Practice examination questions for cubic graphs and factor theorem

1.Jan 2005

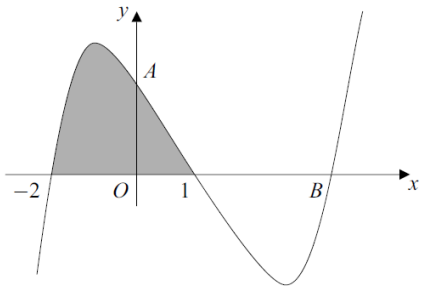
4 (a) The function f is defined for all values of x by $f(x) = x^3 - 3x^2 - 6x + 8$.

(i) Find the remainder when $f(x)$ is divided by $x + 1$. *(2 marks)*

(ii) Given that $f(1) = 0$ and $f(-2) = 0$, write down two linear factors of $f(x)$. *(2 marks)*

(iii) Hence express $x^3 - 3x^2 - 6x + 8$ as the product of three linear factors. *(2 marks)*

(b) The curve with equation $y = x^3 - 3x^2 - 6x + 8$ is sketched below.



(i) The curve intersects the y-axis at the point A. Find the y-coordinate of A. *(1 mark)*

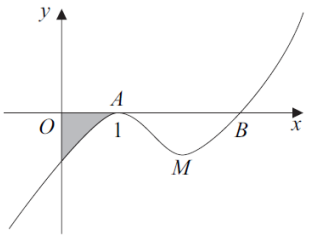
(ii) The curve crosses the x-axis when $x = -2$, when $x = 1$ and also at the point B. Use the results from part (a) to find the x-coordinate of B. *(1 mark)*

(c) (i) Find $\int (x^3 - 3x^2 - 6x + 8)\, dx$. *(4 marks)*

(ii) Hence find the area of the shaded region bounded by the curve and the x-axis. *(3 marks)*

2.June 2005

4 The curve with equation $y = x^3 - 5x^2 + 7x - 3$ is sketched below.



The curve touches the x-axis at the point A (1, 0) and cuts the x-axis at the point B.

(a) (i) Use the factor theorem to show that $x - 3$ is a factor of

$$p(x) = x^3 - 5x^2 + 7x - 3$$ *(2 marks)*

(ii) Hence find the coordinates of B. *(1 mark)*

(b) The point M, shown on the diagram, is a minimum point of the curve with equation $y = x^3 - 5x^2 + 7x - 3$.

(i) Find $\frac{dy}{dx}$. *(2 marks)*

(ii) Hence determine the x-coordinate of M. *(3 marks)*

(c) Find the value of $\frac{d^2y}{dx^2}$ when $x = 1$. *(2 marks)*

(d) (i) Find $\int \left(x^3 - 5x^2 + 7x - 3\right) dx$. *(4 marks)*

(ii) Hence determine the area of the shaded region bounded by the curve and the coordinate axes. *(4 marks)*

3.Jan 2006

6 The polynomial $p(x)$ is given by

$$p(x) = x^3 + x^2 - 10x + 8$$

(a) (i) Using the factor theorem, show that $x - 2$ is a factor of $p(x)$. *(2 marks)*

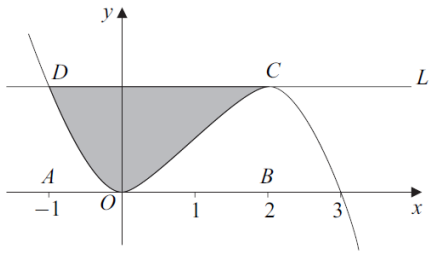
(ii) Hence express $p(x)$ as the product of three linear factors. *(3 marks)*

(b) Sketch the curve with equation $y = x^3 + x^2 - 10x + 8$, showing the coordinates of the points where the curve cuts the axes.

(You are not required to calculate the coordinates of the stationary points.) *(4 marks)*

4.Jan2006

8 The diagram shows the curve with equation $y = 3x^2 - x^3$ and the line L.



The points A and B have coordinates $(-1, 0)$ and $(2, 0)$ respectively. The curve touches the x-axis at the origin O and crosses the x-axis at the point $(3, 0)$. The line L cuts the curve at the point D where $x = -1$ and touches the curve at C where $x = 2$.

