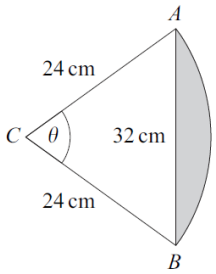
Practice examination questions for radian measure

1. Jan 2005

2 The diagram shows a triangle ABC and the arc AB of a circle whose centre is C and whose radius is 24 cm.



The length of the side AB of the triangle is 32 cm. The size of the angle ACB is θ radians.

(a) Show that $\theta = 1.46$ correct to three significant figures. *(3 marks)*

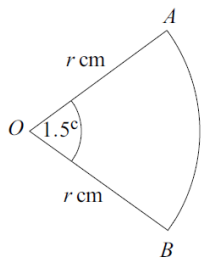
(b) Calculate the length of the arc AB to the nearest cm. *(2 marks)*

(c) (i) Calculate the area of the sector ABC to the nearest cm^2. *(2 marks)*

(ii) Hence calculate the area of the shaded segment to the nearest cm^2. *(3 marks)*

2. June 2005

2 The diagram shows a sector OAB of a circle with centre O and radius r cm.



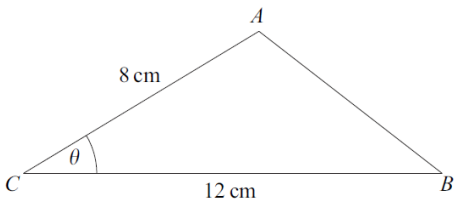
The angle AOB is 1.5 radians. The perimeter of the sector is 56 cm.

(a) Show that $r = 16$. *(3 marks)*

(b) Find the area of the sector. *(2 marks)*

3. Jan 2006

4 The triangle ABC, shown in the diagram, is such that $AC = 8$ cm, $CB = 12$ cm and angle $ACB = \theta$ radians.

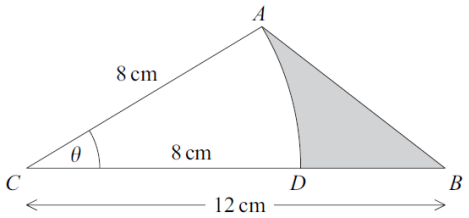


The area of triangle $ABC = 20\,\text{cm}^2$.

(a) Show that $\theta = 0.430$ correct to three significant figures. *(3 marks)*

(b) Use the cosine rule to calculate the length of AB, giving your answer to two significant figures. *(3 marks)*

(c) The point D lies on CB such that AD is an arc of a circle centre C and radius 8 cm. The region bounded by the arc AD and the straight lines DB and AB is shaded in the diagram.



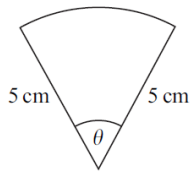
Calculate, to two significant figures:

(i) the length of the arc AD; *(2 marks)*

(ii) the area of the shaded region. *(3 marks)*

4. June 2006

1 The diagram shows a sector of a circle of radius 5 cm and angle θ radians.



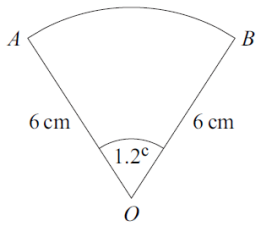
The area of the sector is $8.1\,\text{cm}^2$.

(a) Show that $\theta = 0.648$. *(2 marks)*

(b) Find the perimeter of the sector. *(3 marks)*

5. Jan 2007

1 The diagram shows a sector OAB of a circle with centre O.



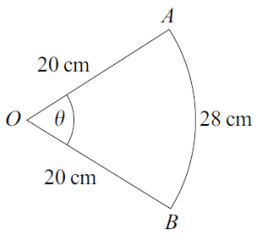
The radius of the circle is 6 cm and the angle AOB is 1.2 radians.

(a) Find the area of the sector OAB. *(2 marks)*

(b) Find the perimeter of the sector OAB. *(3 marks)*

6. June 2007

3 The diagram shows a sector OAB of a circle with centre O and radius 20 cm. The angle between the radii OA and OB is θ radians.



The length of the arc AB is 28 cm.

(a) Show that $\theta = 1.4$. *(2 marks)*

(b) Find the area of the sector OAB. *(2 marks)*

(c) The point D lies on OA. The region bounded by the line BD, the line DA and the arc AB is shaded.

