

(iii) Explain why the statement $\alpha^2 + \beta^2 = -10$ implies that α and β cannot both be real. *(2 marks)*

(b) Find in the form $p + iq$ the values of:

(i) $(\alpha + i) + (\beta + i)$; *(1 mark)*

(ii) $(\alpha + i)(\beta + i)$. *(2 marks)*

(c) Hence find a quadratic equation with roots $(\alpha + i)$ and $(\beta + i)$. *(2 marks)*

3. Jan 2006

5 (a) (i) Calculate $(2 + i\sqrt{5})(\sqrt{5} - i)$. *(3 marks)*

(ii) Hence verify that $\sqrt{5} - i$ is a root of the equation

$$(2 + i\sqrt{5})z = 3z^*$$

where z^* is the conjugate of z. *(2 marks)*

(b) The quadratic equation

$$x^2 + px + q = 0$$

in which the coefficients p and q are real, has a complex root $\sqrt{5} - i$.

(i) Write down the other root of the equation. *(1 mark)*

(ii) Find the sum and product of the two roots of the equation. *(3 marks)*

(iii) Hence state the values of p and q. *(2 marks)*

4. June 2006

6 It is given that $z = x + \mathrm{i}y$, where x and y are real numbers.

(a) Write down, in terms of x and y, an expression for

$$(z + \mathrm{i})^*$$

where $(z + \mathrm{i})^*$ denotes the complex conjugate of $(z + \mathrm{i})$. *(2 marks)*

(b) Solve the equation

$$(z + \mathrm{i})^* = 2\mathrm{i}z + 1$$

giving your answer in the form $a + b\mathrm{i}$. *(5 marks)*

5. Jan 2007

1 (a) Solve the following equations, giving each root in the form $a + b\mathrm{i}$:

(i) $x^2 + 16 = 0$; *(2 marks)*

(ii) $x^2 - 2x + 17 = 0$. *(2 marks)*

(b) (i) Expand $(1 + x)^3$. *(2 marks)*

(ii) Express $(1 + \mathrm{i})^3$ in the form $a + b\mathrm{i}$. *(2 marks)*

(iii) Hence, or otherwise, verify that $x = 1 + \mathrm{i}$ satisfies the equation

$$x^3 + 2x - 4\mathrm{i} = 0$$ *(2 marks)*

6. June 2007

3 It is given that $z = x + \mathrm{i}y$, where x and y are real numbers.

(a) Find, in terms of x and y, the real and imaginary parts of

$$z - 3\mathrm{i}z^*$$

where z^* is the complex conjugate of z. *(3 marks)*

(b) Find the complex number z such that

$$z - 3\mathrm{i}z^* = 16$$ *(3 marks)*

7. Jan 2008

1 It is given that $z_1 = 2 + \mathrm{i}$ and that $z_1{}^*$ is the complex conjugate of z_1.

Find the real numbers x and y such that

$$x + 3\mathrm{i}y = z_1 + 4\mathrm{i}z_1{}^*$$ *(4 marks)*

8. Jan 2008

8 (a) (i) It is given that α and β are the roots of the equation

$$x^2 - 2x + 4 = 0$$

Without solving this equation, show that α^3 and β^3 are the roots of the equation

$$x^2 + 16x + 64 = 0$$ *(6 marks)*

(ii) State, giving a reason, whether the roots of the equation

$$x^2 + 16x + 64 = 0$$

are real and equal, real and distinct, or non-real. *(2 marks)*

(b) Solve the equation

$$x^2 - 2x + 4 = 0$$ *(2 marks)*

(c) Use your answers to parts (a) and (b) to show that

$$(1 + i\sqrt{3})^3 = (1 - i\sqrt{3})^3$$ *(2 marks)*

9. June 2008

2 It is given that $z = x + iy$, where x and y are real numbers.

(a) Find, in terms of x and y, the real and imaginary parts of

$$3iz + 2z^*$$

where z^* is the complex conjugate of z. *(3 marks)*

(b) Find the complex number z such that

$$3iz + 2z^* = 7 + 8i$$ *(3 marks)*

10.Jan 2009

2 The complex number $2 + 3i$ is a root of the quadratic equation

$$x^2 + bx + c = 0$$

where b and c are real numbers.

(a) Write down the other root of this equation. *(1 mark)*

(b) Find the values of b and c. *(4 marks)*

11.June 2009

3 The complex number z is defined by

$$z = x + 2i$$

where x is real.

