Practice examination questions for binomial expansion

1. Jan 2005

6 (a) (i) Using the binomial expansion, or otherwise, express $(2+x)^3$ in the form $8+ax+bx^2+x^3$, where a and b are integers. *(3 marks)*

(ii) Write down the expansion of $(2-x)^3$. *(2 marks)*

(b) Hence show that $(2+x)^3-(2-x)^3=24x+2x^3$. *(2 marks)*

(c) Hence show that the curve with equation

$$y=(2+x)^3-(2-x)^3$$

has no stationary points. *(3 marks)*

2. June 2005

6 (a) Using the binomial expansion, or otherwise, express $(1+x)^4$ in ascending powers of x. *(3 marks)*

(b) (i) Hence show that $(1+\sqrt{5})^4=56+24\sqrt{5}$. *(3 marks)*

(ii) Hence show that $\log_2(1+\sqrt{5})^4=k+\log_2(7+3\sqrt{5})$, where k is an integer. *(3 marks)*

3. June 2006

4 (a) The expression $(1-2x)^4$ can be written in the form

$$1+px+qx^2-32x^3+16x^4$$

By using the binomial expansion, or otherwise, find the values of the integers p and q. *(3 marks)*

(b) Find the coefficient of x in the expansion of $(2+x)^9$. *(2 marks)*

(c) Find the coefficient of x in the expansion of $(1-2x)^4(2+x)^9$. *(3 marks)*

4. Jan 2007

7 (a) The first four terms of the binomial expansion of $(1+2x)^8$ in ascending powers of x are $1+ax+bx^2+cx^3$. Find the values of the integers a, b and c. *(4 marks)*

(b) Hence find the coefficient of x^3 in the expansion of $\left(1+\frac{1}{2}x\right)(1+2x)^8$. *(3 marks)*

5. Jan 2008

6 (a) Using the binomial expansion, or otherwise:

(i) express $(1+x)^3$ in ascending powers of x; *(2 marks)*

(ii) express $(1+x)^4$ in ascending powers of x. *(2 marks)*

(b) Hence, or otherwise:

(i) express $(1+4x)^3$ in ascending powers of x; *(2 marks)*

(ii) express $(1+3x)^4$ in ascending powers of x. *(2 marks)*

(c) Show that the expansion of

$$(1+3x)^4 - (1+4x)^3$$

can be written in the form

$$px^2 + qx^3 + rx^4$$

where p, q and r are integers. *(2 marks)*

6. June 2008

7 (a) The expression $\left(1+\frac{4}{x^2}\right)^3$ can be written in the form

$$1+\frac{p}{x^2}+\frac{q}{x^4}+\frac{64}{x^6}$$

By using the binomial expansion, or otherwise, find the values of the integers p and q. *(3 marks)*

(b) (i) Hence find $\int\left(1+\frac{4}{x^2}\right)^3 \mathrm{d}x$. *(4 marks)*

(ii) Hence find the value of $\int_1^2\left(1+\frac{4}{x^2}\right)^3 \mathrm{d}x$. *(2 marks)*

7. Jan 2009

5 (a) By using the binomial expansion, or otherwise, express $(1+2x)^4$ in the form

$$1+ax+bx^2+cx^3+16x^4$$

where a, b and c are integers. *(4 marks)*

(b) Hence show that $(1+2x)^4+(1-2x)^4 = 2+48x^2+32x^4$. *(3 marks)*

(c) Hence show that the curve with equation

$$y = (1+2x)^4+(1-2x)^4$$

has just one stationary point and state its coordinates. *(4 marks)*

8. Jan 2010

7 (a) The first four terms of the binomial expansion of $(1+2x)^7$ in ascending powers of x are $1+ax+bx^2+cx^3$. Find the values of the integers a, b and c. *(4 marks)*

(b) Hence find the coefficient of x^3 in the expansion of $\left(1-\frac{1}{2}x\right)^2(1+2x)^7$. *(4 marks)*

9. Jan 2011

5 (a) Using the binomial expansion, or otherwise, express $(1-x)^3$ in ascending powers of x. *(2 marks)*

(b) Show that the expansion of

$$(1+y)^4-(1-y)^3$$

is

$$7y+py^2+qy^3+y^4$$

where p and q are constants to be found. *(4 marks)*

(c) Hence find $\int\left[(1+\sqrt{x})^4-(1-\sqrt{x})^3\right]dx$, expressing each coefficient in its simplest form. *(4 marks)*