7. Jan 2008

1 The diagrams show a rectangle of length 6 cm and width 3 cm, and a sector of a circle of radius 6 cm and angle θ radians.

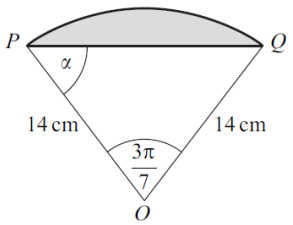
The area of the rectangle is twice the area of the sector.

(a) Show that $\theta = 0.5$. *(3 marks)*

(b) Find the perimeter of the sector. *(3 marks)*

8. June 2008

2 The diagram shows a shaded segment of a circle with centre O and radius 14 cm, where PQ is a chord of the circle.



In triangle OPQ, angle $POQ = \frac{3\pi}{7}$ radians and angle $OPQ = \alpha$ radians.

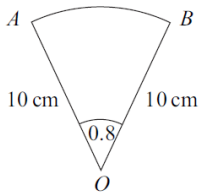
(a) Find the length of the arc PQ, giving your answer as a multiple of π. *(2 marks)*

(b) Find α in terms of π. *(2 marks)*

(c) Find the **perimeter** of the shaded segment, giving your answer to three significant figures. *(2 marks)*

9. Jan 2009

1 The diagram shows a sector OAB of a circle with centre O and radius 10 cm.



The angle AOB is 0.8 radians.

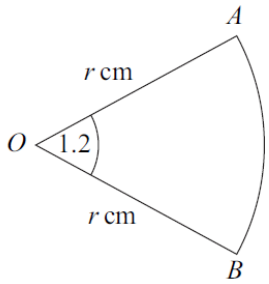
(a) Find the area of the sector. *(2 marks)*

(b) (i) Find the perimeter of the sector OAB. *(3 marks)*

(ii) The perimeter of the sector OAB is equal to the perimeter of a square. Find the area of the square. *(2 marks)*

10.June 2009

6 The diagram shows a sector OAB of a circle with centre O and radius r cm.

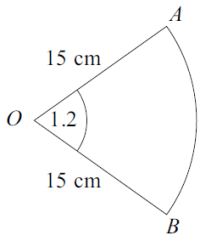


The angle AOB is 1.2 radians. The area of the sector is $33.75\,\text{cm}^2$.

Find the perimeter of the sector. *(6 marks)*

11.Jan 2010

1 The diagram shows a sector OAB of a circle with centre O.

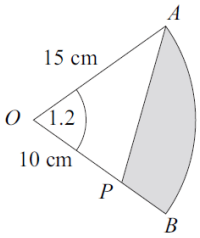


The radius of the circle is 15 cm and angle $AOB = 1.2$ radians.

(a) (i) Show that the area of the sector is $135\,\text{cm}^2$. *(2 marks)*

(ii) Calculate the length of the arc AB. *(2 marks)*

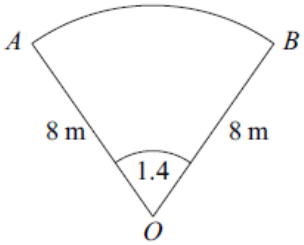
(b) The point P lies on the radius OB such that $OP = 10$ cm, as shown in the diagram below.



Calculate the perimeter of the shaded region bounded by AP, PB and the arc AB, giving your answer to three significant figures. *(5 marks)*

12.June 2010

1 The diagram shows a sector OAB of a circle with centre O.



The radius of the circle is 8 m and the angle AOB is 1.4 radians.

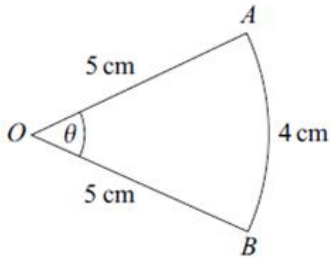
(a) Find the area of the sector OAB. *(2 marks)*

(b) (i) Find the perimeter of the sector OAB. *(3 marks)*

(ii) The perimeter of the sector OAB is equal to the circumference of a circle of radius x m. Calculate the value of x to three significant figures. *(2 marks)*

13.Jan 2011

1 The diagram shows a sector OAB of a circle with centre O and radius 5 cm.



The angle between the radii OA and OB is θ radians.

