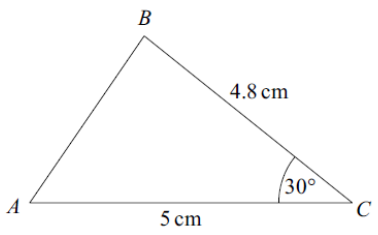
Practice examination questions for trigonometry

1. June 2005

1 The diagram shows a triangle ABC.



The lengths of AC and BC are 5 cm and 4.8 cm respectively.

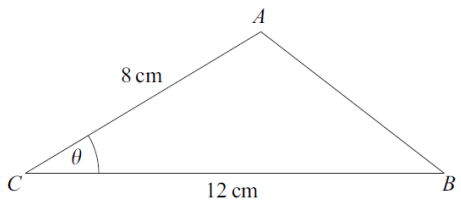
The size of the angle BCA is 30°.

(a) Calculate the area of the triangle ABC. *(2 marks)*

(b) Calculate the length of AB, giving your answer to three significant figures. *(3 marks)*

2. Jan 2006

4 The triangle ABC, shown in the diagram, is such that $AC = 8$ cm, $CB = 12$ cm and angle $ACB = \theta$ radians.



The area of triangle $ABC = 20\text{ cm}^2$.

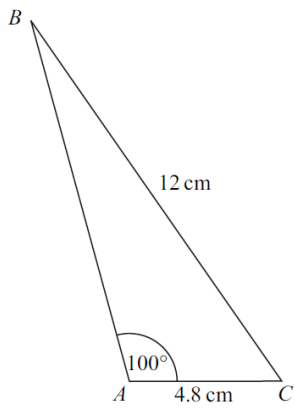
(a) Show that $\theta = 0.430$ correct to three significant figures. *(3 marks)*

(b) Use the cosine rule to calculate the length of AB, giving your answer to two significant figures. *(3 marks)*

(c) The point D lies on CB such that AD is an arc of a circle centre C and radius 8 cm. The region bounded by the arc AD and the straight lines DB and AB is shaded in the diagram.

3. June 2006

2 The diagram shows a triangle ABC.



The lengths of AC and BC are 4.8 cm and 12 cm respectively.

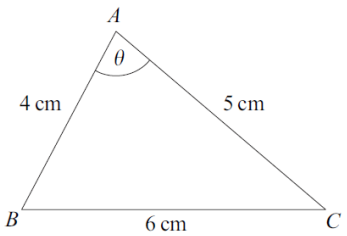
The size of the angle BAC is $100°$.

(a) Show that angle $ABC = 23.2°$, correct to the nearest $0.1°$. *(3 marks)*

(b) Calculate the area of triangle ABC, giving your answer in cm^2 to three significant figures. *(3 marks)*

4. Jan 2007

4 The triangle ABC, shown in the diagram, is such that $BC = 6$ cm, $AC = 5$ cm and $AB = 4$ cm. The angle BAC is θ.



(a) Use the cosine rule to show that $\cos\theta = \frac{1}{8}$. *(3 marks)*

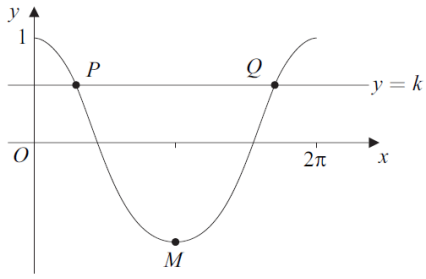
(b) Hence use a trigonometrical identity to show that $\sin\theta = \frac{3\sqrt{7}}{8}$. *(3 marks)*

(c) Hence find the area of the triangle ABC. *(2 marks)*

5. Jan 2007

8 (a) Solve the equation $\cos x = 0.3$ in the interval $0 \leqslant x \leqslant 2\pi$, giving your answers in radians to three significant figures. *(3 marks)*

(b) The diagram shows the graph of $y = \cos x$ for $0 \leqslant x \leqslant 2\pi$ and the line $y = k$.



The line $y = k$ intersects the curve $y = \cos x$, $0 \leqslant x \leqslant 2\pi$, at the points P and Q.
The point M is the minimum point of the curve.

(i) Write down the coordinates of the point M. *(2 marks)*

The line $y = k$ intersects the curve $y = \cos x$, $0 \leqslant x \leqslant 2\pi$, at the points P and Q.
The point M is the minimum point of the curve.