(a) Find, in terms of x, the real and imaginary parts of:

(i) z^2; *(3 marks)*

(ii) $z^2 + 2z^*$. *(2 marks)*

(b) Show that there is exactly one value of x for which $z^2 + 2z^*$ is real. *(2 marks)*

12.Jan 2010

2 The complex number z is defined by

$$z = 1 + \mathrm{i}$$

(a) Find the value of z^2, giving your answer in its simplest form. *(2 marks)*

(b) Hence show that $z^8 = 16$. *(2 marks)*

(c) Show that $(z^*)^2 = -z^2$. *(2 marks)*

13.June 2010

2 It is given that $z = x + \mathrm{i}y$, where x and y are real numbers.

(a) Find, in terms of x and y, the real and imaginary parts of

$$(1 - 2\mathrm{i})z - z^*$$ *(4 marks)*

(b) Hence find the complex number z such that

$$(1 - 2\mathrm{i})z - z^* = 10(2 + \mathrm{i})$$ *(2 marks)*

14.Jan 2011

5 (a) It is given that $z_1 = \frac{1}{2} - \mathrm{i}$.

(i) Calculate the value of ${z_1}^2$, giving your answer in the form $a + b\mathrm{i}$. *(2 marks)*

(ii) Hence verify that z_1 is a root of the equation

$$z^2 + z^* + \frac{1}{4} = 0$$ *(2 marks)*

(b) Show that $z_2 = \frac{1}{2} + \mathrm{i}$ also satisfies the equation in part **(a)(ii)**. *(2 marks)*

(c) Show that the equation in part **(a)(ii)** has two equal **real** roots. *(2 marks)*

15.June 2011

3 It is given that $z = x + \mathrm{i}y$, where x and y are real.

(a) Find, in terms of x and y, the real and imaginary parts of

$$(z - \mathrm{i})(z^* - \mathrm{i})$$ *(3 marks)*

(b) Given that

$$(z - \mathrm{i})(z^* - \mathrm{i}) = 24 - 8\mathrm{i}$$

find the two possible values of z. *(4 marks)*

16.Jan 2012

3 (a) Solve the following equations, giving each root in the form $a + b\mathrm{i}$:

(i) $x^2 + 9 = 0$; *(1 mark)*

(ii) $(x + 2)^2 + 9 = 0$. *(1 mark)*

(b) (i) Expand $(1 + x)^3$. *(1 mark)*

(ii) Express $(1 + 2\mathrm{i})^3$ in the form $a + b\mathrm{i}$. *(3 marks)*

(iii) Given that $z = 1 + 2\mathrm{i}$, find the value of

$$z^* - z^3$$ *(2 marks)*

17.June 2012

3 It is given that $z = x + \mathrm{i}y$, where x and y are real numbers.

(a) Find, in terms of x and y, the real and imaginary parts of

$$\mathrm{i}(z + 7) + 3(z^* - \mathrm{i})$$ *(3 marks)*

(b) Hence find the complex number z such that

$$\mathrm{i}(z + 7) + 3(z^* - \mathrm{i}) = 0$$ *(3 marks)*

18.Jan 2013

2 (a) Solve the equation $w^2 + 6w + 34 = 0$, giving your answers in the form $p + q\mathrm{i}$, where p and q are integers. *(3 marks)*

(b) It is given that $z = \mathrm{i}(1 + \mathrm{i})(2 + \mathrm{i})$.

(i) Express z in the form $a + b\mathrm{i}$, where a and b are integers. *(3 marks)*

(ii) Find integers m and n such that $z + mz^* = n\mathrm{i}$. *(3 marks)*

Answers

1. Jan 2005

3(a)	$z^* = x - \mathrm{i}y$	B1	1	
(b)	$\mathrm{R} = 2x - y$	B1		$\mathrm{i}^2 = -1$ must be used
	$\mathrm{I} = -x + 2y$	B1	2	Condone $\mathrm{I} = \mathrm{i}(x + 2y)$; Answers may appear in (c)
(c)	Equating R and/or I parts	M1		
	Attempt to solve sim equations	m1		
	$z = 1 + 2\mathrm{i}$	A1	3	Allow $x = 1,\ y = 2$
	Total		**6**	