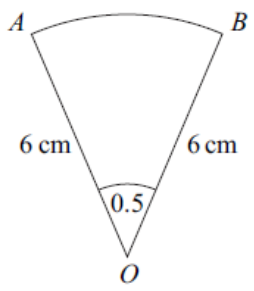
The length of the arc AB is 4 cm.

(a) Find the value of θ. *(2 marks)*

(b) Find the area of the sector OAB. *(2 marks)*

14.June 2011

2 The diagram shows a sector OAB of a circle with centre O.



The radius of the circle is 6 cm and the angle $AOB = 0.5$ radians.

(a) Find the area of the sector OAB. *(2 marks)*

(b) (i) Find the length of the arc AB. *(2 marks)*

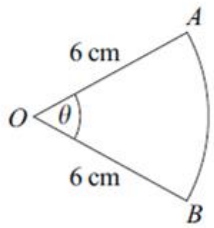
(ii) Hence show that

the perimeter of the sector $OAB = k \times$ the length of the arc AB

where k is an integer. *(2 marks)*

15.Jan 2012

1 The diagram shows a sector OAB of a circle with centre O and radius 6 cm.



The angle between the radii OA and OB is θ radians.

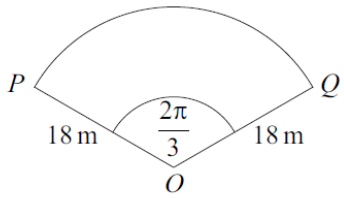
The area of the sector OAB is $21.6\,\text{cm}^2$.

(a) Find the value of θ. *(2 marks)*

(b) Find the length of the arc AB. *(2 marks)*

16.June 2012

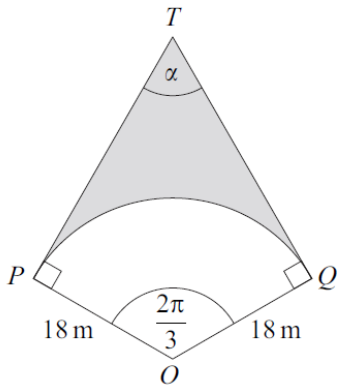
5 The diagram shows a sector OPQ of a circle with centre O.



The radius of the circle is 18 m and the angle POQ is $\frac{2\pi}{3}$ radians.

(a) Find the length of the arc PQ, giving your answer as a multiple of π. *(2 marks)*

(b) The tangents to the circle at the points P and Q meet at the point T, and the angles TPO and TQO are both right angles, as shown in the diagram below.

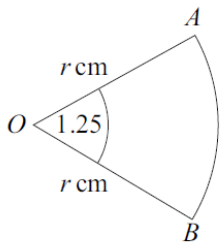


(i) Angle $PTQ = \alpha$ radians. Find α in terms of π. *(1 mark)*

(ii) Find the area of the shaded region bounded by the arc PQ and the tangents TP and TQ, giving your answer to three significant figures. *(6 marks)*

17.Jan 2013

1 The diagram shows a sector OAB of a circle with centre O and radius r cm.



The angle AOB is 1.25 radians. The perimeter of the sector is 39 cm.

(a) Show that $r = 12$. *(3 marks)*

(b) Calculate the area of the sector OAB. *(2 marks)*

Answers

1. Jan 2005

2(a)	$32^2 = 24^2 + 24^2 - 2\times 24\times 24\cos\theta$	M1		
	or $\sin\frac{1}{2}\theta = \frac{\frac{1}{2}(32)}{24}$			
	$\cos\theta = \frac{24^2+24^2-32^2}{2\times 24\times 24}$	m1		
	$\ldots\ldots\ldots = \frac{128}{1152}\{=\frac{1}{9}\}\{=0.11\}$			
	or $\frac{1}{2}\theta = \sin^{-1}\left(\frac{2}{3}\right)$ (= 0.7297..)			
	$\theta = 1.459\ldots = 1.46$ to 3sf	A1	3	CSO AG (be convinced)
(b)	Arc $= r\theta$	M1		
	$= 24 \times 1.459\ldots = 35$ cm	A1	2	Condone absent cm; 35 to 35.04
(c)(i)	Area of sector $= \frac{1}{2}r^2\theta$	M1		Seen
	$= \frac{1}{2}(24)^2(1.459..) = 420.3 = 420$ cm^2	A1	2	Condone absent cm^2; 420 to 420.48
(ii)	Area of triangle $= \frac{1}{2}(24)(24)\sin\theta$	M1		OE
	[= 286. (….)]			
	Shaded area = area of sector – area of triangle	m1		Dep on at least one of the previous two M marks. PI
	$\left[=\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta\right] = 134$ cm^2	A1	3	Condone absent cm^2
	Total		**10**	