(i) Write down the coordinates of the point M. *(2 marks)*

(ii) The x-coordinate of P is α.

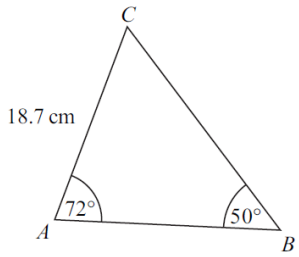
Write down the x-coordinate of Q in terms of π and α. *(1 mark)*

(c) Describe the geometrical transformation that maps the graph of $y = \cos x$ onto the graph of $y = \cos 2x$. *(2 marks)*

(d) Solve the equation $\cos 2x = \cos\frac{4\pi}{5}$ in the interval $0 \leqslant x \leqslant 2\pi$, giving the values of x in terms of π. *(4 marks)*

6. Jan 2008

3 The diagram shows a triangle ABC. The length of AC is 18.7 cm, and the sizes of angles BAC and ABC are 72° and 50° respectively.

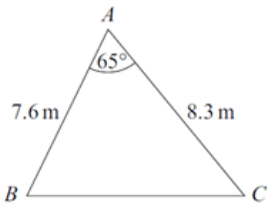


(a) Show that the length of $BC = 23.2$ cm, correct to the nearest 0.1 cm. *(3 marks)*

(b) Calculate the area of triangle ABC, giving your answer to the nearest cm^2. *(3 marks)*

7. June 2008

4 The diagram shows a triangle ABC.



The size of angle BAC is 65°, and the lengths of AB and AC are 7.6 m and 8.3 m respectively.

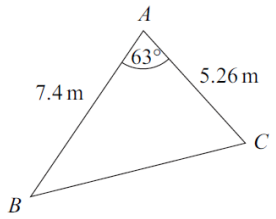
(a) Show that the length of BC is 8.56 m, correct to three significant figures. *(3 marks)*

(b) Calculate the area of triangle ABC, giving your answer in m^2 to three significant figures. *(2 marks)*

(c) The perpendicular from A to BC meets BC at the point D.

8. Jan 2009

3 The diagram shows a triangle ABC.



The size of angle A is 63°, and the lengths of AB and AC are 7.4 m and 5.26 m respectively.

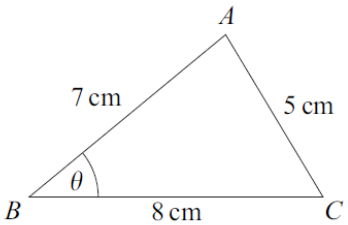
(a) Calculate the area of triangle ABC, giving your answer in m^2 to three significant figures. *(2 marks)*

(b) Show that the length of BC is 6.86 m, correct to three significant figures. *(3 marks)*

(c) Find the value of **sin *B*** to two significant figures. *(2 marks)*

9. June 2009

1 The triangle ABC, shown in the diagram, is such that $AB = 7\,\text{cm}$, $AC = 5\,\text{cm}$, $BC = 8\,\text{cm}$ and angle $ABC = \theta$.

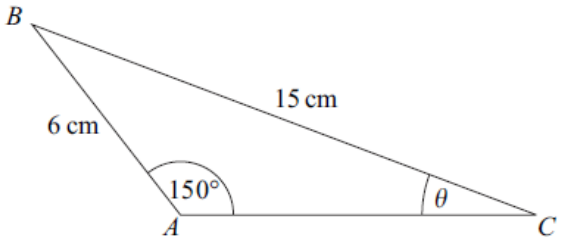


(a) Show that $\theta = 38.2°$, correct to the nearest 0.1°. *(3 marks)*

(b) Calculate the area of triangle ABC, giving your answer, in cm^2, to three significant figures. *(2 marks)*

10. June 2010

3 The triangle ABC, shown in the diagram, is such that $AB = 6\,\text{cm}$, $BC = 15\,\text{cm}$, angle $BAC = 150°$ and angle $ACB = \theta$.

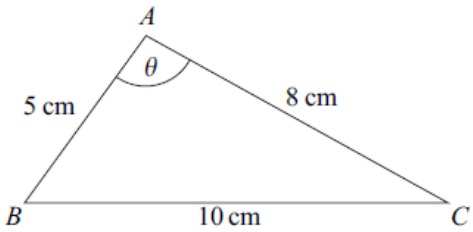


(a) Show that $\theta = 11.5°$, correct to the nearest 0.1°. *(3 marks)*

(b) Calculate the area of triangle ABC, giving your answer in cm^2 to three significant figures. *(3 marks)*

11. Jan 2011

3 The triangle ABC, shown in the diagram, is such that $AB = 5\,\text{cm}$, $AC = 8\,\text{cm}$, $BC = 10\,\text{cm}$ and angle $BAC = \theta$.



