Practice examination questions for logarithms

1. Jan 2005

5 (a) Given that

$$\log_a x = 3\log_a 6 - \log_a 8$$

where a is a positive constant, show that $x = 27$. *(3 marks)*

(b) Write down the value of:

(i) $\log_4 1$; *(1 mark)*

(ii) $\log_4 4$; *(1 mark)*

(iii) $\log_4 2$; *(1 mark)*

(iv) $\log_4 8$. *(1 mark)*

2. June 2005

6 (a) Using the binomial expansion, or otherwise, express $(1+x)^4$ in ascending powers of x. *(3 marks)*

(b) (i) Hence show that $(1+\sqrt{5})^4 = 56 + 24\sqrt{5}$. *(3 marks)*

(ii) Hence show that $\log_2(1+\sqrt{5})^4 = k + \log_2(7+3\sqrt{5})$, where k is an integer. *(3 marks)*

3. Jan 2006

3 (a) Use logarithms to solve the equation $0.8^x = 0.05$, giving your answer to three decimal places. *(3 marks)*

(b) An infinite geometric series has common ratio r. The sum to infinity of the series is five times the first term of the series.

(i) Show that $r = 0.8$. *(3 marks)*

(ii) Given that the first term of the series is 20, find the least value of n such that the nth term of the series is less than 1. *(3 marks)*

4. Jan 2006

7 It is given that n satisfies the equation

$$2\log_a n - \log_a(5n-24) = \log_a 4$$

(a) Show that $n^2 - 20n + 96 = 0$. *(3 marks)*

(b) Hence find the possible values of n. *(2 marks)*

5. June 2006

5 (a) Given that

$$\log_a x = 2\log_a 6 - \log_a 3$$

show that $x = 12$. *(3 marks)*

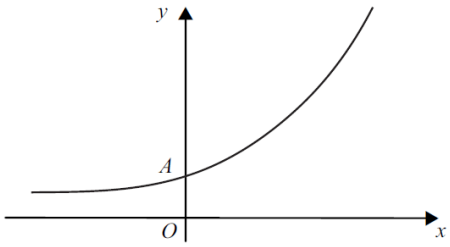
(b) Given that

$$\log_a y + \log_a 5 = 7$$

express y in terms of a, giving your answer in a form not involving logarithms. *(3 marks)*

6. June 2007

6 The diagram shows a sketch of the curve with equation $y = 3(2^x + 1)$.



The curve $y = 3(2^x + 1)$ intersects the y-axis at the point A.

(a) Find the y-coordinate of the point A. *(2 marks)*

(b) Use the trapezium rule with four ordinates (three strips) to find an approximate value for $\int_0^6 3(2^x + 1)\,dx$. *(4 marks)*

(c) The line $y = 21$ intersects the curve $y = 3(2^x + 1)$ at the point P.

(i) Show that the x-coordinate of P satisfies the equation

$$2^x = 6$$ *(1 mark)*

(ii) Use logarithms to find the x-coordinate of P, giving your answer to three significant figures. *(3 marks)*

7. June 2007

8 (a) It is given that n satisfies the equation

$$\log_a n = \log_a 3 + \log_a(2n - 1)$$

Find the value of n. *(3 marks)*

(b) Given that $\log_a x = 3$ and $\log_a y - 3\log_a 2 = 4$:

(i) express x in terms of a; *(1 mark)*

(ii) express xy in terms of a. *(4 marks)*

8. Jan 2008

7 (a) Given that

$$\log_a x = \log_a 16 - \log_a 2$$

write down the value of x. *(1 mark)*

(b) Given that

$$\log_a y = 2\log_a 3 + \log_a 4 + 1$$

express y in terms of a, giving your answer in a form **not** involving logarithms. *(3 marks)*

9. Jan 2008

8 (a) Sketch the graph of $y = 3^x$, stating the coordinates of the point where the graph crosses the y-axis. *(2 marks)*

(b) Describe a single geometrical transformation that maps the graph of $y = 3^x$:

(i) onto the graph of $y = 3^{2x}$; *(2 marks)*

(ii) onto the graph of $y = 3^{x+1}$. *(2 marks)*

(c) (i) Using the substitution $Y = 3^x$, show that the equation

$$9^x - 3^{x+1} + 2 = 0$$

can be written as

$$(Y - 1)(Y - 2) = 0$$ *(2 marks)*

(ii) Hence show that the equation $9^x - 3^{x+1} + 2 = 0$ has a solution $x = 0$ and, by using logarithms, find the other solution, giving your answer to four decimal places. *(4 marks)*

10. June 2008

5 (a) Write down the value of:

(i) $\log_a 1$; *(1 mark)*

(ii) $\log_a a$. *(1 mark)*

(b) Given that

$$\log_a x = \log_a 5 + \log_a 6 - \log_a 1.5$$

find the value of x. *(3 marks)*

11. Jan 2009

6 (a) Write each of the following in the form $\log_a k$, where k is an integer:

(i) $\log_a 4 + \log_a 10$; *(1 mark)*

(ii) $\log_a 16 - \log_a 2$; *(1 mark)*

(iii) $3\log_a 5$. *(1 mark)*

(b) Use logarithms to solve the equation $(1.5)^{3x} = 7.5$, giving your value of x to three decimal places. *(3 marks)*

