Practice examination questions for sequences and series

1. Jan 2005

3 An arithmetic series has fifth term 46 and twentieth term 181.

(a) (i) Show that the common difference is 9. *(3 marks)*

(ii) Find the first term. *(1 mark)*

(b) Find the sum of the first 20 terms of the series. *(2 marks)*

(c) The nth term of the series is u_n.

Given that the sum of the first 50 terms of the series is 11525, find the value of

$$\sum_{n=21}^{50} u_n$$ *(2 marks)*

2. June 2005

3 The nth term of an arithmetic sequence is u_n, where

$$u_n = 90 - 3n$$

(a) Find the value of u_1 and the value of u_2. *(2 marks)*

(b) Write down the common difference of the arithmetic sequence. *(1 mark)*

(c) Given that $\sum_{n=1}^{k} u_n = 0$, find the value of k. *(3 marks)*

3. Jan 2006

5 The nth term of a sequence is u_n.

The sequence is defined by

$$u_{n+1} = pu_n + q$$

where p and q are constants.

The first three terms of the sequence are given by

$$u_1 = 200 \qquad u_2 = 150 \qquad u_3 = 120$$

(a) Show that $p = 0.6$ and find the value of q. *(5 marks)*

(b) Find the value of u_4. *(1 mark)*

(c) The limit of u_n as n tends to infinity is L. Write down an equation for L and hence find the value of L. *(3 marks)*

4. June 2006

3 The first term of an arithmetic series is 1. The common difference of the series is 6.

(a) Find the tenth term of the series. *(2 marks)*

(b) The sum of the first n terms of the series is 7400.

(i) Show that $3n^2 - 2n - 7400 = 0$. *(3 marks)*

(ii) Find the value of n. *(2 marks)*

5. June 2007

4 An arithmetic series has first term a and common difference d.

The sum of the first 29 terms is 1102.

(a) Show that $a + 14d = 38$. *(3 marks)*

(b) The sum of the second term and the seventh term is 13.

Find the value of a and the value of d. *(4 marks)*

6. Jan 2008

2 The arithmetic series

$$51 + 58 + 65 + 72 + \ldots + 1444$$

has 200 terms.

(a) Write down the common difference of the series. *(1 mark)*

(b) Find the 101st term of the series. *(2 marks)*

(c) Find the sum of **the last** 100 terms of the series. *(2 marks)*

7. June 2008

6 The nth term of a sequence is u_n.

The sequence is defined by

$$u_{n+1} = pu_n + q$$

where p and q are constants.

The first three terms of the sequence are given by

$$u_1 = -8 \qquad u_2 = 8 \qquad u_3 = 4$$

(a) Show that $q = 6$ and find the value of p. *(5 marks)*

(b) Find the value of u_4. *(1 mark)*

(c) The limit of u_n as n tends to infinity is L.

(i) Write down an equation for L. *(1 mark)*

(ii) Hence find the value of L. *(2 marks)*

8. Jan 2009

8 The 25th term of an arithmetic series is 38.

The sum of the first 40 terms of the series is 1250.

(a) Show that the common difference of this series is 1.5. *(6 marks)*

(b) Find the number of terms in the series which are less than 100. *(3 marks)*

9. June 2009

3 The nth term of a sequence is u_n.

The sequence is defined by

$$u_{n+1} = ku_n + 12$$

where k is a constant.

The first two terms of the sequence are given by

$$u_1 = 16 \qquad u_2 = 24$$

(a) Show that $k = 0.75$. *(2 marks)*

(b) Find the value of u_3 and the value of u_4. *(2 marks)*

(c) The limit of u_n as n tends to infinity is L.

(i) Write down an equation for L. *(1 mark)*

(ii) Hence find the value of L. *(2 marks)*

10.Jan 2010

4 An arithmetic series has first term a and common difference d.

The sum of the first 31 terms of the series is 310.

(a) Show that $a + 15d = 10$. *(3 marks)*

(b) Given also that the 21st term is twice the 16th term, find the value of d. *(3 marks)*

(c) The nth term of the series is u_n. Given that $\sum_{n=1}^{k} u_n = 0$, find the value of k. *(4 marks)*

11.June 2010

2 The nth term of a sequence is u_n.

The sequence is defined by

$$u_{n+1} = 6 + \tfrac{2}{5}u_n$$

The first term of the sequence is given by $u_1 = 2$.

(a) Find the value of u_2 and the value of u_3. *(2 marks)*

(b) The limit of u_n as n tends to infinity is L.

Write down an equation for L and hence find the value of L. *(3 marks)*

12.June 2011

7 The nth term of a sequence is u_n. The sequence is defined by

$$u_{n+1} = pu_n + q$$

where p and q are constants.