(a) Find the area of the rectangle $ABCD$. *(2 marks)*

(a) Find the area of the rectangle $ABCD$. *(2 marks)*

(b) (i) Find $\int (3x^2 - x^3)\mathrm{d}x$. *(3 marks)*

(ii) Hence find the area of the shaded region bounded by the curve and the line L. *(4 marks)*

(c) For the curve above with equation $y = 3x^2 - x^3$:

(i) find $\frac{\mathrm{d}y}{\mathrm{d}x}$; *(2 marks)*

(ii) hence find an equation of the tangent at the point on the curve where $x = 1$; *(3 marks)*

(iii) show that y is decreasing when $x^2 - 2x > 0$. *(2 marks)*

(d) Solve the inequality $x^2 - 2x > 0$. *(2 marks)*

5.June 2006

6 The polynomial $\mathrm{p}(x)$ is given by $\mathrm{p}(x) = x^3 - 4x^2 + 3x$.

(a) Use the Factor Theorem to show that $x - 3$ is a factor of $\mathrm{p}(x)$. *(2 marks)*

(b) Express $\mathrm{p}(x)$ as the product of three linear factors. *(2 marks)*

(c) (i) Use the Remainder Theorem to find the remainder, r, when $\mathrm{p}(x)$ is divided by $x - 2$. *(2 marks)*

(ii) Using algebraic division, or otherwise, express $\mathrm{p}(x)$ in the form

$$(x - 2)(x^2 + ax + b) + r$$

where a, b and r are constants. *(4 marks)*

6.Jan 2007

1 The polynomial $\mathrm{p}(x)$ is given by

$$\mathrm{p}(x) = x^3 - 4x^2 - 7x + k$$

where k is a constant.

(a) (i) Given that $x + 2$ is a factor of $\mathrm{p}(x)$, show that $k = 10$. *(2 marks)*

(ii) Express $\mathrm{p}(x)$ as the product of three linear factors. *(3 marks)*

(b) Use the Remainder Theorem to find the remainder when $\mathrm{p}(x)$ is divided by $x - 3$. *(2 marks)*

(c) Sketch the curve with equation $y = x^3 - 4x^2 - 7x + 10$, indicating the values where the curve crosses the x-axis and the y-axis. (You are **not** required to find the coordinates of the stationary points.) *(4 marks)*

7.June 2007

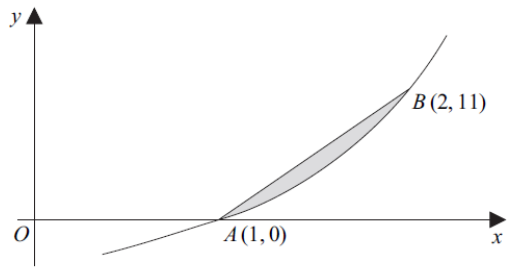
6 (a) The polynomial $f(x)$ is given by $f(x) = x^3 + 4x - 5$.

(i) Use the Factor Theorem to show that $x - 1$ is a factor of $f(x)$. *(2 marks)*

(ii) Express $f(x)$ in the form $(x-1)(x^2 + px + q)$, where p and q are integers. *(2 marks)*

(iii) Hence show that the equation $f(x) = 0$ has exactly one real root and state its value. *(3 marks)*

(b) The curve with equation $y = x^3 + 4x - 5$ is sketched below.



The curve cuts the x-axis at the point $A(1, 0)$ and the point $B(2, 11)$ lies on the curve.

(i) Find $\int (x^3 + 4x - 5)\,dx$. *(3 marks)*

(ii) Hence find the area of the shaded region bounded by the curve and the line AB. *(4 marks)*

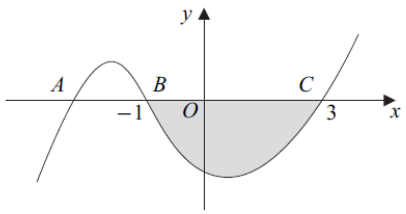
8.Jan 2008

6 (a) The polynomial $p(x)$ is given by $p(x) = x^3 - 7x - 6$.