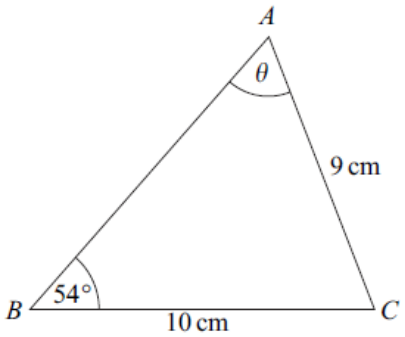
(a) Show that $\theta = 97.9°$, correct to the nearest 0.1°. *(3 marks)*

(b) (i) Calculate the area of triangle ABC, giving your answer, in cm^2, to three significant figures. *(2 marks)*

(ii) The line through A, perpendicular to BC, meets BC at the point D. Calculate the length of AD, giving your answer, in cm, to three significant figures. *(3 marks)*

12. June 2011

1 The triangle ABC, shown in the diagram, is such that $AC = 9\,\text{cm}$, $BC = 10\,\text{cm}$, angle $ABC = 54°$ and the acute angle $BAC = \theta$.

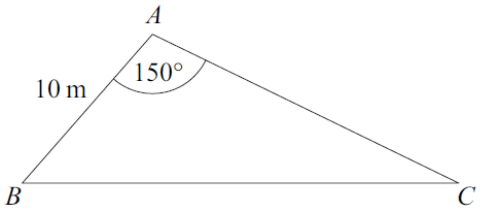


(a) Show that $\theta = 64°$, correct to the nearest degree. *(3 marks)*

(b) Calculate the area of triangle ABC, giving your answer to the nearest square centimetre. *(3 marks)*

13. Jan 2012

4 The triangle ABC, shown in the diagram, is such that AB is 10 metres and angle BAC is 150°.



The area of triangle ABC is $40\,\text{m}^2$.

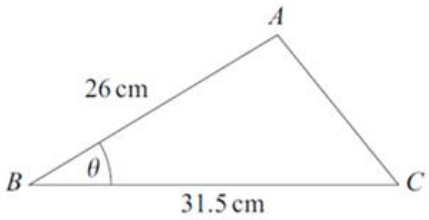
(a) Show that the length of AC is 16 metres. *(2 marks)*

(b) Calculate the length of BC, giving your answer, in metres, to two decimal places. *(3 marks)*

(c) Calculate the smallest angle of triangle ABC, giving your answer to the nearest 0.1°. *(3 marks)*

14. June 2012

2 The triangle ABC, shown in the diagram, is such that $AB = 26\,\text{cm}$ and $BC = 31.5\,\text{cm}$.



The acute angle ABC is θ, where $\sin\theta = \frac{5}{13}$.

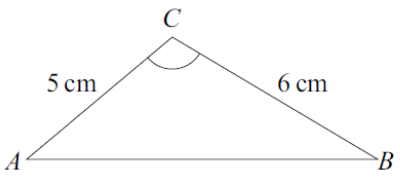
(a) Calculate the area of triangle ABC. *(2 marks)*

(b) Find the exact value of $\cos\theta$. *(1 mark)*

(c) Calculate the length of AC. *(3 marks)*

15. Jan 2013

3 The diagram shows a triangle ABC.



The lengths of AC and BC are 5 cm and 6 cm respectively.

The area of triangle ABC is $12.5\,\text{cm}^2$, and angle ACB is **obtuse**.