The first two terms of the sequence are given by $u_1 = 60$ and $u_2 = 48$.

The limit of u_n as n tends to infinity is 12.

(a) Show that $p = \frac{3}{4}$ and find the value of q. *(5 marks)*

(b) Find the value of u_3. *(1 mark)*

13.Jan 2012

6 An arithmetic series has first term a and common difference d.

The sum of the first 25 terms of the series is 3500.

(a) Show that $a + 12d = 140$. *(3 marks)*

(b) The fifth term of this series is 100.

Find the value of d and the value of a. *(4 marks)*

(c) The nth term of this series is u_n. Given that

$$33\left(\sum_{n=1}^{25} u_n - \sum_{n=1}^{k} u_n\right) = 67\sum_{n=1}^{k} u_n$$

find the value of $\sum_{n=1}^{k} u_n$. *(3 marks)*

14.June 2012

1 The arithmetic series

$$23 + 32 + 41 + 50 + \ldots + 2534$$

has 280 terms.

(a) Write down the common difference of the series. *(1 mark)*

(b) Find the 100th term of the series. *(2 marks)*

(c) Find the sum of the 280 terms of the series. *(2 marks)*

Answers

1. Jan 2005

3(a)(i)	$a+19d=181$; $a+4d=46$ $\Rightarrow 15d=181-46$ $\Rightarrow d=9$	M1 A1 A1	 3	$a+(n-1)d$ used; PI AG (be convinced)
(ii)	$a=10$	B1	1	
(b)	$S_{20}=\frac{20}{2}[2a+(20-1)d]$ $\ldots=1910$	M1 A1	 2	OE
(c)	$\sum_{n=1}^{50}u_n-\sum_{n=1}^{20}u_n$ $\ldots=11525-$"1910"$=9615$	M1 A1√	 2	OE ft on $11525-$c's S_{20}
	Total		**8**	

2. June 2005

3(a)	$u_1=87;\ u_2=84$	B1;B1 √	2	ft on $u_2=u_1-3$ **SC** B1 for 90, 87
(b)	Common difference (d) is -3	B1	1	
(c)	$\sum_{n=1}^{k}u_n$ = sum of AP $\ldots\ldots=\frac{k}{2}[174+(k-1)(-3)]$ $0=\frac{k}{2}[177-3k]\Rightarrow 177=3k$ $\Rightarrow k=59$	M1 A1√ A1	 3	 OE ft on u_1 and use of $d=3$ (For M1A1 ft condone n in place of k) Just the **single** value 59
ALTI	$=\sum_{n=1}^{k}90-\sum_{n=1}^{k}3n=90k-3\left[\frac{k}{2}(k+1)\right]$ $0=90k-1.5k(k+1)\Rightarrow k=59$	M1;A1 A1		M1 split and either $90k$ or $\left[\frac{k}{2}(k+1)\right]$ (For 1st two marks condone n in place of k)
	Total		**6**	

3. Jan 2006

5(a)	$150=200p+q$ $120=150p+q$ $p=0.6$ $q=30$	M1 A1 m1 A1 B1	 5	Either equation Both (condone embedded values for the M1A1) Valid method to solve two simultaneous eqns in p and q to find either p or q AG (condone if left as a fraction)
(b)	$u_4=102$	B1F√	1	Ft on $(72+q)$
(c)	$L=pL+q$; $L=0.6L+30$ $L=\frac{q}{1-p}$ $L=75$	M1 m1 A1F√	 3	 Ft on $2.5q$
	Total		**9**	

4. June 2006

3(a)	(Tenth term) = $a + (10-1)\,d$	M1		
	…… = 1 + 9(6) = 55	A1	2	NMS or rep. addn. B2 CAO SC if M0 award B1 for $6n-5$ OE
(b)(i)	$S_n = \frac{n}{2}[2+(n-1)6]$	M1		Formula for $\{S_n\}$ with either $a = 1$ or $d = 6$ substituted
	$\frac{n}{2}[2+6n-6] = 7400$	A1		Eqn formed with some expansion of brackets
	$3n^2 - 2n = 7400 \Rightarrow 3n^2 - 2n - 7400 = 0$	A1	3	CSO AG
(ii)	$(3n+148)(n-50) = 0$	M1		Formula/factorisation OE
	$\Rightarrow n = 50$	A1	2	NMS single ans. 50.. B2 CAO NMS 50 and –49.3(3…) B1 CAO
	Total		**7**	