(i) Use the Factor Theorem to show that $x + 1$ is a factor of $p(x)$. *(2 marks)*

(ii) Express $p(x) = x^3 - 7x - 6$ as the product of three linear factors. *(3 marks)*

(b) The curve with equation $y = x^3 - 7x - 6$ is sketched below.



The curve cuts the x-axis at the point A and the points $B(-1, 0)$ and $C(3, 0)$.

(i) State the coordinates of the point A. *(1 mark)*

(ii) Find $\int_{-1}^{3} (x^3 - 7x - 6)\,dx$. *(5 marks)*

(iii) Hence find the area of the shaded region bounded by the curve $y = x^3 - 7x - 6$ and the x-axis between B and C. *(1 mark)*

(iv) Find the gradient of the curve $y = x^3 - 7x - 6$ at the point B. *(3 marks)*

(v) Hence find an equation of the normal to the curve at the point B. *(3 marks)*

9.June 2008

6 The polynomial $p(x)$ is given by $p(x) = x^3 + x^2 - 8x - 12$.

(a) Use the Remainder Theorem to find the remainder when $p(x)$ is divided by $x - 1$. *(2 marks)*

(b) (i) Use the Factor Theorem to show that $x + 2$ is a factor of $p(x)$. *(2 marks)*

(ii) Express $p(x)$ as the product of linear factors. *(3 marks)*

(c) (i) The curve with equation $y = x^3 + x^2 - 8x - 12$ passes through the point $(0, k)$. State the value of k. *(1 mark)*

(ii) Sketch the graph of $y = x^3 + x^2 - 8x - 12$, indicating the values of x where the curve touches or crosses the x-axis. *(3 marks)*

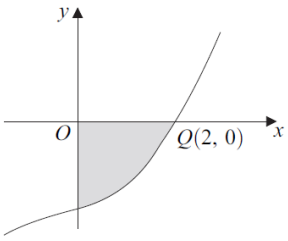
10.Jan 2009

6 (a) The polynomial $p(x)$ is given by $p(x) = x^3 + x - 10$.

(i) Use the Factor Theorem to show that $x - 2$ is a factor of $p(x)$. *(2 marks)*

(ii) Express $p(x)$ in the form $(x - 2)(x^2 + ax + b)$, where a and b are constants. *(2 marks)*

(b) The curve C with equation $y = x^3 + x - 10$, sketched below, crosses the x-axis at the point $Q(2, 0)$.



(i) Find the gradient of the curve C at the point Q. *(4 marks)*

(ii) Hence find an equation of the tangent to the curve C at the point Q. *(2 marks)*

(iii) Find $\int (x^3 + x - 10)\, dx$. *(3 marks)*

(iv) Hence find the area of the shaded region bounded by the curve C and the coordinate axes. *(2 marks)*

11.June 2009

4 (a) The polynomial $p(x)$ is given by $p(x) = x^3 - x + 6$.

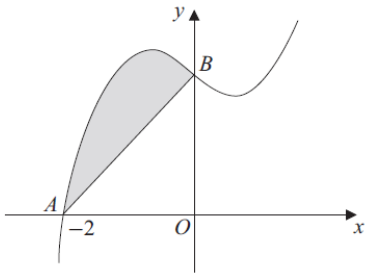
(i) Find the remainder when $p(x)$ is divided by $x - 3$. *(2 marks)*

(ii) Use the Factor Theorem to show that $x + 2$ is a factor of $p(x)$. *(2 marks)*

(iii) Express $p(x) = x^3 - x + 6$ in the form $(x + 2)(x^2 + bx + c)$, where b and c are integers. *(2 marks)*

(iv) The equation $p(x) = 0$ has one root equal to -2. Show that the equation has no other real roots. *(2 marks)*

(b) The curve with equation $y = x^3 - x + 6$ is sketched below.