10.Jan 2012

5 (a) (i) Describe the geometrical transformation that maps the graph of $y=\left(1+\frac{x}{3}\right)^6$ onto the graph of $y=(1+2x)^6$. *(2 marks)*

(ii) The curve $y=\left(1+\frac{x}{3}\right)^6$ is translated by the vector $\begin{bmatrix}3\\0\end{bmatrix}$ to give the curve $y=g(x)$. Find an expression for $g(x)$, simplifying your answer. *(2 marks)*

(b) The first four terms in the binomial expansion of $\left(1+\frac{x}{3}\right)^6$ are $1+ax+bx^2+cx^3$. Find the values of the constants a, b and c, giving your answers in their simplest form. *(4 marks)*

11.Jan 2013

8 (a) Expand $\left(1+\frac{4}{x}\right)^2$. *(1 mark)*

(b) The first four terms of the binomial expansion of $\left(1+\frac{x}{4}\right)^8$ in ascending powers of x are $1+ax+bx^2+cx^3$. Find the values of the constants a, b and c. *(4 marks)*

(c) Hence find the coefficient of x in the expansion of $\left(1+\frac{4}{x}\right)^2\left(1+\frac{x}{4}\right)^8$. *(4 marks)*

Answers

1. Jan 2005

6(a)(i)	$(2+x)^3 =$			Any valid method; must contain all
	$(2^3)+3(2^2)(x)+3(2)(x^2)+(x^3)$	M1		components
	$\ldots = 8 + 12x + 6x2 + x3$ (*)	A1		Accept $a = 12$
		A1	3	Accept $b = 6$
(ii)	$(2-x)^3 = 8-12x+6x^2-x^3$ (**)	M1		Clear $x \to -x$ in (i) OE
		A1√	2	ft numerical a and b
(b)	$(2+x)^3-(2-x)^3 = (*)-(**)$	M1		Subtracts the 2 expressions in (a)
	$\ldots\ldots = 24x + 2x^3.$	A1	2	CSO AG (be convinced)
(c)	$\frac{dy}{dx} = 24+6x^2$	M1		A power of x decreased by 1
	For st. pt. $24+6x^2 = 0$	A1		
	Not possible since $24+6x^2 > 0$	E1	3	Any valid explanation
	Total		**10**	

2. June 2005

6(a)	$(1+x)^4 = 1+4x+6x^2+4x^3+x^4$	M1 A2,1	3	Full method A1 if four terms correct or just one slip
(b)(i)	$(1+\sqrt{5})^4 = 1+4\sqrt{5}+6(\sqrt{5})^2+$ $+4(\sqrt{5})^3+(\sqrt{5})^4$	M1		Substitute. √5 for x.
	$=1+4\sqrt{5}+6(5)+4(5\sqrt{5})+(25)$	A1ft		Two of 3 terms shown in brackets
	$\ldots\ldots\ldots = 56 + 24\sqrt{5}$	A1	3	**AG** CSO Be convinced
(ii)	$\log_2(1+\sqrt{5})^4 = \log_2[8(7+3\sqrt{5})]$	M1		
	$= \log_2 8 + \log_2(7+3\sqrt{5})$	m1		
	$= 3+\log_2(7+3\sqrt{5})$	A1	3	CSO **SC** B1 Change to base 10 and verify
	Total		**9**	

3. June 2006

4(a)	$(1-2x)^4 = (1)^4+4(1)^3(-2x)+$ $6(1^2)(-2x)^2+[4(1)(-2x)^3+(-2x)^4]$	M1		Any valid method as far as term(s) in x and term(s) in x^2.
	$= [1] - 8x + 24x^2 + [-32x^3 + 16x^4]$	A1		$p = -8$ Accept $-8x$ even within a series.
		A1	3	$q = 24$ Accept $24x^2$ even within a series.