(a) Find the size of angle ACB, giving your answer to the nearest $0.1°$. *(3 marks)*

(b) Find the length of AB, giving your answer to two significant figures. *(3 marks)*

Answers

1. June 2005

1(a)	Area $= \frac{1}{2} \times 5 \times 4.8 \times \sin 30^\circ$	M1		Use of $\frac{1}{2}ab\sin C$ OE
	$= 6$ cm^2.	A1	2	Condone absent cm^2. [**Note:** Calculator set in wrong mode, penalise only once on the paper.]
(b)	$AB^2 = 5^2 + 4.8^2 - 2 \times 5 \times 4.8 \cos 30^\circ$	M1		RHS of cosine rule used
	$= 25 + 23.04 - 41.569$	m1		Correct order of evaluation
	$= 6.4707..$ $\Rightarrow$ AB $= \sqrt{6.47...} = 2.5437$ $= 2.54$ cm to 3 sf	A1	3	Accept 'better' than 2.54 Condone absent cm
	Total		**5**	

2. Jan 2006

	[Note: Calc. set in wrong mode, penalise only once on the paper.] Condone missing units throughout the question.			
4(a)	Area of triangle $= \frac{1}{2}(12)(8)\sin\theta$	M1		Use of $\frac{1}{2}ab\sin C$ or full equivalent
	$\sin\theta = \frac{20}{48}$ [=0.41(666…)]	A1		OE (giving 0.412 to 0.42)
	$\Rightarrow \theta = 0.4297(7\ldots) = 0.430$ to 3sf	A1	3	AG(need to see >3sf value)
(b)	$\{AB^2 =\}8^2 + 12^2 - 2 \times 8 \times 12 \times \cos\theta$	M1		
	$= 64 + 144 - 174.5\ldots.$	m1		Accept 33 to 34 inclusive if three values not separate
	$\Rightarrow AB = 5.78\ldots = 5.8$ cm to 2sf	A1	3	If not 2sf condone 5.78 to 5.79 inclusive. Condone ±
(c)(i)	Arc $AD = 8\theta$; $= 3.44.. = 3.4$ cm to 2sf	M1; A1	2	If not 2sf condone 3.438 to 3.44 inclusive
(ii)	Area of sector $= \frac{1}{2}r^2\theta$	M1		Stated or used [or 13.7(6..) seen]
	Shaded area = Area of triangle – sector area	M1		Difference of areas
	Shaded area $= 20 - 0.5 \times 8^2 \times \theta$ $= 6.2$ cm^2 to 2sf	A1	3	Condone 6.24 to 6.2472
	Total		**11**	

3. June 2006

2(a)	$\frac{\sin B}{4.8} = \frac{\sin 100}{12}$	M1		Use of the sine rule
	$\sin B = \frac{4.8\sin 100}{12}$ [= 0.39(392…)]	m1		Rearrangement
	(angle ABC) = 23.19(8…) {= 23.2°.}	A1	3	AG Need >1dp eg 23.19 or 23.20

(b)	Angle $C = 80° - 23.2° = 56.8°$	M1		Valid method to find a relevant angle eg C (PI eg by correct sin C) or 23.2°+10°
	Area of triangle = $0.5\times12\times4.8\times\sin C$	M1		OE eg $0.5\times4.8\times12\times\cos(B+10)$
	….. = 24.09…. = 24.1 cm^2. (to 3sf)	A1	3	Condone missing/wrong units
	Total		**6**	

4. Jan 2007

4(a)	$6^2 = 4^2 + 5^2 - 2(4)(5)\cos\theta$	M1		Use of the cosine rule
	$\cos\theta = \dfrac{4^2+5^2-6^2}{2(4)(5)}$	m1		Rearrangement
	$\cos\theta = \dfrac{5}{40} = \dfrac{1}{8}$	A1	3	CSO **AG** (be convinced)
(b)	$\cos^2\theta + \sin^2\theta = 1$	M1		Stated or used (PI)
	$\sin^2\theta = \dfrac{63}{64}$	A1		Or better
	$\sin\theta = \dfrac{\sqrt{63}}{8} = \dfrac{\sqrt{9\times7}}{8} = \dfrac{3\sqrt{7}}{8}$	A1	3	**AG** (be convinced)
(c)	Area of triangle = $0.5\times4\times5\times\sin\theta$.	M1		
	….. = $\dfrac{30\sqrt{7}}{8}$ cm^2.	A1	2	OE (Condone 9.92)
	Total		**8**	