The curve cuts the x-axis at the point $A\,(-2, 0)$ and the y-axis at the point B.

(i) State the y-coordinate of the point B. *(1 mark)*

(ii) Find $\int_{-2}^{0} (x^3 - x + 6)\,dx$. *(5 marks)*

(iii) Hence find the area of the shaded region bounded by the curve $y = x^3 - x + 6$ and the line AB. *(3 marks)*

12.Jan 2010

1 The polynomial $p(x)$ is given by $p(x) = x^3 - 13x - 12$.

(a) Use the Factor Theorem to show that $x + 3$ is a factor of $p(x)$. *(2 marks)*

(b) Express $p(x)$ as the product of three linear factors. *(3 marks)*

13.June 2010

3 The polynomial $p(x)$ is given by

$$p(x) = x^3 + 7x^2 + 7x - 15$$

(a) (i) Use the Factor Theorem to show that $x + 3$ is a factor of $p(x)$. *(2 marks)*

(ii) Express $p(x)$ as the product of three linear factors. *(3 marks)*

(b) Use the Remainder Theorem to find the remainder when $p(x)$ is divided by $x - 2$. *(2 marks)*

(c) (i) Verify that $p(-1) < p(0)$. *(1 mark)*

(ii) Sketch the curve with equation $y = x^3 + 7x^2 + 7x - 15$, indicating the values where the curve crosses the coordinate axes. *(4 marks)*

14.Jan 2011

5 (a) (i) Sketch the curve with equation $y = x(x-2)^2$. *(3 marks)*

(ii) Show that the equation $x(x-2)^2 = 3$ can be expressed as

$$x^3 - 4x^2 + 4x - 3 = 0$$ *(1 mark)*

(b) The polynomial $\mathrm{p}(x)$ is given by $\mathrm{p}(x) = x^3 - 4x^2 + 4x - 3$.

(i) Find the remainder when $\mathrm{p}(x)$ is divided by $x+1$. *(2 marks)*

(ii) Use the Factor Theorem to show that $x-3$ is a factor of $\mathrm{p}(x)$. *(2 marks)*

(iii) Express $\mathrm{p}(x)$ in the form $(x-3)(x^2+bx+c)$, where b and c are integers. *(2 marks)*

(c) Hence show that the equation $x(x-2)^2 = 3$ has only one real root and state the value of this root. *(3 marks)*

15.June 2011

5 The polynomial $\mathrm{p}(x)$ is given by $\mathrm{p}(x) = x^3 - 2x^2 + 3$.

(a) Use the Remainder Theorem to find the remainder when $\mathrm{p}(x)$ is divided by $x-3$. *(2 marks)*

(b) Use the Factor Theorem to show that $x+1$ is a factor of $\mathrm{p}(x)$. *(2 marks)*

(c) (i) Express $\mathrm{p}(x) = x^3 - 2x^2 + 3$ in the form $(x+1)(x^2+bx+c)$, where b and c are integers. *(2 marks)*

(ii) Hence show that the equation $\mathrm{p}(x) = 0$ has exactly one real root. *(2 marks)*

16.Jan 1012

5 The polynomial $\mathrm{p}(x)$ is given by $\mathrm{p}(x) = x^3 + cx^2 + dx - 12$, where c and d are constants.

(a) When $\mathrm{p}(x)$ is divided by $x+2$, the remainder is -150.