(b)	x term is $\binom{9}{1}2^8 x$	M1		OE
	Coefficient of x term is $= 9\times 2^8 = 2304$ $(=k)$	A1	2	Condone $2304x$
(c)	$(1-2x)^4(2+x)^9 = (1+px+..)(2^9+kx...)$	M1		Uses (**a**) and (**b**) oe (PI)
	$= \ldots$ $= \ldots + kx + px(2^9) + \ldots$	M1		Multiply the two expansions to get x terms
	Coefficient of x is $k + 512p$ $= 2304 - 4096 = -1792$	A1ft	3	ft on candidate's values of k and p. Condone $-1792x$ SC If 0/3 award B1ft for $p+k$ evaluated
	Total		**8**	

4. Jan 2007

7(a)	$(1+2x)^8$ $=1+\binom{8}{1}(2x)^1+\binom{8}{2}(2x)^2+\binom{8}{3}(2x)^3+$	M1		Any valid method. PI by correct value for a, b or c
	$= 1 + 16x + 112x^2 + 448x^3 + \ldots.$	A1A1		A1 for each of a, b, c
	$\{a = 16,\ b = 112,\ c = 448\}$	A1	4	
(b)	x^3 terms <u>from expn.</u> of $\left(1+\frac{1}{2}x\right)(1+2x)^8$			
	are cx^3 and $\frac{1}{2}x(bx^2)$	M1		Either
	$cx^3 + \frac{1}{2}x(bx^2)$	A1		b, c or candidate's values for b and c from **(a)**
	Coefficient of x^3 is $c + 0.5\,b = 504$	A1ft	3	Ft on candidate's $(c + 0.5b)$ provided b and c are positive integers >1
	Total		**7**	

5. Jan 2008

6(a)(i)	$(1+x)^3 = 1 + 3x + 3x^2 + x^3$	M1		Any valid method to expand $(1+x)^3$ fully
		A1	2	
(ii)	$(1+x)^4 = 1 + 4x + 6x^2 + 4x^3 + x^4$	M1		Any valid method to expand $(1+x)^4$ fully
		A1	2	
(b)(i)	$(1+4x)^3 = 1 + 3(4x) + 3(4x)^2 + (4x)^3$	M1		
	$(1+4x)^3 = 1 + 12x + 48x^2 + 64x^3$	A1√	2	Ft on one numerical slip in (a)(i)
(ii)	$(1+3x)^4$ $= 1 + 4(3x) + 6(3x)^2 + 4(3x)^3 + (3x)^4$	M1		
	$= 1 + 12x + 54x^2 + 108x^3 + 81x^4$	A1√	2	Ft on one numerical slip in (a)(ii)

(c)	$(1+3x)^4-(1+4x)^3 = 1 + 12x + 54x^2 + 108x^3 + 81x^4 - (1+12x+48x^2+64x^3)$	M1		Subtracts the answers to (b) with correct number of terms and combines at least two pairs of like terms.
	$= 6x2 + 44x3 + 81x4$	A1	2	CAO **SC**: If no attempt in (b) but full expansions given in working for (c), mark retrospectively.
	Total		**10**	

6. June 2008

7(a)	$\left(1+\frac{4}{x^2}\right)^3 =$			
	$\left[1^3\right]+3(1^2)\left(\frac{4}{x^2}\right)+3(1)\left(\frac{4}{x^2}\right)^2+\left[\left(\frac{4}{x^2}\right)^3\right]$	M1		Any valid method as far as term(s) in $1/x^2$ and term(s) in $1/x^4$
	$= [1]+\frac{12}{x^2}+\frac{48}{x^4}+\left[\frac{64}{x^6}\right]$	A1		$p = 12$ Accept $\frac{12}{x^2}$ even within a series
		A1	3	$q = 48$ Accept $\frac{48}{x^4}$ even within a series
(b)(i)	$\int\left(1+\frac{4}{x^2}\right)^3 \mathrm{d}x$			
	$=\int\ (1+\frac{p}{x^2}+\frac{q}{x^4}+\frac{64}{x^6})\ \mathrm{d}x$	M1		Integral of an 'expansion', at least 3 terms PI by the next line
	$= x - px^{-1} - \frac{q}{3}x^{-3} - \frac{64}{5}x^{-5}\ (+c)$	m1 A2F,1	4	At least two powers correctly obtained Ft on c's non-zero integer values for p and q (A1F for two terms correct; can be unsimplified) Condone missing c but check that signs have been simplified at some stage before the award of both A marks.
	$= x - 12x^{-1} - 16x^{-3} - \frac{64}{5}x^{-5}\ (+c)$			
(ii)	$\left(2-\frac{p}{2}-\frac{q}{3(8)}-\frac{64}{5(32)}\right)-$			
	$\left(1-p-\frac{q}{3}-\frac{64}{5}\right)$	M1		F(2) − F(1), where F(x) is cand's answer or the correct answer to (b)(i).
	$= 33.4$	A1	2	CSO
	Total		**9**	