(c) Given that $\log_2 p = m$ and $\log_8 q = n$, express pq in the form 2^y, where y is an expression in m and n. *(3 marks)*

12. June 2009

9 (a) (i) Find the value of p for which $\sqrt{125} = 5^p$. *(2 marks)*

(ii) Hence solve the equation $5^{2x} = \sqrt{125}$. *(1 mark)*

(b) Use logarithms to solve the equation $3^{2x-1} = 0.05$, giving your value of x to four decimal places. *(3 marks)*

(c) It is given that

$$\log_a x = 2(\log_a 3 + \log_a 2) - 1$$

Express x in terms of a, giving your answer in a form not involving logarithms. *(4 marks)*

13. Jan 2010

3 (a) Find the value of x in each of the following:

(i) $\log_9 x = 0$; *(1 mark)*

(ii) $\log_9 x = \frac{1}{2}$. *(1 mark)*

(b) Given that

$$2\log_a n = \log_a 18 + \log_a(n-4)$$

find the possible values of n. *(5 marks)*

14. Jan 2011

8 (a) Given that $2\log_k x - \log_k 5 = 1$, express k in terms of x. Give your answer in a form not involving logarithms. *(4 marks)*

(b) Given that $\log_a y = \frac{3}{2}$ and that $\log_4 a = b + 2$, show that $y = 2^p$, where p is an expression in terms of b. *(3 marks)*

15. June 2011

4 (a) Sketch the curve with equation $y = 4^x$, indicating the coordinates of any point where the curve intersects the coordinate axes. *(2 marks)*

(b) Describe the geometrical transformation that maps the graph of $y = 4^x$ onto the graph of $y = 4^x - 5$. *(2 marks)*

(c) (i) Use the substitution $Y = 2^x$ to show that the equation $4^x - 2^{x+2} - 5 = 0$ can be written as $Y^2 - 4Y - 5 = 0$. *(2 marks)*

(ii) Hence show that the equation $4^x - 2^{x+2} - 5 = 0$ has only one real solution. Use logarithms to find this solution, giving your answer to three decimal places. *(4 marks)*

16. June 2011

9 The first term of a geometric series is 12 and the common ratio of the series is $\frac{3}{8}$.

(a) Find the sum to infinity of the series. *(2 marks)*

(b) Show that the sixth term of the series can be written in the form $\frac{3^6}{2^{13}}$. *(3 marks)*

(c) The nth term of the series is u_n.

(i) Write down an expression for u_n in terms of n. *(1 mark)*

(ii) Hence show that

$$\log_a u_n = n \log_a 3 - (3n - 5) \log_a 2$$ *(4 marks)*

17. Jan 2012

7 (a) Sketch the graph of $y = \frac{1}{2^x}$, indicating the value of the intercept on the y-axis. *(2 marks)*

(b) Use logarithms to solve the equation $\frac{1}{2^x} = \frac{5}{4}$, giving your answer to three significant figures. *(3 marks)*

(c) Given that

$$\log_a(b^2) + 3 \log_a y = 3 + 2 \log_a \left(\frac{y}{a}\right)$$

express y in terms of a and b.

Give your answer in a form not involving logarithms. *(5 marks)*

18. June 2012

8 (a) Sketch the curve with equation $y = 7^x$, indicating the coordinates of any point where the curve intersects the coordinate axes. *(2 marks)*

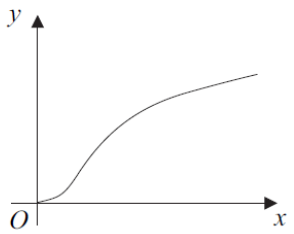
(b) The curve C_1 has equation $y = 7^x$.
The curve C_2 has equation $y = 7^{2x} - 12$.

(i) By forming and solving a quadratic equation, prove that the curves C_1 and C_2 intersect at exactly one point. State the y-coordinate of this point. *(4 marks)*

(ii) Use logarithms to find the x-coordinate of the point of intersection of C_1 and C_2, giving your answer to three significant figures. *(2 marks)*

19. June 2012

9 The diagram shows part of a curve whose equation is $y = \log_{10}(x^2 + 1)$.



(a) Use the trapezium rule with five ordinates (four strips) to find an approximate value for

$$\int_0^1 \log_{10}(x^2 + 1)\,dx$$

giving your answer to three significant figures. *(4 marks)*

(b) The graph of $y = 2\log_{10}x$ can be transformed into the graph of $y = 1 + 2\log_{10}x$ by means of a translation. Write down the vector of the translation. *(1 mark)*

(c) (i) Show that $\log_{10}(10x^2) = 1 + 2\log_{10}x$. *(2 marks)*

(ii) Show that the graph of $y = 2\log_{10}x$ can also be transformed into the graph of $y = 1 + 2\log_{10}x$ by means of a **stretch**, and describe the stretch. *(4 marks)*

(iii) The curve with equation $y = 1 + 2\log_{10}x$ intersects the curve $y = \log_{10}(x^2 + 1)$ at the point P. Given that the x-coordinate of P is positive, find the gradient of the line OP, where O is the origin. Give your answer in the form $\log_{10}\left(\frac{a}{b}\right)$, where a and b are integers. *(4 marks)*