5. June 2007

8(a)	$\{x =\}\ \cos^{-1}(0.3) = 1.266\ldots\quad \{=\beta\}$	M1		$\cos^{-1}(0.3)$ PI by eg 72° or 73°
	$\{x =\}\ 2\pi - \beta$	m1		Condone degrees or mix.
	$x =$ 1.27, 5.02	A1	3	Accept 1.26 to 1.27 with 5.01 to 5.02 inclusive
(b)(i)	$M(\pi, -1)$	B1;B1	2	B1 for each coordinate
(ii)	$\{x_Q =\}\ 2\pi - \alpha$	B1	1	OE (unsimplified)
(c)	Stretch **(I)** in x-direction **(II)** scale factor $\dfrac{1}{2}$ **(III)**	M1 A1	2	Need**(I)** & one of **(II)**,**(III)**
(d)	$\cos 2x = \cos\dfrac{4\pi}{5} \Rightarrow 2x = \dfrac{4\pi}{5}$	B1		OE. (From correct work)
	$\Rightarrow x = \dfrac{2\pi}{5}\ (=\alpha)$			Condone decimals/degrees
	$x = \pi - \alpha$; OE	M1		OE eg $2x = 2\pi - \dfrac{4\pi}{5}$ Correct quadrant; condone degrees/decimals/mix
	$x = \pi + \alpha$; $x = 2\pi - \alpha$; OE	m1		Need both (OE for $2x$=) with no extras (quadrants) within the given interval. Condone degrees/decimals/mix

	$x = \frac{2\pi}{5}, \frac{3\pi}{5}, \frac{7\pi}{5}, \frac{8\pi}{5}$	A1	4	Need all 4 solutions for x but condone unsimplified provided in terms of π Ignore extra values outside the given interval.
	Total		**12**	

6. Jan 2008

3(a)	$\frac{BC}{\sin 72} = \frac{18.7}{\sin 50}$ [=24.4….]	M1		Use of the sine rule
	$BC = \frac{18.7 \sin 72}{\sin 50}$	m1		Rearrangement
	(BC)=23.21(6..){= 23.2 to nearest 0.1cm}	A1	3	AG Need >1dp if using cm eg 23.21 or 23.22; at least 1dp if using mm.
(b)	Angle C = 180° – (50°+72°) = 58°	M1		Valid method to find either angle C (PI eg by sin C =0.848(04..)) or side AB
	Area of triangle = 0.5×18.7×23.2..×sin C	M1		OE eg 0.5×18.7×AB×sin72°
	….. = 184 cm^2	A1	3	Accept 183.8 to 184.2 Condone missing/wrong units
	Total		**6**	

7. June 2008

4(a)	$\{BC^2 =\}\ 7.6^2 + 8.3^2 - 2\times 7.6\times 8.3\cos 65$ ….. $= 57.76 + 68.89 - 53.3175...$	M1 m1		RHS of cosine rule used Correct order of evaluation
	$BC = \sqrt{73.33..} = 8.563..$ (= 8.56 m)	A1	3	AG; must see $\sqrt{73.33....}$ or > 3sf value
(b)	Area triangle $= \frac{1}{2}\times 7.6\times 8.3\times \sin 65$ = 28.58… = 28.6 (m^2)	M1 A1	 2	Use of $\frac{1}{2}bc\sin A$ OE Condone > 3sf
(c)	Area of triangle = 0.5× BC × AD AD = [Ans (b)] ÷ [0.5×Ans (a)] AD = 6.67.. = 6.7 (m)	M1 m1 A1	 3	Or valid method to find sin$B$ or sin$C$ Or AD = 7.6sin$B$; Or AD = 8.3sin$C$ If not 6.7 accept 6.65 to 6.69 inclusive.
	Total		**8**	

8. Jan 2009

3(a)	{Area =} $\frac{1}{2}\times 7.4\times 5.26\times \sin 63°$	M1		
	= 17.3(407…) {m^2}	A1	2	Accept any value from 17.3 to 17.341
(b)	$\{BC^2 =\}\ 5.26^2 + 7.4^2 - 2\times 5.26\times 7.4\cos 63$	M1		RHS of cosine rule used
	….. = 27.66(76) + 54.76 − 35.34(22…)	m1		Correct order of evaluation
	$\Rightarrow BC = \sqrt{47.08(5...)} = 6.861(8..)$			
	BC = 6.86 {m} to 3sf	A1	3	AG. Cand. must show a 4th sf in either $\sqrt{47.08(5...)}$ or 6.861(8) before giving the printed answer 6.86