2. June 2005

2(a)	Arc $= r\theta$	M1		For $r\theta$ or 16θ or 16×1.5 OE multiplicatio
	$1.5r + r + r$ (= 56)	M1		For realising that perimeter is sum of two radii and arc.
	$3.5r = 56 \Rightarrow r = 16$	A1	3	**AG** Completion (condone verification)
(b)	Area of sector $= \frac{1}{2}r^2\theta$	M1		$\frac{1}{2}r^2\theta$ OE seen
	$= \frac{1}{2}16^2(1.5) = 192$ cm^2.	A1	2	Condone absent cm^2.
	Total		**5**	

3. Jan 2006

	[Note: Calc. set in wrong mode, penalise only once on the paper.] Condone missing units throughout the question.			

4(a)	Area of triangle $=\frac{1}{2}(12)(8)\sin\theta$	M1		Use of $\frac{1}{2}ab\sin C$ or full equivalent
	$\sin\theta=\frac{20}{48}$ [=0.41(666…)]	A1		OE (giving 0.412 to 0.42)
	$\Rightarrow \theta = 0.4297(7\ldots) = 0.430$ to 3sf	A1	3	AG(need to see >3sf value)
(b)	$\{AB^2 =\}8^2+12^2-2\times8\times12\times\cos\theta$	M1		
	$= 64 + 144 - 174.5\ldots.$	m1		Accept 33 to 34 inclusive if three values not separate
	$\Rightarrow AB = 5.78\ldots = 5.8$ cm to 2sf	A1	3	If not 2sf condone 5.78 to 5.79 inclusive. Condone ±
(c)(i)	Arc $AD = 8\theta$;	M1;		
	$= 3.44.. = 3.4$ cm to 2sf	A1	2	If not 2sf condone 3.438 to 3.44 inclusive
(ii)	Area of sector $=\frac{1}{2}r^2\theta$	M1		Stated or used [or 13.7(6..) seen]
	Shaded area = Area of triangle – sector area	M1		Difference of areas
	Shaded area $= 20-0.5\times8^2\times\theta$ $= 6.2\ \text{cm}^2$ to 2sf	A1	3	Condone 6.24 to 6.2472
	Total		**11**	

4. June 2006

1(a)	Area of sector $=\frac{1}{2}r^2\theta=\frac{1}{2}\times5^2\times\theta$	M1		$\frac{1}{2}r^2\theta$ seen or used
	$12.5\theta = 8.1 \Rightarrow \theta = 0.648$	A1	2	AG Condone $\theta = 0.648$ used to show that area = 8.1
(b)	Arc $= 5\theta$;	M1		5θ
	….. = 3.24 cm	A1		PI by a correct perimeter
	$\Rightarrow$ Perimeter = 10 + arc = 13.24 cm	A1√	3	CSO Condone missing/wrong units; condone 3sf i.e. 13.2 if no obvious error NMS 3/3
	Total		**5**	

5. Jan 2007

1(a)	{Area of sector =} $\frac{1}{2}r^2\theta$	M1		
	$=0.5\times36\times1.2=21.6\ \text{cm}^2$	A1	2	Condone missing/wrong units throughout the paper
(b)	Arc $= r\theta$	M1		
	$= 6\times1.2 = 7.2$	A1		
	Perimeter = 12 + 7.2 = 19.2 cm	A1ft	3	Ft on incorrect evaluation of 6×1.2
	Total		**5**	

6. June 2007

3(a)	Arc $= r\theta$	M1		For $r\theta$ or 20θ or PI by 20×1.4
	$28 = 20\theta \Rightarrow \theta = 1.4$	A1	2	AG
(b)	Area of sector $=\frac{1}{2}r^2\theta$	M1		$\frac{1}{2}r^2\theta$ OE seen
	$=\frac{1}{2}20^2(1.4)=280\ (\text{cm}^2.)$	A1	2	Condone absent cm^2.