2. June 2005

6(a)(i)	$\alpha + \beta = 4,\ \alpha\beta = 13$	B1B1	2	
(ii)	$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$	M1		
	$\ldots = 4^2 - 26 = -10$	A1	2	convincingly shown (AG)
(iii)	The square of a real number is positive (or zero)	E1		
	The sum of two such squares is positive (or zero)	E1	2	
(b)(i)	$(\alpha + \mathrm{i}) + (\beta + \mathrm{i}) = 4 + 2\mathrm{i}$	B1F	1	ft wrong value in (a)(i)
(ii)	$(\alpha + \mathrm{i})(\beta + \mathrm{i}) = 12 + 4\mathrm{i}$	M1A1F	2	ditto
(c)	Correct coeff of x or constant term	M1		Using c's answers in (b)
	$x^2 - (4 + 2\mathrm{i})x + (12 + 4\mathrm{i}) = 0$	A1F	2	ft wrong answers in (b)
	Total		**11**	

3. Jan 2006

5(a)(i)	Full expansion of product	M1		
	Use of $\mathrm{i}^2 = -1$	m1		
	$\left(2 + \sqrt{5}\mathrm{i}\right)\left(\sqrt{5} - \mathrm{i}\right) = 3\sqrt{5} + 3\mathrm{i}$	A1	3	$\sqrt{5}\sqrt{5} = 5$ must be used – Accept not fully simplified
(ii)	$z^* = x - \mathrm{i}y\ (= \sqrt{5} + \mathrm{i})$	M1		
	Hence result	A1	2	Convincingly shown (AG)
(b)(i)	Other root is $\sqrt{5} + \mathrm{i}$	B1	1	
(ii)	Sum of roots is $2\sqrt{5}$	B1		
	Product is 6	M1A1	3	
(iii)	$p = -2\sqrt{5},\ q = 6$	B1 B1√	2	ft wrong answers in (ii)
	Total		**11**	

4. June 2006

6(a)	$(z + \mathrm{i})^* = x - \mathrm{i}y - \mathrm{i}$	B2	2	
(b)	$\ldots = 2\mathrm{i}x - 2y + 1$	M1		$\mathrm{i}^2 = -1$ used at some stage
	Equating R and I parts	M1		involving at least 5 terms in all
	$x = -2y + 1,\ -y - 1 = 2x$	A1√		ft one sign error in (a)
	$z = -1 + \mathrm{i}$	m1A1√	5	ditto; allow $x = -1,\ y = 1$
	Total		**7**	

5. Jan 2007

1(a)(i)	Roots are $\pm 4i$	M1A1	2	M1 for one correct root or two correct factors
(ii)	Roots are $1 \pm 4i$	M1A1	2	M1 for correct method
(b)(i)	$(1+x)^3 = 1 + 3x + 3x^2 + x^3$	M1A1	2	M1A0 if one small error
(ii)	$(1+i)^3 = 1 + 3i - 3 - i = -2 + 2i$	M1A1	2	M1 if $i^2 = -1$ used
(iii)	$(1+i)^3 + 2(1+i) - 4i$	M1		with attempt to evaluate
	$\ldots = (-2+2i) + (2-2i) = 0$	A1	2	convincingly shown (**AG**)
	Total		**10**	

6. June 2007

3(a)	Use of $z^* = x - iy$	M1		
	$z - 3iz^* = x + iy - 3ix - 3y$	m1		Condone sign error here
	$R = x - 3y$, $I = -3x + y$	A1	3	Condone inclusion of i in I Allow if correct in (b)
(b)	$x - 3y = 16$, $-3x + y = 0$	M1		
	Elimination of x or y	m1		
	$z = -2 - 6i$	A1F	3	Accept $x = -2$, $y = -6$; ft $x + 3y$ for $x - 3y$
	Total		**6**	

7. Jan 2008

1	$z_1 + 4i\, z_1^* = (2+i) + 4i\,(2-i)$	M1		Use of conjugate
	$\ldots = (2+i) + (8i+4)$	M1		Use of $i^2 = -1$
	$\ldots = 6 + 9i$, so $x = 6$ and $y = 3$	M1A1	4	M1 for equating Real and imaginary parts
	Total		**4**	

8. Jan 2008

8(a)(i)	$\alpha + \beta = 2$, $\alpha\beta = 4$	B1B1		
	$\alpha^3 + \beta^3 = (2)^3 - 3(4)(2) = -16$	M1A1		
	$\alpha^3\beta^3 = (4)^3 = 64$, hence result	M1A1	6	convincingly shown (AG)
(ii)	Discriminant 0, so roots equal	B1E1	2	or by factorisation
(b)	$x = \dfrac{2 \pm \sqrt{4-16}}{2}$	M1		or by completing square
	$\ldots = 1 \pm \frac{1}{2} i\sqrt{12}$	A1	2	
(c)	$\alpha, \beta = 1 \pm i\sqrt{3}$			
	and $\alpha^3 = \beta^3$, hence result	E2	2	
	Total		**12**	