20. Jan 2013

4 Given that

$$\log_a N - \log_a x = \frac{3}{2}$$

express x in terms of a and N, giving your answer in a form not involving logarithms. *(3 marks)*

21. Jan 2013

7 (a) Describe a geometrical transformation that maps the graph of $y = 4^x$ onto the graph of $y = 3 \times 4^x$. *(2 marks)*

(b) Sketch the curve with equation $y = 3 \times 4^x$, indicating the value of the intercept on the y-axis. *(2 marks)*

(c) The curve with equation $y = 4^{-x}$ intersects the curve $y = 3 \times 4^x$ at the point P. Use logarithms to find the x-coordinate of P, giving your answer to three significant figures. *(5 marks)*

Answers

1. Jan 2005

5(a)	$\log_a x = \log_a 6^3 - \log_a 8$ $\log_a x = \log_a (6^3 \div 8)$ $x = 6^3 \div 8 = 27$	M1 M1 A1	3	A law of logs used correctly A different law of logs used correctly CSO AG (be convinced) **ALT** $\log_a x = 3\log_a 6 - 3\log_a 2$ (M1) $\frac{1}{3}\log_a x = \log_a \frac{6}{2}$ (M1) $x^{\frac{1}{3}} = 3 \Rightarrow x = 27$ (A1) CSO
(b)(i)	$\log_4 1 = 0$	B1		
(ii)	$\log_4 4 = 1$	B1		SC in (b): For all four answers ¼; 1; ½ ; 2
(iii)	$\log_4 2 = 0.5$	B1		give 0/4; otherwise mark each
(iv)	$\log_4 8 = 1.5$	B1	4	independently.
	Total		**7**	

2. June 2005

6(a)	$(1+x)^4 = 1 + 4x + 6x^2 + 4x^3 + x^4$	M1 A2,1	3	Full method A1 if four terms correct or just one slip
(b)(i)	$\left(1+\sqrt{5}\right)^4 = 1 + 4\sqrt{5} + 6(\sqrt{5})^2 +$ $+4(\sqrt{5})^3 + (\sqrt{5})^4$	M1		Substitute. √5 for x.
	$= 1 + 4\sqrt{5} + 6(5) + 4(5\sqrt{5}) + (25)$	A1ft		Two of 3 terms shown in brackets
	$\ldots\ldots\ldots = 56 + 24\sqrt{5}$	A1	3	**AG** CSO Be convinced
(ii)	$\log_2 \left(1+\sqrt{5}\right)^4 = \log_2[8(7+3\sqrt{5})]$	M1		
	$= \log_2 8 + \log_2 (7+3\sqrt{5})$	m1		
	$= 3 + \log_2 (7+3\sqrt{5})$	A1	3	CSO **SC** B1 Change to base 10 and verify
	Total		**9**	

3. Jan 2006

3(a)	$\log 0.8^x = \log 0.05$	$x = \log_{0.8} 0.05$ **(M1)**	M1		**NMS**: **SC** B2 for 13.425 or better (B1 for 13.4 or 13.43; 13.42)
	$x \log_{10} 0.8 = \log_{10} 0.05$ oe		A1		
	$x = 13.425$ to 3dp	13.425 **(A2)** **(else A1** for 1 or 2dp**)**	A1	3	Condone greater accuracy

(b)(i)	$\frac{a}{1-r}$	M1		$S_\infty = \frac{a}{1-r}$ **used**
	$\frac{a}{1-r} = 5a \Rightarrow a = 5a(1-r)$	A1		Or better
	$\Rightarrow 1 = 5(1-r) \Rightarrow r = \frac{4}{5} = 0.8$	A1	3	AG (be convinced)
(ii)	n^{th} term $= 20\times(0.8)^{n-1}$	M1		Condone $20\times(0.8)^n$.
	n^{th} term $< 1 \Rightarrow 0.8^{n-1} < \frac{1}{20}$ oe	A1		$0.8^{n-1} < 0.05$ or $0.8^{n-1} = k$, where $k = 0.05$ or k rounds **up** to 0.050
	Least n is 15	A1F	3	If not 15, ft on integer part of [answer (a)+2] provided $n>2$ **SC** 3/3 for 15 if no error **SC** n^{th} term$=16^{n-1}$M1A0A0
	Total		**9**	

4. Jan 2006

7(a)	$2\log_a n - \log_a(5n-24) = \log_a 4$			
	$\Rightarrow \log_a n^2 - \log_a(5n-24) = \log_a 4$	M1		A law of logs used
	$\Rightarrow \log_a\left[\frac{n^2}{5n-24}\right] = \log_a 4$	M1		A second law of logs used leading to both sides being single log terms or single log term on LHS with RHS=0
	$\Rightarrow \frac{n^2}{5n-24} = 4$			
	$\Rightarrow n^2 - 20n + 96 = 0$	A1	3	**CSO.** AG
(b)	$\Rightarrow (n-8)(n-12) = 0$	M1		Accept alternatives eg formula, completing of sq..
	$\Rightarrow n = 8,\ 12$	A1	2	
	Total		**5**	