(c)(i)	Area triangle $=\frac{1}{2}\times 15\times 20\times \sin 1.4$ (= 147.8….)	M1		Use of $\frac{1}{2}ab\sin C$ OE
	Shaded area = Area of sector – area of triangle	M1		
	= 280 – 147.8 = 132 (cm^2.) (3sf)	A1ft	3	Ft on [ans (b) – 147.8…] to 3sf provided [….] > 0
(ii)	$\{BD^2 =\}15^2+20^2-2\times 15\times 20\cos 1.4$	M1		RHS of cosine rule used
	$= 225+400-101.98...$	m1		Correct order of evaluation
	$\Rightarrow BD=\sqrt{523.019...}=22.86..$ = 22.9 (cm) to 3 sf	A1	3	Condone absent cm
	Total		**10**	

7. Jan 2008

1(a)	Area of sector $=\frac{1}{2}r^2\theta=\frac{1}{2}\times 6^2\times\theta$	M1		$\frac{1}{2}r^2\theta$ seen or used
	$6\times 3=2\times\frac{1}{2}\times 6^2\times\theta$	m1		OE Forming equation
	$36\theta=18\Rightarrow\theta=0.5$	A1	3	AG
(b)	Arc = 6θ;	M1		$r\theta$ seen or used
	….. = 3 cm	A1		PI by a correct perimeter
	$\Rightarrow$ Perimeter = 12 + arc = 15 cm	A1F	3	Ft wrong evaluation of 6θ. Condone missing/wrong units throughout the question.
	Total		**6**	

8. June 2008

2(a)	Arc $PQ=r\theta$	M1		$r\theta$
	= 6π (cm)	A1	2	Condone missing units throughout the paper
(b)	$\alpha+\alpha+\frac{3\pi}{7}=\pi$	M1		OE
	$\alpha=\frac{2\pi}{7}$	A1	2	Accept equivalent fractions eg $\frac{4\pi}{14}$ and condone 0.286π or better
(c)	Chord $PQ=2\times 14\times\cos\alpha$	M1		OE eg $2\times 14\times\sin\frac{3\pi}{14}$ or 17.45-17.5 inclusive or $\sqrt{14^2+14^2-2\times 14^2\times\cos\frac{3\pi}{7}}$
	Perimeter = 17.45… + 6π = 36.307… = 36.3 (cm)	A1	2	Condone > 3sf
	Total		**6**	

9. Jan 2009

1(a)	{Area of sector =} $\frac{1}{2}r^2\theta$	M1		$\frac{1}{2}r^2\theta$ stated or used for area of sector. PI
	$=\frac{1}{2}\times 10^2\times 0.8=40$ {cm^2}	A1	2	
(b)(i)	{Arc =} $r\theta$	M1		$r\theta$ stated or used for arc length. PI
	…. = 8	A1		PI

	Perimeter = $20 + r\theta = 28$ (cm)	A1ft	3	ft on $20 + r\times\theta$
(ii)	Area of square = $\left[\frac{\text{c's answer for (b)(i)}}{4}\right]^2$	M1		PI
	= 49 {cm^2}	A1cao	2	
	Total		**7**	

10.June 2009

6	{Area of sector =} $\frac{1}{2}r^2\theta$	M1		$\frac{1}{2}r^2\theta$ seen or used for the area; PI
	$r^2 = \frac{33.75}{\frac{1}{2}\theta}$ (= 56.25)	m1		Correct rearrangement to $r^2 = \ldots$ or $r = \ldots$
	$r = 7.5$	A1		PI eg by a correct arc length
	{Arc =} $r\theta$	M1		$r\theta$ seen or used for the arc length
	…. = 9	A1F		ft on 1.2 × cand's r provided the **two** M's scored; if not explicit, PI by ft on 3.2 × cand's r for perimeter
	{Perimeter =} 24 {cm}	A1	6	CAO
	Total		**6**	