Show that $2c - d + 65 = 0$. *(3 marks)*

(b) Given that $x-3$ is a factor of $\mathrm{p}(x)$, find another equation involving c and d. *(2 marks)*

(c) By solving these two equations, find the value of c and the value of d. *(3 marks)*

17.June 2012

3 The polynomial $p(x)$ is given by

$$p(x) = x^3 + 2x^2 - 5x - 6$$

(a) (i) Use the Factor Theorem to show that $x + 1$ is a factor of $p(x)$. *(2 marks)*

(ii) Express $p(x)$ as the product of three linear factors. *(3 marks)*

(b) Verify that $p(0) > p(1)$. *(2 marks)*

(c) Sketch the curve with equation $y = x^3 + 2x^2 - 5x - 6$, indicating the values where the curve crosses the x-axis. *(3 marks)*

18.Jan 2013

5 The polynomial $p(x)$ is given by

$$p(x) = x^3 - 4x^2 - 3x + 18$$

(a) Use the Remainder Theorem to find the remainder when $p(x)$ is divided by $x + 1$. *(2 marks)*

(b) (i) Use the Factor Theorem to show that $x - 3$ is a factor of $p(x)$. *(2 marks)*

(ii) Express $p(x)$ as a product of linear factors. *(3 marks)*

(c) Sketch the curve with equation $y = x^3 - 4x^2 - 3x + 18$, stating the values of x where the curve meets the x-axis. *(3 marks)*

Answers

1.Jan 2005

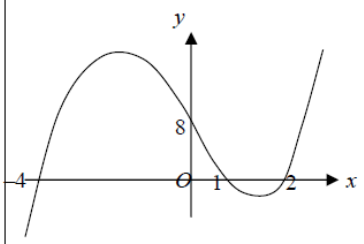
Q	Solution	Marks	Total	Comments
4(a)(i)	f(–1) = –1 – 3 + 6 + 8 (Remainder) = 10	M1 A1	 2	Or long division up to remainder term
(ii)	$x-1$ is a factor $x+2$ is a factor	B1 B1	 2	May be earned retrospectively From part (iii)
(iii)	Attempt at third factor f(x) = $(x-1)(x+2)(x-4)$	M1 A1	 2	Multiplying/ dividing/factor theorem $(x+4) \Rightarrow$ M1, A0
(b)(i)	At A , $y = 8$	B1	1	Or (0,8)
(ii)	At B , $x = 4$	B1	1	Or (4,0) NO ft of wrong factor
(c)(i)	$\frac{x^4}{4} - x^3 - 3x^2 + 8x \quad (+c)$	M1 A1 A1 A1	 4	Increase one power by 1 One term correct (unsimplified) Two other terms correct (unsimplified) All correct (unsimplified) (condone missing + c)
(ii)	Realisation that limits are –2 and 1 Area = $\left[\frac{1}{4} - 1 - 3 + 8\right] - [4 + 8 - 12 - 16]$ $= 20\frac{1}{4}$	B1 M1 A1	 3	Condone wrong way round Attempt to sub their limits into their (c)(i) CSO. Must use F(1) – F(–2) correctly
	Total		**15**	

2.June 2005

Q	Solution	Marks	Total	Comments
4(a)(i)	p(3) = 27 – 45 + 21 – 3 p(3) = 0 $\Rightarrow$ x–3 is a factor	M1 A1	 2	Finding p(3) Shown = 0 plus a statement Or $(x-3)(x^2-2x+1)$
(ii)	B is point (3,0)	B1	1	Must have coordinates
(b)(i)	$\frac{dy}{dx} = 3x^2 - 10x + 7$	M1 A1	 2	One term correct All correct with NO +c etc
(ii)	$3x^2 - 10x + 7 = 0$ $\Rightarrow (x-1)(3x-7) = 0$ $\Rightarrow$ at M, $x = \frac{7}{3}$	M1 m1 A1	 3	Putting their $\frac{dy}{dx} = 0$ Attempt to use quad formula or factorise **CSO** factors correct etc
(c)	$\frac{d^2y}{dx^2} = 6x - 10$ sub $x = 1$, $\Rightarrow \frac{d^2y}{dx^2} = -4$	B1√ B1√	 2	ft their $\frac{dy}{dx}$ ft their $\frac{d^2y}{dx^2}$
(d)(i)	$\frac{x^4}{4} - \frac{5x^3}{3} + \frac{7x^2}{2} - 3x \quad (+c)$	M1 A1 A1 A1	 4	Increase one power by 1 One term correct Two other terms correct All correct (condone missing + c)