5. June 2006

5(a)	$\log_a x = \log_a 6^2 - \log_a 3$	M1		One law of logs used correctly
	$\log_a x = \log_a\left(\frac{6^2}{3}\right)$	M1		A second law of logs used correctly
	$\log_a x = \log_a \frac{36}{3} \Rightarrow x = 12$	A1	3	CSO AG
(b)	$\log_a y + \log_a 5 = 7 \Rightarrow \log_a 5y = 7$	M1		
	$\Rightarrow 5y = a^7$ or $y = \frac{1}{5}a^7$ or $a = (5y)^{1/7}$	m1 A1	3	Eliminates logs Accept these forms
	Total		**6**	

6. June 2007

6(a)	$y_A = 3(2^0+1)$ $= 6$	M1 A1	2	Substituting $x = 0$ PI

(b)	$h = 2$ Integral = $h/2$ {……} {….} = $f(0)+2[f(2)+f(4)]+f(6)$ {} = $6+2[3\times5+3\times17]+3\times65$ = 6 + 2[15 + 51] + 195 Integral = 333	B1 M1 A1 A1	 4	PI OE summing of areas of the three traps. Condone 1 numerical slip {ft on (**a**) for f(0) if not recovered} [Sum of 3 traps. = 21 + 66 + 246] CAO
(c)(i)	$21 = 3(2^x+1) \Rightarrow 2^x = 6$	B1	1	AG (be convinced)
(ii)	$\log_{10} 2^x = \log_{10} 6$ $x\log_{10} 2 = \log_{10} 6$ $x = \frac{\lg 6}{\lg 2} = 2.5849... = 2.58$ to 3sf	M1 m1 A1	 3	Take ln or $\log_{10}$ of both sides of $a^x = b$ or other relevant base if clear. The equation $a^x = b$ used must be correct. Use of $\log 2^x = x\log 2$ OE Both method marks must have been awarded.
	Total		**10**	

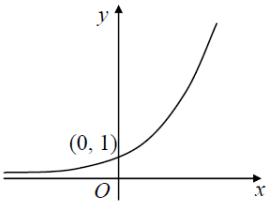
7. June 2007

8(a)	$\log_a n = \log_a 3(2n-1)$ $\Rightarrow n = 3(2n-1)$ $\Rightarrow 3 = 5n \Rightarrow n = \frac{3}{5}$	M1 m1 A1	 3	OE Log law used PI by next line OE, but must **not** have any logs.
(b)(i)	$\log_a x = 3 \Rightarrow x = a^3$	B1	1	
(ii)	$\log_a y - \log_a 2^3 = 4$ $\log_a \frac{y}{2^3} = 4$ $\begin{cases} xy = a^7 \times a^{(3\log_a 2)} \\ \text{or} \\ y = a^4 \times a^{(3\log_a 2)} \end{cases}$ $\frac{y}{2^3} = a^4$ $\begin{cases} xy = a^7 \times 2^3 \\ \text{or} \\ y = a^4 \times 2^3 \end{cases}$ $by = a^3 \times 8a^4$ or $8a^7$	M1 M1 ml A1	 4	$3\log 2 = \log 2^3$ seen or used any time in **(ii)** Correct method leading to an equation involving y (or xy) and a log but **not** involving + or − Correct method to eliminate **ALL** logs e.g. using $\log_a N = k \Rightarrow N = a^k$ or using $a^{\log_a c} = c$
	Total		**8**	

8. Jan 2008

7(a)	$x = 8$	B1	1	No clear log law errors seen. Condone answer left as $\frac{16}{2}$
(b)	$\log_a y = \log_a 3^2 + \log_a 4 + 1$ $\log_a y = \log_a (3^2 \times 4) + 1$ $\log_a y = \log_a (3^2 \times 4) + \log_a a = \log_a 36a$ $\Rightarrow y = 36a$	M1 M1 A1	 3	One law of logs used correctly Either a second law of logs used correctly or the 1 written as $\log_a a$ CSO
	Total		**4**	

9. Jan 2008

8(a)		B1 B1	 2	Shape (graph must clearly go below the intersection pt.). Condone if x-axis is a tangent Only intersection with y-axis at (0, 1) stated/indicated … (accept 1 on y-axis as equivalent) 0
(b)(i)	Stretch **(I)** in x-direction **(II)** scale factor 0.5 **(III)**	M1 A1	 2	Need**(I)** & one of **(II)**,**(III)** M0 if >1 transformation
(ii)	Translation; $\begin{bmatrix} -1 \\ 0 \end{bmatrix}$	B1; B1	 2	Must be 'Translation' or 'translate(d)' for 1st B mark Accept **full** equivalent to vector in words provided linked to 'translation/ move/shift' and **negative** x-direction (Note: B0 B1 is possible)
	ALTn: Stretch **(I)** in y-direction **(II)** scale factor 3 **(III)**			[Mark the alternative as in (b)(i).]
(c)(i)	$9^x = (3^2)^x = 3^{2x} = (3^x)^2 = Y^2$; $3^{x+1} = 3^x \times 3^1 = 3Y$ $9^x - 3^{x+1} + 2 = 0 \Rightarrow Y^2 - 3Y + 2 = 0$ $\Rightarrow (Y-1)(Y-2) = 0$	 M1 A1	 2	 Justifying either $9^x = Y^2$ or $3^{x+1} = 3Y$ AG
(ii)	$Y = 1 \Rightarrow 3^x = 1 \Rightarrow x = 0$ $Y = 2 \Rightarrow 3^x = 2$ $\log_{10} 3^x = \log_{10} 2$	B1 M1	 	AG (Accept direct substitution if convinced) Takes logs of both, PI by 'correct' value(s) later. or $x = \log_3 2$ seen
	$x \log_{10} 3 = \log_{10} 2$ $x = \dfrac{\lg 2}{\lg 3} = 0.630929.... = 0.6309$ to 4dp	m1 A1	 4	Use of $\log 3^x = x \log 3$ or $\log_3 2 = \dfrac{\lg 2}{\lg 3}$ OE (PI by $\log_3 2 = 0.630$ or 0.631 or better) Must show that logarithms have been used otherwise 0/3
	Total		**12**	