9. June 2008

2(a)	Use of $z^* = x - iy$	M1		
	Use of $i^2 = -1$	M1		
	$3iz + 2z^* = (2x - 3y) + i\,(3x - 2y)$	A1	3	Condone inclusion of i in I part
(b)	Equating R and I parts	M1		
	$2x - 3y = 7$, $3x - 2y = 8$	m1		with attempt to solve
	$z = 2 - i$	A1	3	Allow $x = 2$, $y = -1$
	Total		**6**	

10.Jan 2009

2(a)	Other root is $2 - 3i$	B1	1	
(b)	Sum of roots = 4	B1F		ft error in (a)
	So $b = -4$	B1F		ft wrong value for sum
	Product is 13	B1		
	So $c = 13$	B1F	4	ft wrong value for product
	Alternative:			
	Substituting $2 + 3i$ into equation	M1		
	Equating R and I parts	m1		
	$12 + 3b = 0$, so $b = -4$	A1		
	$-5 + 2b + c = 0$, so $c = 13$	A1F	(4)	ft wrong value for b
	Total		**5**	

11.June 2009

3(a)(i)	$z^2 = (x^2 - 4) + i(4x)$	M1A1		M1 for use of $i^2 = -1$
	R and I parts clearly indicated	A1F	3	Condone inclusion of i in I part ft one numerical error
(ii)	$z^2 + 2z^* = (x^2 + 2x - 4) + i(4x - 4)$	M1A1F	2	M1 for correct use of conjugate ft numerical error in (i)
(b)	$z^2 + 2z^*$ real if imaginary part zero	M1		
	... ie if $x = 1$	A1F	2	ft provided imaginary part linear
	Total		**7**	

12.Jan 2010

2(a)	$z^2 = 1 + 2i + i^2 = 2i$	M1A1	2	M1 for use of $i^2 = -1$
(b)	$z^8 = (2i)^4$	M1		or equivalent complete method
	... $= 16i^4 = 16$	A1	2	convincingly shown (AG)
(c)	$(z^*)^2 = (1 - i)^2$	M1		for use of $z^* = 1 - i$
	... $= -2i = -z^2$	A1	2	convincingly shown (AG)
	Total		**6**	

13.June 2010

2(a)	Use of $z^* = x - iy$	M1		
	Use of $i^2 = -1$	M1		
	$(1 - 2i)z - z^* = 2y + i(2y - 2x)$	A2,1	4	A1 if one numerical error made
(b)	$2y = 20$, $2y - 2x = 10$	M1		equate and attempt to solve
	so $z = 5 + 10i$	A1	2	allow $x = 5$, $y = 10$
	Total		**6**	

14.Jan 2011

5(a)(i)	$z_1^2 = \frac{1}{4} - i + i^2 = -\frac{3}{4} - i$	M1A1	2	M1 for use of $i^2 = -1$
(ii)	LHS $= -\frac{3}{4} - i + \frac{1}{2} + i + \frac{1}{4} = 0$	M1A1	2	AG; M1 for z^* correct

(b)	LHS $= -\frac{3}{4} + \mathrm{i} + \frac{1}{2} - \mathrm{i} + \frac{1}{4} = 0$	M1A1	2	AG; M1 for $z_2^{\,2}$ correct
(c)	z real $\Rightarrow z^* = z$	M1		Clearly stated
	Discr't zero or correct factorisation	A1	2	AG
	Total		**8**	

15.June 2011

3(a)	Use of $z^* = x - \mathrm{i}y$	M1		
	$(z - \mathrm{i})(z^* - \mathrm{i}) = (x^2 + y^2 - 1) - 2\mathrm{i}x$	m1A1	3	A1 may be earned in (b)
(b)	Equating R and I parts	M1		
	$-2x = -8$ so $x = 4$	A1		
	$16 + y^2 - 1 = 24$ so $y = \pm 3$ ($z = 4 \pm 3\mathrm{i}$)	m1A1	4	A0 if $x = -4$ used
	Total		**7**	