(ii)	Realisation that limits are 0 and 1	B1		Condone wrong way round
	$\left[\frac{1}{4}-\frac{5}{3}+\frac{7}{2}-3\right]-0$	M1		Attempt to sub their limits into their (d)(i)
	$=-\frac{11}{12}$	A1		**CSO**. Must use F(1) – F(0) correctly
	Area $=\frac{11}{12}$	E1	4	**CSO**. Convincing argument
	Total		**18**	

3.Jan 2006

6(a)(i)	$p(2)=8+4-20+8$	M1		Finding p(2) M0 *long division*
	$=0$, $\Rightarrow$ $x-2$ is a factor	A1	2	Shown = 0 **AND** conclusion/ statement about x –2 being a factor
(ii)	Attempt at quadratic factor	M1		or factor theorem again for 2nd factor
	x^2+3x-4	A1		or (x+4) or (x–1) proved to be a factor
	$p(x)=(x-2)(x+4)(x-1)$	A1	3	
(b)		B1		Graph through (0,8) 8 marked
		B1√		Ft "their factors" 3 roots marked on x-axis
		M1		Cubic curve through their 3 points
		A1	4	Correct including x- intercepts correct Condone max on y-axis etc or slightly wrong concavity at ends of graph
	Total		**9**	

4.Jan 2006

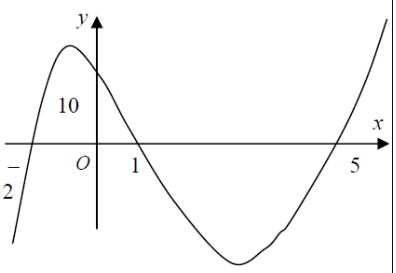
8(a)	$y_D=3+1=4$ or $y_C=12-8=4$	M1		Attempt at either y coordinate
	Area $ABCD$ $=3\times4=12$	A1	2	
(b)(i)	$x^3-\frac{x^4}{4}$ $(+C)$	M1		Increase one power by 1
		A1		One term correct unsimplified
		A1	3	All correct unsimplified (condone no +C)
(ii)	Sub limits –1 and 2 into their (b) (i) ans	M1		May use both –1, 0 and 0, 2 instead
	$[8-4]-\left[-1-\frac{1}{4}\right]$ $=5\frac{1}{4}$	A1		
	Shaded area = "their" (rectangle– integral)	M1		Alt method: difference of two integrals
	$=12-5\frac{1}{4}=6\frac{3}{4}$	A1	4	**CSO**. Attempted M2, A2
(c)(i)	$\frac{dy}{dx}=6x-3x^2$	M1		One term correct
		A1	2	All correct (no +C etc)
(ii)	When $x=1$, $y=2$ when $x=1$,	B1		May be implied by correct tgt equation
	$\frac{dy}{dx}=3$ as 'their' grad of tgt	M1√		Ft their derivative when $x=1$
	Tangent is $y-2=3(x-1)$	A1	3	Any correct form $y=3x-1$ etc
(iii)	Decreasing when $\frac{dy}{dx}=6x-3x^2<0$	M1		Watch no fudging here!! May work backwards in proof.

(iii)	Decreasing when $\frac{dy}{dx} = 6x - 3x^2 < 0$	M1		Watch no fudging here!! May work backwards in proof.
	$3(2x - x^2) < 0 \quad \Rightarrow x^2 - 2x > 0$	A1	2	**AG** (be convinced no step incorrect)
(d)	Two critical points 0 and 2	M1		Marked on diagram or in solution
	$x > 2$, $x < 0$ ONLY	A1	2	or M1 A0 for $0 < x < 2$ or $0 > x > 2$ **SC** B1 for $x > 2$ (or $x < 0$)
	Total		**18**	