5. June 2007

4(a)	$\{S_{29} =\}\frac{29}{2}[2a+28d]$	M1		Formula for S_n with $n = 29$ substituted and with a and d
	$29\,(a + 14d) = 1102$	m1		Equation formed then some manipulation
	$a + 14d = \frac{1102}{29} \Rightarrow a + 14d = 38$	A1	3	CSO AG
(b)	$u_2 = a + d \quad u_7 = a + 6d$	B1		Either expression correct
	$u_2 + u_7 = 13 \Rightarrow 2a + 7d = 13$	M1		Forming equation using u_2 & u_7 both in form $a + kd$
	e.g. $21d = 63;\ 3a = -12$	m1		Solving $a + 14d = 38$ with candidate's '$2a + 7d = 13$' to at least stage of elimination of either a or d
	$a = -4 \quad d = 3$	A1	4	Both correct
	Total		**7**	

6. Jan 2008

2(a)	$(d) = 7$	B1	1	7
(b)	(101st term) = $a + (101-1)\,d$ …… = 51 + 100(7) = 751	M1 A1F	2	Ft on c's answer for d. NMS/rep. addn., give both marks for '751'. **SC** if M0, award B1 for $7n+44$ OE
(c)	$S_n = \frac{100}{2}[751+1444]$ **or** $S_n = \frac{100}{2}[2\times751+(100-1)7]$	M1		Formula for $\{S_n\}$ with [any **3** of a = c's 751 (condoning '751'$\pm d$) **or** d = c's 7 **or** n =100 **or** l = 1444 substituted] or [$S_{200}-S_k$ with k=100, (condoning k=99 or 101) stated/used with correct ft substitution in S_{200} or S_k]
	= 109 750	A1	2	
	Total		**5**	

7. June 2008

6(a)	$8 = -8p + q$	M1		Either equation. PI eg by combined eqn.
	$4 = 8p + q$	A1		Both (condone embedded values for the M1A1)
		m1		Valid method to solve two simultaneous equations in p **and** q to find either p or q
	$q = 6$	A1		AG (condone if left as a fraction)
	$p = -0.25$	B1	5	OE
(b)	$u_4 = 5$	B1F	1	Ft on $(6 + 4p)$
(c)(i)	$L = pL + q$; $(L = -0.25L + 6)$	M1	1	OE
(ii)	$L = \dfrac{q}{1-p}$	m1		Rearranging
	$L = \dfrac{6}{1.25} = 4.8$	A1F	2	Ft on $\dfrac{6}{1-p}$ Dependent on previous two marks
	Total		**9**	

8. Jan 2009

8(a)	$\{S_{40} =\}\ \dfrac{40}{2}\left[2a + (40-1)d\right]$	M1		
	$20(2a + 39d) = 1250$	A1		
	$\{25^{\text{th}}$ term $=\}\ a + (25 - 1)d$	M1		
	$a + 24d = 38$	A1		
		m1		Dep on both previous two Ms. Solving two equations in a and d simultaneously
	$18d = 27 \Rightarrow d = 1.5$	A1cso	6	AG Be convinced SC Using the given answer for d: mark out of a maximum of 4/6 as M1A1M1A1 {conclusion also needed in last A mark} (m0A0)
(b)	$a = 38 - 24 \times 1.5$	M1		PI if using $a = 2$ **in (b)**
	$= 2$			If using eg $a = 38$ award this M mark at stage: no. of terms $\dfrac{100-38}{1.5} + 1 + 24$
	$a + (n - 1)1.5 < 100$	M1		
	$n < \dfrac{100-a}{1.5} + 1$			
	$n < 66.333\ldots$ $\Rightarrow$ number of terms < 100 is 66	A1	3	NMS mark as B3 for 66 else B0
	Total		**9**	

9. June 2009

3(a)	$24 = 16k + 12$ $k = 12 \div 16 = 0.75$	M1 A1	 2	Condone with 0.75 (OE) subst for k AG; OE fraction; if verification must explicitly state the conclusion
(b)	$u_3 = 30$ $u_4 = 34.5$	B1 B1F	 2	 ft on 0.75 × cand's u_3 + 12
(c)(i)	$L = 0.75L + 12$	M1	1	Replacing u_{n+1} and u_n by L
(ii)	$L = \frac{12}{1-k} = \frac{12}{1-0.75}$ $L = 48$	m1 A1	 2	PI, but previous M **must** be scored SC: (c)(i) incorrect and then in (c)(ii) $L = 0.75L + 12$ leading to $L = 48$ scores B2
	Total		**7**	