11.Jan 2010

1(a)(i)	{Area of sector =} $\frac{1}{2}r^2\theta$	M1		Stated or explicitly used
	$= \frac{1}{2}\times 15^2 \times 1.2 = 135$ (cm^2)	A1	2	AG Must see some substitution
(ii)	{Arc =} $r\theta$	M1		PI
	…. = 18 (cm)	A1	2	
(b)	$PB = 5$ (cm)	B1		Accept even if only on a diagram or within an expression for the perimeter
	{AP^2 =} $15^2 + 10^2 - 2\times 15\times 10\cos 1.2$	M1		RHS of cosine rule used
	$= 325 - 300\cos 1.2 = 216.2926\ldots$	m1		Correct order of evaluation
	$AP = 14.7(068..)$	A1		PI eg within an expression for perimeter
	Perimeter = 5 + 18 + 14.7…. = 37.7 (cm)	A1	5	3sf or better
	Total		**9**	

12.June 2010

1(a)	{Area of sector =} $\frac{1}{2}r^2\theta$	M1		$\frac{1}{2}r^2\theta$ seen or used for the area
	$= \frac{1}{2}\times 8^2 \times 1.4 = 44.8$ {m^2}	A1	2	Must be exact, not rounded to
(b)(i)	{Arc =} $r\theta$	M1		$r\theta$ seen or used for the arc length
	…. = 11.2	A1		PI Condone AWRT 11.2
	Perimeter of sector = 16+11.2 = 27.2 {m}	A1F	3	Ft on c's evaluation of 8×1.4
(ii)	$27.2 = 2\pi x$	M1		[c's numerical answer for (b)(i)] = $2\pi x$
	$x = \frac{27.2}{2\pi} = 4.329\ldots = 4.33$ to 3sf	A1	2	Condone >3sf
	Total		**7**	

13.Jan 2011

1(a)	Arc $= r\theta$	M1		arc $= r\theta$ seen or used. PI by correct θ
	$4 = 5\theta \Rightarrow \theta = \frac{4}{5} = 0.8$	A1	2	$(\theta =)\ \frac{4}{5}$ OE
(b)	Area of sector $= \frac{1}{2}r^2\theta$	M1		Area $= \frac{1}{2}r^2\theta$ seen or used within **(b)**. PI
	$= \frac{1}{2}\times 5^2 \times 0.8 = 10\ (\text{cm}^2)$	A1F	2	Ft on 12.5×c's exact value for θ in part (a) provided $5 \le$ c's area ≤ 20
	Total		**4**	

14.June 2011

Q	Solution	Marks	Total	Comments
2(a)	{Area of sector =} $\frac{1}{2}r^2\theta = \frac{1}{2}\times 6^2 \times 0.5$	M1		$\frac{1}{2}r^2\theta$ seen within (a) or used for the area
	$= 9\ (\text{cm}^2)$	A1	2	Condone missing/incorrect units
(b)(i)	{Arc =} $r\theta = 6\times 0.5$	M1		$r\theta$ seen within (b) or used for the arc length
	$= 3$ (cm)	A1	2	Condone missing/incorrect units
(ii)	Perimeter of sector = 6+6+arc length	M1		PI by value of 12+c's (b)(i) answer
	= 15 (cm) (= 5×3) Perimeter (of sector) = 5×(length of) arc	A1	2	Completion, including concluding statement
	Total		**6**	

15.Jan 2012

1(a)	{Area of sector =} $\frac{1}{2}r^2\theta = \frac{1}{2}\times 6^2 \times \theta$	M1		$\frac{1}{2}r^2\theta$ seen in **(a)** or used for the area
	$21.6 = 18\theta$ so $\theta = 1.2$	A1	2	Must be exact, not rounded to
(b)	{Arc =} $r\theta$	M1		$r\theta$ seen in **(b)** or used for the arc length
	…. = 7.2 {cm}	A1F	2	Ft on 6×c's value for θ provided 4<arc<10.
	Total		**4**	

16.June 2012

5(a)	{Arc =} $r\theta$	M1		$r\theta$ seen or used for the arc length
	$= 18\times\frac{2\pi}{3} = 12\pi$ (m)	A1	2	12π
(b)(i)	$\alpha = \frac{\pi}{3}$	B1	1	$\frac{1}{3}\pi$ OE expression which simplifies to $\frac{1}{3}\pi$