5.June 2006

Q	Solution	Marks	Total	Comments
6(a)	$p(3) = 27 - 36 + 9$	M1		Finding p(3) and **not** long division
	$p(3) = 0 \Rightarrow x - 3$ is a factor	A1	2	Shown $= 0$ **plus a statement**
(b)	$x(x^2 - 4x + 3)$ or $(x-3)(x^2 - x)$ attempt	M1		Or $p(1) = 0 \Rightarrow x - 1$ is a factor attempt
	$p(x) = x(x-1)(x-3)$	A1	2	Condone $x + 0$ or $x - 0$ as factor
(c)(i)	$p(2) = 8 - 16 + 6$	M1		Must use p(2) and **not** long division
	(Remainder is) -2	A1	2	
(ii)	Attempt to multiply out and compare	M1		Or long division (2 terms of quotient)
	coefficients $a = -2$	A1		$x^2 - 2x...$
	$b = -1$	A1		-1
	$r = -2$	A1	4	Withhold final A1 for long division unless written as $(x-2)(x^2 - 2x - 1) - 2$
	SC B1 for $r = -2$ if M0 scored			
	Total		**10**	

6.Jan 2007

Q	Solution	Marks	Total	Comments
1(a)(i)	$p(-2) = -8 - 16 + 14 + k$	M1		or long division or $(x+2)(x^2 - 6x + 5)$
	$p(-2) = 0 \Rightarrow -10 + k = 0 \quad \Rightarrow k = 10$	A1	2	**AG** likely withhold if $p(-2) = 0$ not seen
	Must have statement if k=10 substitute			
(ii)	$p(x) = (x+2)(x^2 + ... \quad 5)$	M1		Attempt at quadratic or second linear
	$p(x) = (x+2)(x^2 - 6x + 5)$	A1		factor $(x-1)$ or $(x-5)$ <u>from factor theorem</u>
	$\Rightarrow p(x) = (x+2)(x-1)(x-5)$	A1	3	Must be written as product
(b)	$p(3) = 27 - 36 - 21 + k$	M1		long division scores M0
	(Remainder) $= k - 30 = \underline{-20}$	A1	2	Condone $k - 30$
(c)		B1		Curve thro' 10 marked on y-axis
		B1√		**FT** their 3 roots marked on x-axis
		M1		Cubic shape with a max and min
		A1	4	Correct graph (roughly as on left) going beyond -2 and 5 (condone max anywhere between $x = -2$ and 1 and min between 1 and 5)
	Total		**11**	

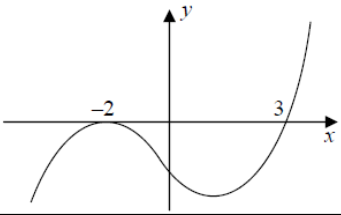
7.June 2007

Q	Solution	Marks	Total	Comments
6(a)(i)	$f(1)=1+4-5$	M1		must find $f(1)$ NOT long division
	$\Rightarrow f(1)=0 \Rightarrow (x-1)$ is factor	A1	2	shown $=0$ plus a statement
(ii)	Attempt at x^2+x+5	M1		long division leading to $x^2 \pm x + ...$ or equating coefficients
	$f(x)=(x-1)(x^2+x+5)$	A1	2	$p=1,\ q=5$ by inspection scores B1, B1
(iii)	$(x=)\ 1$ is real root	B1		
	Consider b^2-4ac for their x^2+x+5	M1		not the cubic!
	$b^2-4ac=1^2-4\times5=-19<0$			
	Hence no real roots (or only real root is 1)	A1	3	CSO; all values correct plus a statement
(b)(i)	$\int ...\,dx=\frac{x^4}{4}+2x^2-5x\ (+c)$	M1 A1 A1	3	one term correct unsimplified second term correct unsimplified all correct unsimplified
(ii)	$[4+8-10]-\left[\frac{1}{4}+2-5\right]$	M1		correct use of limits 1 and 2; $F(2)-F(1)$ attempted
	$=4\frac{3}{4}$	A1		
	Area of $\Delta=\frac{1}{2}\times11=5\frac{1}{2}$	B1		correct unsimplified
	$\Rightarrow$ shaded area $=5\frac{1}{2}-4\frac{3}{4}$			combined integral of $7x-6-x^3$ scores M1 for limits correctly used then
	$=\frac{3}{4}$	A1	4	A3 correct answer with all working correct
	Total		**14**	