16.Jan 2012

3(a)(i)	$x = \pm 3\mathrm{i}$	B1	1	$\pm 3\mathrm{i}$ ($a = 0, b = \pm 3$)
(ii)	$x = -2 \pm 3\mathrm{i}$	B1F	1	If not correct, ft wrong answer(s) to (i) provided (i) has a non-zero b value
(b)(i)	$(1 + x)^3 = 1 + 3x + 3x^2 + x^3$	B1	1	Terms simplified in any order.
(ii)	$(1 + 2\mathrm{i})^3 = 1 + 3(2\mathrm{i}) + 3(2\mathrm{i})^2 + (2\mathrm{i})^3$			Replacing x in (b)(i) by 2i, squaring and cubing correctly, only ft on c's wrong non-zero coefficients from (b)(i).
	$= 1+3(2\mathrm{i}) + 3(4\mathrm{i}^2) + (8\mathrm{i}^3)$	B1F		
	$= 1 + 3(2\mathrm{i}) + 3(4)(-1) + (8)(-\mathrm{i})$	M1		Use of $\mathrm{i}^2 = -1$ at least once.
	$= -11 - 2\mathrm{i}$	A1	3	$-11 - 2\mathrm{i}$ ($a = -11$, $b = -2$)
(iii)	$z^* - z^3 = 1 - 2\mathrm{i} - (-11 - 2\mathrm{i})$	M1		Use of $z^* = 1 - 2\mathrm{i}$
	$= 12$	A1F	2	If not correct, only ft on $1 - 2\mathrm{i} -$ c's (b)(ii) if c's (b)(ii) answer is of the form $a + b\mathrm{i}$ with $a \neq 0$ and $b \neq 0$
	Total		**8**	

17.June 2012

3(a)	$\mathrm{i}(z+7) + 3(z^* - \mathrm{i}) =$			
	$\mathrm{i}(x + \mathrm{i}y + 7) + 3(x - \mathrm{i}y - \mathrm{i})$	M1		M1 for use of $z^* = x - \mathrm{i}y$
	$= \mathrm{i}x - y + 7\mathrm{i} + 3x - 3\mathrm{i}y - 3\mathrm{i}$	M1		M1 for $\mathrm{i}^2 y = -y$
	$= 3x - y + \mathrm{i}(x - 3y + 4)$	A1	3	If the five terms correct but not grouped into Real and Imaginary parts, allow A1 retrospectively provided the correct two expressions used in the M1 line in (b)
(b)	$3x - y = 0, \quad x - 3y + 4 = 0$	M1		Attempting to equate all Real parts to zero and all Imaginary parts to zero
	$x - 9x + 4 = 0$ (or eg $y - 9y + 12 = 0$)	A1		A correct equation in either x or y PI by correct final answer
	Solving to give $z = \frac{1}{2} + \frac{3}{2}\mathrm{i}$	A1	3	Allow $x = \frac{1}{2}$, $y = \frac{3}{2}$
	Total		**6**	

18.Jan 2013

2(a)	$(w=)\ \dfrac{-6\pm\sqrt{36-4(34)}}{2}\ \left\{=\dfrac{-6\pm\sqrt{-100}}{2}\right\}$	M1		Correct substitution into quadratic formula OE
	$=\dfrac{-6\pm 10\text{i}}{2}$	B1		$\sqrt{-100}=10\text{i}$ or $\sqrt{-100}/2=5\text{i}$
	$=-3\pm 5\text{i}$	A1	3	$-3\pm 5\text{i}$ $(p=-3,\ q=\pm 5)$ NMS mark as 3/3 or 0/3
(b)(i)	$z=\text{i}(1+\text{i})(2+\text{i})=\text{i}(2+3\text{i}+\text{i}^2)=2\text{i}+3\text{i}^2+\text{i}^3$	M1		Attempt to expand all brackets.
	$=2\text{i}+3(-1)+\text{i}(-1)$	B1		$\text{i}^2=-1$ used at least once
	$=-3+\text{i}$	A1	3	$-3+\text{i}$ $(a=-3,\ b=1)$
(ii)	$z^*=-3-\text{i}$	B1F		OE Ft c's $a-b\text{i}$
	$-3+\text{i}+m(-3-\text{i})=n\text{i}$ $\Rightarrow -3-3m=0;\ 1-m=n$	M1		Equating **both** real parts and the imag. parts, PI by next line
	$\Rightarrow\ m=-1,\ n=2$	A1	3	Both correct
	Total		**9**	