(ii)	{Area of sector =} $\frac{1}{2}r^2\theta = \frac{1}{2}\times 18^2 \times \frac{2\pi}{3}$	M1		$\frac{1}{2}r^2\theta$ seen or used for the sector area If not exact accept 3sf or better
	$= 108\pi$ (=339.(29..))	A1		PI by final correct answer
	$\tan\frac{\pi}{3} = \frac{TP}{18}$ {or $\tan\frac{\alpha}{2} = \frac{18}{TP}$} {or $PQ = 2\times 18\sin\frac{\pi}{3}$} {or $\frac{1}{2}PQ = 18\sin\frac{\pi}{3}$} $\left\{\text{or } \cos\frac{\pi}{3} = \frac{18}{OT}\right\}$ $\left\{\text{or } \sin\frac{\alpha}{2} = \frac{18}{OT}\right\}$	M1		OE Correct method (PI) to find either TP or TQ (=TP) or OT or PQ or $\frac{1}{2}PQ$. If α not $\pi/3$ then ft c's value for α in (b)(i). If c finds two of TP/TQ, OT and $PQ/\frac{1}{2}PQ$ and gets one correct, one wrong, mark correct one ie M1A1 (M1A0 possible if no correct length)
	$TP = 18\sqrt{3}$ =31.1769… exact or 31.1 to 31.2 incl} {or $PQ = 18\sqrt{3}$ =31.1769... exact or 31.1 to 31.2 incl} {or $OT = 36$}; {$\frac{1}{2}PQ = 9\sqrt{3}$ or 15.5 to 15.6 incl}	A1		Correct TP or TQ or PQ or $\frac{1}{2}PQ$ or OT either exact value or in range indicated PI by value 561 to 561.3 inclusive for the area of the kite.
	Area of kite $PTQO = 2\times\frac{1}{2}\times 18\times TP$ {or Area $= \frac{1}{2}(18^2)\sin\frac{2\pi}{3} + \frac{1}{2}TP^2\sin\alpha$} {or area kite $= \frac{1}{2}\times PQ\times\left[18\div\cos\frac{\pi}{3}\right]$} {or area kite $= \frac{1}{2}\times 2\times 18\sin\frac{\pi}{3}\times OT$} {$=18^2\sqrt{3}$} {$= 2\times 162\sqrt{3}$}; {$243\sqrt{3} + 81\sqrt{3}$}	M1		OE valid method to find area of kite, down to a correct expression with no more than 1 unknown length; ft on c's value of α. For method using > one unknown length this M is dependent on previous M for length PI by value $324\sqrt{3}$ or a numerical expression which simplifies to $324\sqrt{3}$; or a value 561 to 561.3 inclusive for the area of the kite. Can also be implied by award of the final A1
	Alternative Area triangle $PTQ = \frac{1}{2}TP^2\sin\alpha$ and Area triangle $POQ = \frac{1}{2}18^2\sin(2\pi/3)$	(M1)		OE Alternative: Award this method mark if **both** area of triangle PTQ (=$243\sqrt{3}$) and area of triangle POQ (=$81\sqrt{3}$) are found with or without finding area of kite
	Area of shaded region = 561.(18...) -108π = 221.89… = 222 (m^2) to 3sf	A1	6	If not 222, condone value from 221.7 to 222.0 inclusive
	Alternative Area of shaded region = $243\sqrt{3} - (108\pi - 81\sqrt{3})$ = 221.89…= 222 (m^2) to 3sf	(A1)	(6)	
	Total		**9**	

17.Jan 2013

1(a)	Arc = $r\theta$ (= 1.25r)	M1		Within (a), $r\theta$ or 15 used for the arc length PI
	P = $r + r + r\theta$ =39	m1		Use of $r + r + r\theta$ for the perimeter. m0 if no indication that '15' comes from $r\theta$.
	$3.25r = 39$ $r = \frac{39}{3.25} = 12$	A1	3	CSO AG
(b)	{Area of sector =} $\frac{1}{2}r^2\theta$	M1		Within (b), $\frac{1}{2}r^2\theta$ stated or used for the sector area.
	$= \frac{1}{2}\times 12^2\times 1.25$ = 90 (cm^2)	A1	2	NMS: 90 scores 2 marks
	Total		**5**	