10. June 2008

5(a)(i)	$\log_a 1 = 0$	B1	1	
(ii)	$\log_a a = 1$	B1	1	
(b)	$\log_a x = \log_a (5 \times 6) - \log_a 1.5$ $\log_a x = \log_a \left(\dfrac{5 \times 6}{1.5}\right)$ $\log_a x = \log_a 20 \Rightarrow x = 20$	M1 M1 A1	 3	One law of logs used correctly A second law of logs used correctly
	Total		**5**	

11. Jan 2009

6(a)(i)	$\log_a 40$	B1	1	Accept '$k = 40$'
(ii)	$\log_a 8$	B1	1	Accept '$k = 8$'
(iii)	$\log_a 125$	B1	1	Accept '$k = 125$' but not '$k = 5^3$'
(b)	$\log_{10}\left[(1.5)^{3x}\right] = \log_{10} 7.5$	M1		Correct statement having taken logs of both sides of $(1.5)^{3x} = 7.5$ OE PI or $3x = \log_{1.5} 7.5$ seen
	$3x\log_{10} 1.5 = \log_{10} 7.5$	m1		$\log 1.5^{3x} = 3x\log 1.5$ OE
	$x = \dfrac{\lg 7.5}{3\lg 1.5} = 1.65645\ldots = 1.656$ to 3dp	A1	3	Both method marks must have been awarded with clear use of logarithms seen
(c)	$\log_2 p = m \Rightarrow p = 2^m$; $\log_8 q = n \Rightarrow q = 8^n$	M1		Either $p = 2^m$ or $q = 8^n$ seen or used
	$p = 2^m$ and $q = 2^{3n}$	m1		Writing $8^n = 2^{3n}$ and having $p = 2^m$
	$pq = 2^m \times \left(2^3\right)^n = 2^m \times 2^{3n}$ so $pq = 2^{m+3n}$	A1	3	Accept $y = m + 3n$
	Total		**9**	

12. June 2009

9(a)(i)	$\sqrt{125} = \sqrt{25\times 5} = 5\sqrt{5}$	M1		OE eg $\sqrt{125} = \sqrt{5^3}$ or $5^{1.5}$ seen
	$5^p = \sqrt{125} \Rightarrow p = 1.5$	A1	2	Correct value of p must be explicitly stated
	Alternative for (a)(i):			
	$p\log 5 = \frac{1}{2}\log 125$	(M1)		OE eg $p\log 5 = \log 11.18$ or eg $p = \log_5\sqrt{125}$
	$p\log 5 = \frac{3}{2}\log 5 \Rightarrow p = \frac{3}{2}$	(A1)		Correct value of p must be explicitly stated
(ii)	$5^{2x} = \sqrt{125} = 5^p \Rightarrow x = 0.5p = 0.75$	B1F	1	Must be 0.5 × c's value of p SC: $x = 0.75$ with c's ans (a)(i) $5^{1.5}$ scores B1F
(b)	$3^{2x-1} = 0.05$ $(2x-1)\log 3 = \log 0.05$	M1		Take logs of both sides and use 3rd law of logs. PI eg by $2x - 1 = \log_3 0.05$ seen
	$x = \dfrac{\log_{10} 0.05}{2\log_{10} 3} + \dfrac{1}{2}$	m1		Correct rearrangement to $x = \ldots$ PI
	$= -0.8634(165\ldots) = -0.8634$ to 4dp	A1	3	Condone > 4dp. Must see logs clearly **used** in solution, so NMS scores 0/3

(c)	$\log_a x = 2(\log_a 3 + \log_a 2) - 1$			
	$= 2\log_a(3\times 2) - 1$	M1		A valid law of logs used
	$= \log_a(6^2) - 1$	M1		Another valid law of logs used
	$= \log_a 36 - \log_a a$	B1		$\log_a a = 1$ quoted or used or $\log_a \frac{x}{k} = -1 \Rightarrow \frac{x}{k} = a^{-1}$ OE
	$\log_a x = \log_a\left(\frac{36}{a}\right) \Rightarrow x = \frac{36}{a}$	A1	4	CSO Must be $x = \frac{36}{a}$ or $x = 36a^{-1}$
	Total		**10**	