7. Jan 2009

5(a)	$(1+2x)^4 = 1+4(2x)+6(2x)^2+4(2x)^3+(2x)^4$	M1		(1), 4, 6, 4, (1) OE unsimplified with correct powers of x Algebraic multiplication must be a full method
	$= 1 + 8x$	A1		Accept $a = 8$ provided 1st term is 1
	$+ 24x^2$	A1		$b = 24$
	$+ 32x^3\ \{+ 16x^4\}$	A1	4	$c = 32$
(b)	$(1-2x)^4 = 1 - 8x + 24x^2 - 32x^3\ \{+ 16x^4\}$	M1 A1ft		Replace x by $-x$ even in M1 line of (a) PI ft c's non zero values for a, b and c

	$(1+2x)^4+(1-2x)^4$ $=1+8x+24x^2+32x^3+16x^4$ $+1-8x+24x^2-32x^3+16x^4$ $=2+48x^2+32x^4$	A1cso	3	AG Be convinced
(c)	$\frac{dy}{dx}=96x+128x^3$	M1		A correct power of x OE
	For st. pt. $96x+128x^3=0$	A1		
	$32x(3+4x^2)=0$ Since $3+4x^2>0$ there is only one stationary point	E1		Any valid explanation of curve having just one stationary point
	The coordinates of the stationary point are (0, 2)	B1	4	(0, 2) as the **only** stationary point
	Total		**11**	

8. Jan 2010

7(a)	$(1+2x)^7$ $=1+\binom{7}{1}(2x)^1+\binom{7}{2}(2x)^2+\binom{7}{3}(2x)^3+$	M1		Any valid method. PI by a correct value for either a or b or c
	$=1+14x+84x^2+280x^3+\ldots$ $\{a=14,\ b=84,\ c=280\}$	A1 × 3	4	A1 for each of a, b, c **SC** $a=7,\ b=21,\ c=35$ either explicitly or within expn (M1A0)
(b)	$\left(1-\frac{1}{2}x\right)^2=1-x+\frac{1}{4}x^2$	B1		Correct expansion stated explicitly or used later
(b)	$\left(1-\frac{1}{2}x\right)^2=1-x+\frac{1}{4}x^2$	B1		Correct expansion stated explicitly or used later
	x^3 terms from expn of $\left(1-\frac{1}{2}x\right)^2(1+2x)^7$ are cx^3 and $-x(bx^2)$ and $\frac{1}{4}x^2(ax)$	M1		Any one of the three, or ft on c's non-zero values for a, b or c. Must be from products of terms using c's two expansions
	$cx^3-x(bx^2)+\frac{1}{4}x^2(ax)$	A1F		ft c's two expansions provided all three combinations of terms are present
	Coefficient of x^3 is $c-b+0.25a=199.5$	A1	4	OE eg 399/2 Condone $199.5x^3$
	Total		**8**	

9. Jan 2011

5(a)	$(1-x)^3=1-3x+3x^2-x^3$	M1 A1	2	3 terms correct or 1 (±)3 (±)3 (±)1 seen All correct
(b)	$(1+y)^4=1+4y+6y^2+4y^3+y^4$	M1 A1		4 terms correct, accept unsimplified All 5 terms correct and simplified at some stage
	$(1+y)^4-(1-y)^3=$ $(4y+3y)+(6y^2-3y^2)+(4y^3+y^3)+y^4$ $=7y+3y^2+5y^3+y^4$ (as required with p=3 and q=5)	A2,1	4	A2 Be convinced as part answer is given (A1 for three terms found correctly or if found correct values for p and q but did not show $7y+y^4$.)

(c)	$\int\left[\left(1+\sqrt{x}\right)^4-\left(1-\sqrt{x}\right)^3\right]\,dx=$			
	$\int\left(7\sqrt{x}+3x+5x\sqrt{x}+x^2\right)\,dx$	M1		Use of part (b)….$y\to\sqrt{x}$ OE before any integration
	$\int\left(7x^{0.5}+3x+5x^{1.5}+x^2\right)\,dx$			
	$=\frac{7x^{1.5}}{1.5}+\frac{3x^2}{2}+\frac{5x^{2.5}}{2.5}+\frac{x^3}{3}\ (+c)$	m1		Correct integration of an x^k term where k is non-integer
	$=\frac{14}{3}x^{1.5}+\frac{3}{2}x^2+2x^{2.5}+\frac{1}{3}x^3\ (+c)$	A2,1F	4	Coeffs simplified; condone absent $(+c)$ Ft on c's p and q ie 2nd term $+\frac{p}{2}x^2$ and 3rd term is $+\frac{2q}{5}x^{2.5}$. (A1F for three of these four ft terms or for four correct ft terms unsimplified)
	Total		**10**	