(c)	$\frac{\sin B}{5.26} = \frac{\sin 63}{BC}$	M1		Sine rule involving 'sin B' [If valid cosine rule used to find cos B, no marks awarded until stage of converting to sin B]
	$\sin B = 0.68$ to 2sf	A1	2	If not 0.68, accept AWRT any value from 0.682 to 0.684 inclusive
	ALTn			
	$\frac{1}{2} \times 7.4 \times (6.86..) \times \sin B =$ c's ans (a)	(M1)		(6.86..) could be c's ans (b)
	$\sin B = 0.68$ to 2sf	(A1)		If not 0.68, accept AWRT any value from 0.682 to 0.684 inclusive
	Total		**7**	

9. June 2009

1(a)	$5^2 = 7^2 + 8^2 - 2 \times 7 \times 8\cos\theta$	M1		Use of the cosine rule – must be correct (PI by the correct line below)
	$\cos\theta = \frac{7^2 + 8^2 - 5^2}{2 \times 7 \times 8}$ $(= \frac{88}{112} = 0.7857\ldots)$	m1		Rearrangement
	$\theta = 38.21\ldots = 38.2°$ (to nearest 0.1°)	A1	3	CSO (Must see either exact value for $\cos\theta$ or at least 4sf value for either $\cos\theta$ or θ before the printed answer 38.2°) AG
(b)	Area $= \frac{1}{2} \times 7 \times 8 \sin\theta$	M1		OE eg Area $= \sqrt{10(10-5)(10-8)(10-7)}$ $(= \sqrt{300})$
	$= 17.3$ {cm^2} to 3sf	A1	2	Condone 17.31 to 17.33 inclusive
	Total		**5**	

10. June 2010

3(a)	$\frac{6}{\sin\theta} = \frac{15}{\sin 150}$	M1		Sine rule OE PI
	$\sin\theta = \frac{6 \times \sin 150}{15}$ $\{= 0.2\}$	m1		Rearrangement
	$\theta = 11.53(6..) = 11.5°$ {to nearest 0.1°}	A1	3	AG Must see at least 4sf value or an exact value for $\sin\theta$ (0.2, 3/15, OE) before seeing the printed value 11.5
(b)	Angle $B = 180 - (150 + \theta) = 18.5$ {to 3sf}	B1		Award for B = any value between 18 and 19 inclusive [18.463041….]
	Area $= \frac{1}{2} \times 6 \times 15 \sin B$	M1		
	$= 14.3$ {cm^2} to 3sf	A1	3	Accept a value 14.2 to 14.3 inclusive Note: For methods involving AC, for the M1 need both a correct method to find AC and a correct area formula
	Total		**6**	

11. Jan 2011

3(a)	$10^2 = 8^2 + 5^2 - 2 \times 8 \times 5\cos\theta$	M1		Use of the cosine rule PI by next line
	$\cos\theta = \frac{8^2 + 5^2 - 10^2}{2 \times 8 \times 5}$ $(= -\frac{11}{80} = -0.1375)$	m1		Rearrangement
	$\theta = 97.90(32...) = 97.9°$ (to nearest 0.1°)	A1	3	CSO (Must see either exact value for $\cos\theta$ or at least 4sf value for either $\cos\theta$ or θ before the printed answer 97.9°) AG

(b)(i)	Area $= \frac{1}{2}\times 8\times 5\sin\theta$	M1		OE
	$= 19.810\ldots = 19.8$ (cm^2) to 3sf	A1	2	Condone > 3sf
(ii)	Area of triangle $= 0.5\times BC\times AD$	M1		Or valid method to find sin B or sin C or B or C
	$AD =$ [Ans.(b)(i)]÷[$0.5\times BC$]	m1		Or $AD = 5\sin B$; or $AD = 8\sin C$ OE
	$AD = \frac{19.810..}{5} = 3.962.. = 3.96$ (cm) to 3sf	A1	3	Condone > 3sf
	Total		**8**	

12. June 2011

1(a)	$\frac{10}{\sin\theta} = \frac{9}{\sin 54}$	M1		Sine rule, with $\sin\theta$ being the only unknown
	$\sin\theta = \frac{10\times\sin 54}{9}$ $\left\{=\frac{8.09...}{9}\right\}$ $\left\{=\frac{10}{11.12...}\right\}$	m1		Correct rearrangement to '$\sin\theta = \ldots$' or to '$\theta = \sin^{-1}(\ldots)$'
	$\sin\theta = 0.898(9...)$, $\theta = 64.01(48\ldots)$			
	$\theta = 64°$ {to nearest degree}	A1	3	AG m1 must have been awarded and must see at least 3sf value either for $\sin\theta$ so that $0.898\le\sin\theta\le 0.8993$ or for θ so that $64.0\le\theta\le 64.1$ as well as seeing 'θ(OE)= 64'
(b)	Angle $C = 180 - (54+\theta) = 62$ {to 2sf}	B1		$C = 62$. AWRT 62. PI if '$C = 180 - (54+\theta)$' and accurate later work.
	Area $= \frac{1}{2}\times 10\times 9\sin 62$	M1		OE Ft c's value for C ($C\neq 54$, $C\neq\theta$)
	$= 39.73.. = 40$ {cm^2 to nearest sq cm}	A1	3	If not 40 condone a value 39.7 to 39.8 inclusive.
	Total		**6**	