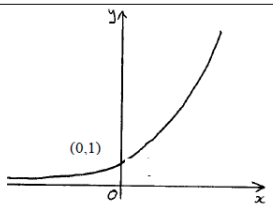
13. Jan 2010

3(a)(i)	$(x =)\ 1$	B1	1	CAO
(ii)	$(x =)\ 3$	B1	1	CAO
(b)	$\log_a n^2 = \log_a 18(n-4)$	M1 M1		A valid law of logs applied to correct logs A second valid law of logs applied to correct logs
	$n^2 - 18n + 72 = 0$	A1		ACF of these terms eg $n^2 - 18n = -72$
	$(n-6)(n-12) = 0$	m1		Valid method to solve quadratic, dep on both the previous Ms
	$n = 6,\ n = 12$	A1	5	Both values required **SC** NMS max (out of 5) B3 for both 6 and 12 without uniqueness considered; max B1 for either 6 or 12 only
	Total		**7**	

14. Jan 2011

8(a)	$\log_k x^2 - \log_k 5 = 1$	M1		A valid law of logs used correctly
	$\log_k \frac{x^2}{5} = 1$	M1		Another valid law of logs used correctly or correct method to reach $\log f(x) = \log 5k$
	$\log_k \frac{x^2}{5} = \log_k k$ [or $\log x^2 = \log 5k$]	A1		PI by next line
	$\Rightarrow \frac{x^2}{5} = k$ ie $k = \frac{x^2}{5}$	A1	4	Accept either of these two forms.
(b)	$\log_a y = \frac{3}{2}$; $\log_4 a = b + 2$			
	$\Rightarrow y = a^{\frac{3}{2}}$ $\Rightarrow a = 4^{b+2}$	M1		For either equation
	$y = (4^{b+2})^{\frac{3}{2}}$	m1		Elimination of a from two correct equations not involving logarithms
	$y = 2^{3(b+2)}$; $y = 2^{3b+6}$	A1	3	CSO Either form acceptable
	Total		**7**	

15. June 2011

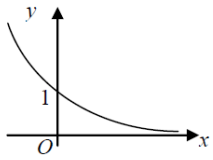
4(a)		B1		Any graph only crossing the y–axis at (0, 1) stated /indicated … (accept 1 on y-axis as equivalent) and not drawn **below** x-axis
		B1	2	Correct shaped graph, must clearly go below the intersection pt and an indication of correct behaviour of curve for large positive and large negative values of x. Ignore any scaling on axes.
(b)	Translation;	B1		Accept 'transl…' as equivalent [T or Tr is NOT sufficient]
	$\begin{bmatrix} 0 \\ -5 \end{bmatrix}$	B1	2	If vector not given, accept **full** equivalent to vector in words provided linked to 'transl../ move/shift' (B0B0 if >1 transformation)
(c)(i)	$4^x = (2^2)^x = 2^{2x} = (2^x)^2 = Y^2$ $2^{x+2} = 2^x \times 2^2 = 4Y$	M1		Justifying either $4^x = Y^2$ or $2^{x+2} = 4Y$ with no errors seen
(c)(i)	$4^x = (2^2)^x = 2^{2x} = (2^x)^2 = Y^2$ $2^{x+2} = 2^x \times 2^2 = 4Y$	M1		Justifying either $4^x = Y^2$ or $2^{x+2} = 4Y$ with no errors seen
	$4^x - 2^{x+2} - 5 = 0 \Rightarrow Y^2 - 4Y - 5 = 0$	A1	2	AG Be convinced; must have justified both of the above.
(ii)	$(Y-5)(Y+1) = 0$	M1		Correct factorising or use of quadratic formula or completing sq. PI by both solns 5& −1 seen
	(Since) 2^x>0 (for all real x,) 2^x=5 so only one (real) solution	E1		Rejection of 2^x (condone Y) negative, with justification, (condone "2^x not negative") followed by statement
	$\log 2^x = \log 5 \Rightarrow x \log 2 = \log 5$	M1		Eqn of form $p^x = q \Rightarrow x \log p = \log q$ provided $p > 0$ & $q > 0$ OE eg $x = \log_2 5$
	$x = 2.3219\ldots = 2.322$ (to 3dp)	A1	4	Condone > 3dp but must see explicit use of logs and must only be the one solution.
	Total		**10**	

16. June 2011

9(a)	$\{S_\infty =\}\ \dfrac{a}{1-r} = \dfrac{12}{1-\frac{3}{8}}$	M1		$\dfrac{a}{1-r}$ used
	$\{S_\infty =\}$ 19.2	A1	2	19.2 OE NMS mark as 2/2 or 0/2
(b)	{6th term = } ar^{6-1}	M1		Stated or used
	$= 12 \times \left(\dfrac{3}{8}\right)^5 = 2 \times 2 \times 3 \times \left(\dfrac{3}{2 \times 2 \times 2}\right)^5$	m1		Changing 8 and 12 in correct expression to correct products/powers of 2 and 3
	$= \dfrac{2 \times 2 \times 3 \times 3^5}{(2^3)^5} = \dfrac{2^2 \times 3^6}{2^{15}} = \dfrac{3^6}{2^{13}}$	A1	3	AG Be convinced
(c)(i)	$\{u_n =\}\ 12 \times \left(\dfrac{3}{8}\right)^{n-1}$	B1	1	OE. eg $32(3/8)^n$