10.Jan 2010

4(a)	$\{S_{31} =\} \frac{31}{2}[2a + (31-1)d]$ $31(a + 15d) = 310$ $a + 15d = 310/31$; $a + 15d = 10$	M1 m1 A1	 3	 Forming eqn and eliminating fraction or bracket AG Completion to printed answer
(b)	$a + (21 - 1)d = 2[a + (16 - 1)d]$ $\Rightarrow a = -10d$; $\Rightarrow -10d + 15d = 10$	M1 m1		$a + (n - 1)d$ used for at least one term Solving $a + 15d = 10$ simultaneously with an eqn in a and d obtained from $a+20d = k[a+15d]$ with $k=2$ or with $k=1/2$
(c)	$u_1 = a = -20$ $\sum_{n=1}^{k} u_n = S_k = \frac{k}{2}[2a + (k-1)d]$ $\frac{k}{2}[-40 + 2k - 2] = 0$ $k = 21$	B1F M1 A1F A1	 4	ft on c's value for d in $a + 15d = 10$ or in another correct (dep on m1) equation in a and d The value for a must appear within c's soln for (c) Condone n for k in M1 and A1F lines provided n replaced by k at a later stage '= 0' can be implied by later line; ft on c's non-zero values for a and d Condone presence of $k = 0$ **SC** NMS $k = 21$ and with $d = 2$ found earlier award B2. If $k = 21$ but never see $d = 2$, award 0/4
	Total		**10**	

11.June 2010

2(a)	$u_2 = 6.8$ $u_3 = 8.72$	B1 B1F	 2	OE eg 34/5 Ft on 6+0.4×c's u_2

(b)	$L = 6 + 0.4L$	M1		Replacing u_{n+1} and u_n by L
	$L = \dfrac{6}{1-0.4}$	m1		PI provided M scored
	$L = 10$	A1	3	Must form an equation in L otherwise 0/3
	Total		**5**	

12.June 2011

7(a)	$48 = 60p + q$ $12 = 12p + q$	M1 M1		M1 for each equation in ACF (Condone embedded values for the M1M1)
	$36 = 48p$ or $p = \dfrac{36}{48}$	m1		Valid method to solve the correct two simultaneous eqns in p <u>and</u> q to at least the stage $ap = b$ or $cq = d$
	$p = \dfrac{3}{4}$	A1		AG (condone if left as equiv. decimal)
	$q = 3$	B1	5	Can award if seen **<u>explicitly</u>** in (b) and no contradiction [ie not attempted in (a)]
(b)	$u_3 = 36 + q = 39$	B1F	1	If not 39, ft on (36 + c's q)
	Total		**6**	

13.Jan 2012

6(a)	$\{S_{25} =\}\ \dfrac{25}{2}[2a + (25-1)d]$	M1		$\dfrac{25}{2}[2a + (25-1)d]$ OE
	$\dfrac{25}{2}[2a + 24d] = 3500$ $25(2a+24d) = 7000$ or $[\dfrac{50a + 600d}{2} = 3500]$	m1		Forming equation and attempt to remove fraction or to expand brackets or better
	$50a + 600d = 7000$ (or better) so $a + 12d = 140$	A1	3	CSO AG Be convinced.
(b)	5th term $= a + 4d$ $a + 12d = 140,\ a + 4d = 100$	M1		$a + (5-1)d$ used correctly
	$\Rightarrow 8d = 40$	M1		Solving $a + 12d = 140$ simultaneously with either $a+4d = 100$ or $a+5d = 100$ as far as eliminating either a or d.

(c)	$33\left(3500 - \sum_{n=1}^{k} u_n\right) = 67\sum_{n=1}^{k} u_n$	M1		Recognition that $\sum_{n=1}^{25} u_n = 3500$
	$33 \times 3500 = 67\sum_{n=1}^{k} u_n + 33\sum_{n=1}^{k} u_n$	m1		Correct rearrangement PI
	$100 \times \sum_{n=1}^{k} u_n = 33 \times 3500 \Rightarrow \sum_{n=1}^{k} u_n = 1155$	A1	3	
	Total		**10**	

14.June 2012

1(a)	(common difference) = 9	B1	1	9
(b)	(100th term) = 23 + (100 − 1) d	M1		23 + (100 −1) d or better seen (or used with d = 9 or with d = c's answer (a))
	= 914	A1	2	914 NMS mark as B2 or B0
(c)	(Sum of series) = $\frac{280}{2}(23+2534)$			Substitution of n = 280, l = 2534, a = 23 (or c's value of a used in (b)), d = 9 (or c's answer to (a)) into $\frac{n}{2}(a+l)$
	{or $\frac{280}{2}\left[2\times 23+(280-1)(9)\right]$}	M1		PI or $\frac{n}{2}\left[2a+(n-1)d\right]$ PI
	= 357 980	A1	2	357 980 NMS mark as B2 or B0
	Total		**5**	