13. Jan 2012

4(a)	Area $= \frac{1}{2}\times 10\times AC\sin 150$	M1		$\frac{1}{2}\times 10\times AC\sin 150$
	$40 = 2.5AC$ so $AC = 16$ (m)	A1	2	AG Be convinced
(b)	$\{BC^2=\}10^2 + 16^2 - 2\times 10\times 16\times\cos 150$	M1		RHS of cosine rule used
	$= 100 + 256 + 277.128\ldots$	m1		Correct order of evaluation
	$BC = \sqrt{633.128....} = 25.162\ldots = 25.16$m	A1	3	AWRT 25.16
(c)	$\frac{10}{\sin C} = \frac{BC}{\sin 150}$ (or $\frac{BC}{\sin 150} = \frac{AC}{\sin B}$)	M1		A correct equation using sine rule or cosine rule or area formula for either B or C. Subst of BC or AC not required for this M.
	$\sin C = \frac{10\sin 150}{"25.16"}$ (=0.1987..)	m1		Correct rearrangement to either sin$C$ or cos$C$ or sin$B$ or cos$B$ equal to numerical expression ft on c's numerical value for BC. PI by correct C or (by correct B if Mscored)
	(or $\sin B = \frac{16\sin 150}{"25.16"}$ (=0.317… or 0.318))			
	Smallest angle, (C =) 11.5° to 1dp	A1	3	Accept a value 11.4 to 11.5 inclusive.
			8	

14. June 2012

2(a)	(Area) = $\frac{1}{2}(26)(31.5)\sin\theta$	M1		$\frac{1}{2}(26)(31.5)\sin(\theta)$ stated or used
	$\frac{1}{2}(26)(31.5)\times\frac{5}{13}$ = 157.5 (cm^2)	A1	2	OE eg $\frac{315}{2}$ Condone AWRT 157.50 NMS: 157.5 or AWRT 157.50 scores B2
(b)	$(\cos\theta =)\frac{12}{13}$	B1	1	$\frac{12}{13}$ OE exact fraction
(c)	$\{AC^2 =\}$ $31.5^2 + 26^2 - 2 \times 31.5 \times 26 \times \cos(\theta)$ $= 992.25 + 676 - 1512$	M1		RHS of cosine rule
	$= 1668.25 - 1512 = 156.25$	m1		Correct order of evaluation. Do not award if evaluation leads to or would lead to RHS value being outside interval 120 to 195
	$AC = \sqrt{156.25}$ = 12.5 (cm)	A1	3	12.5 OE with no sight of premature approximation clearly used
	(Alternative) $\{AC^2 =\}$ $(26 \sin\theta)^2 + (31.5 - 26\cos\theta)^2$ $= 10^2 + 7.5^2$ $AC = \sqrt{156.25}$ = 12.5 (cm)	(M1) (m1) (A1)	(3)	
	Total		**6**	

15. Jan 2013

3(a)	$\frac{1}{2}\times 5\times 6\sin C = 12.5$	M1		(Area=) $\frac{1}{2}\times 5\times 6\sin C$
	$\sin C = 0.833(3..)$	A1		AWRT 0.83 or 5/6 OE PI by e.g. seeing 56 or better
	(C is obtuse) $C = 123.6°$	A1	3	AWRT 123.6
(b)	$\{AB^2 =\} 5^2 + 6^2 - 2\times 5\times 6\cos C$	M1		RHS of cosine rule used
	$= 61 - 60\times(-0.553\ldots) = 94.1(66\ldots)$	m1		Correct ft evaluation, to at least 2 sf, of AB^2 or AB using c's value of C.
	(AB =) 9.7 (cm to 2sf)	A1	3	If not 9.7 accept AWRT 9.70 or AWRT 9.71
	Total		**6**	