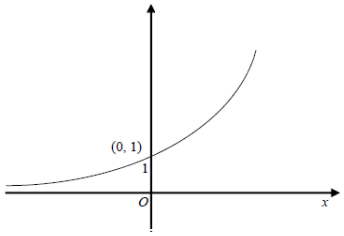
(ii)	$\log u_n = \log 12 + \log\left(\frac{3}{8}\right)^{n-1}$ $\log u_n = \log 12 + (n-1)\log\left(\frac{3}{8}\right)$ $\log u_n = \log 12 + (n-1)[\log 3 - \log 8]$	M1 M1		Log laws log(PQ)=logP + logQ ; $\log\left(\frac{P}{Q}\right) = \log P - \log Q$ $\log(P)^k = k\log P$ Using (c)(i) and taking logs: one log law used correctly, on a correct expression for u_n. a second different log law used correctly, indep of prev M error and ft on cand's (c)(i) provided cand's u_n expression has a power involving n.
	$\log u_n = \log 3 + 2\log 2 + (n-1)[\log 3 - 3\log 2]$ $\log u_n = n\log 3 - 3n\log 2 + 5\log 2$ $\log_a u_n = n\log_a 3 - (3n-5)\log_a 2$	m1 A1	 4	A third different log law used correctly (or equivalent valid step) to reach a correct RHS whose terms are all multiples of log2 and log3. Dep on both prev two Ms CSO AG Be convinced, no slips although we will condone the absence of the bases a even in the final line.
	Total		**10**	

17. Jan 2012

7(a)		B1 B1	 2	Correct shaped graph in 1st two quadrants only and indication of correct behaviour of curve for large positive and negative vals. of x. Ignore any scaling on axes. y-intercept indicated as 1 on diagram or stated as intercept=1 or as coords (0, 1).
(b)	$\frac{1}{2^x} = \frac{5}{4} \Rightarrow 2^{-x} = \frac{5}{4}$ (or $2^x = \frac{4}{5}$ or $2^{2-x} = 5$) $\log 2^{-x} = \log 1.25 \Rightarrow -x\log 2 = \log 1.25$ $[\log 2^x = \log 0.8 \Rightarrow x\log 2 = \log 0.8]$ $[\log 2^{2-x} = \log 5 \Rightarrow (2-x)\log 2 = \log 5]$ $[2^x=0.8,\ x=\log_2 0.8]$; $[0.5^x=1.25,\ x=\log_{0.5} 1.25]$ $x = -0.321928\ldots$ so $x = -0.322$ (to 3sf)	M1 M1 A1	 3	Correct 'rearrangement' to eg $2^x = \frac{4}{5}$ or $2^{-x} = \frac{5}{4}$ or $0.5^x = 1.25$ PI or $\log 1 - \log 2^x = \log(5/4)$ or better Takes logs of both sides of eqn of form either $2^x = k$ or $2^{-x} = k$ OE and uses 3rd law of logs or log to base 2 (or base ½) correctly Condone >3sf [Logs must be seen to be used otherwise **max** of M1M0A0]
(c)	$\log_a b^2 + 3\log_a y = 3 + 2\log_a\left(\frac{y}{a}\right)$ $\log_a b^2 + 3\log_a y = 3 + 2[\log_a y - \log_a a]$ $\log_a b^2 + \log_a y = 3 - 2\log_a a$ $\log_a b^2 y = 3 - 2\log_a a$ $\log_a b^2 y = 3 - 2(1)$ [or $\log_a b^2 y + \log_a a^2 = 3$]	M1 M1 M1		A log law used correctly; condone missing base a. A different log law used correctly condone missing base a. Either a further different log law used correctly condone missing base a or $\log_a a = 1$ stated/used.

	$\Rightarrow \log_a b^2 y = 1 \;\Rightarrow\; b^2 y = a$	m1		$\log_a Z = k \Rightarrow Z = a^k$ used or a correct method to eliminate logs (dep on no misapplication of any log law OE in the whole solution) Rearrangements which require only two of the above Ms to eliminate logs correctly: award the remaining M with the m mark.
	$\Rightarrow \; y = ab^{-2}$	A1	5	ACF of RHS
	Total		**10**	

18. June 2012

8(a)		B1		Correct shape, curve in 1st two quadrants only, crossing positive y-axis once and asymptotic to negative x-axis.
		B1	2	Coordinates (0, 1). Accept y-intercept indicated as 1 on diagram or stated as 'intercept = 1' B0 if graph clearly drawn crossing axes at more than one point
(b)(i)	$y^2 - 12 = y$ OE; $7^{2x} - 12 = 7^x$ OE	M1		Eliminates either x or y correctly
	$(y-4)(y+3)\ (=0)$; $(7^x - 4)(7^x + 3)\ (=0)$	A1		Correct factors or $y = \frac{1 \pm \sqrt{49}}{2}$ or better or $7^x = \frac{1 \pm \sqrt{49}}{2}$ or better
	Since $y\ (=7^x) > 0$, $[\,y\ (=7^x) \neq -3]$ (there is exactly one point of intersection)	E1		Clear indication that c's negative solution(s) has/have been considered and rejected
	y-coordinate is 4	B1	4	
(ii)	$7^x = 4$ so $x \log 7 = \log 4$ [or $x = \log_7 4$]	M1		OE ft on $7^x = k$, where k is positive, to either $x \log 7 = \log k$ or $x = \log_7 k$
	$x = 0.712(414\ldots) = 0.712$ to 3SF	A1	2	Condone > three significant figures. If use of logarithms not explicitly seen then score 0/2
	Total		**8**	