10.Jan 2012

5(a)(i)	Stretch(**I**) in x-direction(**II**) scale factor $\frac{1}{6}$ (**III**)	M1 A1	 2	Need (**I**) **and** either (**II**) or (**III**) Need (**I**) and (**II**) and (**III**)
(ii)	$(g(x)=)=\left(1+\frac{x-3}{3}\right)^6$	M1		OE Replaces $\frac{x}{3}$ by $\frac{x-3}{3}$
	$=\left(\frac{x}{3}\right)^6$ or $\frac{x^6}{3^6}$ or $\frac{x^6}{729}$	A1	2	Must be simplified
(b)	$\left(1+\frac{x}{3}\right)^6=1+\binom{6}{1}\frac{x}{3}+\binom{6}{2}\left(\frac{x}{3}\right)^2+\binom{6}{3}\left(\frac{x}{3}\right)^3$			
	…			
	$=(1+)\ 2x$	B1		a=2. Condone '2x'
	$+\frac{6!}{4!2!}\left(\frac{x}{3}\right)^2+\frac{6!}{3!3!}\left(\frac{x}{3}\right)^3$	M1		Either (1 6) 15 20 seen or $\binom{6}{2}$, $\binom{6}{3}$ written (PI) in terms of factorials (OE)
	$=(1+2x)$ $+\frac{15}{9}x^2+\frac{20}{27}x^3$			
	(a=2) $b=\frac{5}{3},\ c=\frac{20}{27}$	A1 A1	 4	$b=\frac{5}{3}$ (or $1\frac{2}{3}$). Condone …$+\frac{5}{3}x^2$ $c=\frac{20}{27}$. Condone …$+\frac{20}{27}x^3$ Accept equivalent recurring decimals Ignore terms with higher powers of x **SC** If A0A0 award A1 for either $+15\frac{x^2}{9}$, $+20\frac{x^3}{27}$ seen or $+\frac{15x^2}{9}$, $+\frac{20x^3}{27}$ seen
	Total		**8**	

11.Jan 2013

8(a)	$\left(1+\frac{4}{x}\right)^2 = 1+\frac{8}{x}+\frac{16}{x^2}$ (or $1+8x^{-1}+16x^{-2}$)	B1	1	Unsimplified equivalent answers, e.g. $1+\frac{4}{x}+\frac{4}{x}+\left(\frac{4}{x}\right)^2$ etc. must be correctly simplified in part (c) to one of the two forms in 'solution' to retrospectively score the B1 here
(b)	$\left(1+\frac{x}{4}\right)^8 = \{1+\}\binom{8}{1}\left(\frac{x}{4}\right)+\binom{8}{2}\left(\frac{x}{4}\right)^2+\binom{8}{3}\left(\frac{x}{4}\right)^3+\ldots$	M1		Any valid method. PI by a correct value for either a or b or c
	$=\{1+\}2x+\frac{7}{4}x^2+\frac{7}{8}x^3+\ldots$	A1A1A1		A1 for each of a, b, c
	$\{a=2,\ b=1.75$ OE, $c=0.875$ OE$\}$		4	**SC** $a=8,\ b=28,\ c=56$ or $a=32,\ b=448,\ c=3584$ either explicitly or within expn (M1A0)
(c)	$(1+\frac{8}{x}+\frac{16}{x^2})\left(1+2x+\frac{7}{4}x^2+\frac{7}{8}x^3\right)$	M1		Product of c's two expansions either stated explicitly or used
	x terms from expansion of $\left(1+\frac{4}{x}\right)^2\left(1+\frac{x}{4}\right)^8$ are ax and '8'bx and '16'cx	m1		Any **two** of the three, **ft** from products of non-zero terms using c's two expansions. May just use the coefficients.
	ax + '8'bx + '16'cx	A1F		Ft on c's non-zero **values** for a, b **and** c and also ft on c's non-zero coeffs. of $1/x$ and $1/x^2$ in part (a). Accept x's missing i.e. sum of coeffs. PI by the correct final answer.
	Coefficient of x is 2+14+14 = 30	A1	4	OE Condone answer left as $30x$. Ignore terms in other powers of x in the expansion.
	Total		**9**	