19. June 2012

9(a)	$h = 0.25$	B1		PI
	$f(x) = \log_{10}(x^2 + 1)$ $I \approx h/2\{\ldots\}$ $\{.\} = f(0) + f(1) +$ $2\,[f(0.25) + f(0.5) + f(0.75)]$	M1		OE summing of areas of the 'trapezia'
	$\{.\} =$ $\log 1 + \log 2 + 2\left[\log\frac{17}{16} + \log\frac{5}{4} + \log\frac{25}{16}\right]$ $= 0 + 0.3010\ldots +$	A1		OE Accept 1sf evidence
	$2\,(0.0263\ldots + 0.0969\ldots + 0.1938\ldots)$ $= 0.3010\ldots + 2(0.317058\ldots) = 0.935147\ldots$ $(I \approx)\ 0.125\,[0.935147\ldots] = 0.117$ (to 3SF)	A1	4	CAO Must be 0.117

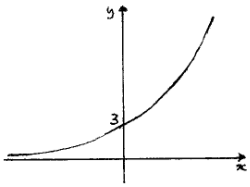
(b)	$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$	B1	1	
(c)(i)	$\log_{10}(10x^2) = \log_{10} 10 + \log_{10} x^2$ $= 1 + 2\log_{10} x$	M1 A1	2	Condone missing bases for M mark. Accept $\log x^2$ replaced by 2logx in M1 line AG. Bases must be included or statement '$\log_{10} 10 = 1$' given. **Condone missing bases in (c)(ii) & (c)(iii)**
(ii)	$y = 1 + 2\log_{10} x = \log_{10}(10x^2)$ Either $y = 2\log_{10}(\sqrt{10}\,x)$ (to compare $y = 2\log x$) or both $y = \log_{10} x^2$ and $y = \log_{10}(\sqrt{10}\,x)^2$ (Stretch) parallel to x-axis, sf $\frac{1}{\sqrt{10}}$ OE	M1 A1 B2,1,0	4	PI Writing in correct form so that stretch details can be stated directly B2 for correct direction and scale factor ACF (B1 for correct exact scale factor ACF) (or B1 for 'x-direction, scale factor 1/10 ') (or B1for 'x-direction, scale factor $\sqrt{10}$ ') Apply ISW if dec follows exact values. (OE scale factor must be in exact form)
(iii)	$\log_{10}(10x^2) = \log_{10}(x^2 + 1)$ $(10x^2 = x^2 + 1,\ 9x^2 = 1$ and since $x > 0$) $x = \frac{1}{3}$ (y-coordinate of P) $y = \log_{10}\frac{10}{9}$ Or $y = \log\left(\frac{1}{9} + 1\right)$ Gradient of OP = $3\log_{10}\frac{10}{9} = \log_{10}\frac{1000}{729}$	M1 A1 A1 A1	4	PI by $10x^2 = x^2 + 1$ or correct x $x = \frac{1}{3}$ OE stated or used; accept $\sqrt{\frac{1}{9}}, \frac{1}{\sqrt{9}}$ PI by $3\log\frac{10}{9}$ OE for the gradient of OP $\log\frac{1000}{729}$; Accept 'a=1000, b=729'
	Total		**15**	

20. Jan 2013

4	$\log_a N - \log_a x = \frac{3}{2}$ $\log_a \frac{N}{x} = \frac{3}{2}$ $\frac{N}{x} = a^{\frac{3}{2}}$ $x = a^{-\frac{3}{2}}N$	M1 m1 A1	3	A log law used correctly. PI by next line. Logarithm(s) eliminated correctly ACF of RHS
	Total		**3**	

21. Jan 2013

7(a)	Stretch(**I**) in y-direction(**II**) scale factor 3(**III**)	M1 A1	2	OE Need (**I**) **and** either (**II**) or (**III**) All correct. Need (**I**) and (**II**) and (**III**) [>1 transformation scores 0/2]

(b)		B1 B1	2	Shape with indication of correct asymptotic behaviour in 2nd quadrant below pt of intersection with y-axis Only intersection is with y-axis, and only intercept is 3 stated/indicated
(c)	$3 \times 4^x = 4^{-x}$ $\log 3 + \log 4^x = \log 4^{-x}$	M1 m1		OE eqn. in x Log Law 1 (or Law 2 applied to $\frac{4^x}{4^{-x}} = 3$ or $\frac{1}{3}$ OE) used correctly or correct rearrangement to $4^{2x}=1/3$ OE simplified e.g. $16^x=3^{-1}$ or $4^x=(1/\sqrt{3})$
	$\log 3 + x \log 4 = -x \log 4$ $x = \frac{-\log 3}{2 \log 4} \quad \left(= \frac{-\log 3}{\log 16}\right)$ $x = -0.396(2406\ldots) \quad = -0.396$ (to 3sf)	m1 A1 A1	5	Log Law 3 applied correctly twice (dependent on both M1 & m1) or a correct method using logs to solve an eqn. of form $a^{kx}=b$, $b>0$ (including case $k=1$) (dependent on M1 and valid method to a^{kx}) Correct expression for x or for $-x$ e.g. $x = \frac{1}{2}\log_4\left(\frac{1}{3}\right)$ PI by correct 3sf value or better If logs not used explicitly then max of M1m1m0.
	Total		**9**	