8.Jan 2008

Q	Solution	Marks	Total	Comments
6(a)(i)	$p(-1)=-1+7-6$	M1		Finding $p(-1)$
	$=0$ therefore $x+1$ is a factor	A1	2	Shown to $=0$ **plus statement**
(ii)	$p(x)=(x+1)(x^2-x-6)$	M1		Long division/inspection (2 terms correct)
		A1		Quadratic factor correct
	$p(x)=(x+1)(x+2)(x-3)$	A1	3	May earn M1,A1 for correct second factor then A1 for $(x+1)(x+2)(x-3)$
(b)(i)	$A(-2,0)$	B1	1	Condone $x=-2$
(ii)	$\frac{x^4}{4}-\frac{7x^2}{2}-6x\quad(+c)$	M1		One term correct
		A1		Another term correct
	(may have $+c$ or not)	A1		All correct unsimplified
(iii)	Area of shaded region = 32	B1√	1	FT their (b)(ii) but positive value needed
(iv)	$\frac{dy}{dx}=3x^2-7$	M1		One term correct
		A1		All correct (no $+c$ etc)
	When $x=-1$, gradient $=-4$	A1	3	CSO
(v)	Gradient of normal $=\frac{1}{4}$	B1√		
	$y=$ "their gradient" $(x\pm1)$	M1		Must be finding **normal**, not tangent
	$y=\frac{1}{4}(x+1)$	A1	3	CSO; any correct form eg $4y-x=1$
	Total		**18**	

9.June 2008

6(a)	Remainder $= p(1) = 1+1-8-12$ $= -18$	M1 A1	 2	Use of p(1) NOT long division
(b)(i)	$p(-2) = -8+4+16-12$ $= 0 \Rightarrow (x+2)$ is factor	M1 A1	 2	NOT long division $p(-2)$ shown $= 0$ **and** statement
(ii)	Quad factor by comparing coefficients or $(x^2 + kx \pm 6)$ by inspection	M1		Or full long division or attempt at Factor Theorem using $f(\pm 3)$
	$p(x) = (x+2)(x^2 - x - 6)$	A1		Correct quadratic factor or $(x-3)$ shown to be factor by Factor Theorem
	$p(x) = (x+2)^2(x-3)$ or $(x+2)(x+2)(x-3)$	A1	3	CSO; SC: B1 for $(x+2)(x***)(x-3)$ by inspection or without working
(c)(i)	$(k =)\ -12$	B1	1	Condone $y = -12$ or $(0, -12)$
(ii)		M1 A1 A1	 3	Cubic shape (one max and one min) Maximum at $(-2, 0)$ and through $(3, 0)$ – at least one of these values marked "correct" graph as shown (touching smoothly at –2, 3 marked and minimum to right of y-axis)
	Total		**11**	

10.Jan 2009

Q	Solution	Marks	Total	Comments
6(a)(i)	$p(2) = 8 + 2 - 10$ $\Rightarrow p(2) = 0 \Rightarrow (x-2)$ is factor	M1 A1	 2	Must find p(2) NOT long division Shown $= 0$ plus a statement
(ii)	Attempt at long division (generous)	M1		Obtaining a quotient $x^2 + cx + d$ or equating coefficients (full method)
	$p(x) = (x-2)(x^2 + 2x + 5)$	A1	2	$a = 2,\ b = 5$ by inspection B1, B1
(b)(i)	$\frac{dy}{dx} = 3x^2 + 1$	M1 A1		One term correct All correct – no $+c$ etc
	When $x = 2$ $\frac{dy}{dx} = 3 \times 4 + 1$	m1		Sub $x = 2$ into their $\frac{dy}{dx}$
	Therefore gradient at Q is 13	A1	4	CSO
(ii)	$y = 13(x-2)$	M1 A1	 2	Tangent (NOT normal) attempted ft their gradient answer from (b)(i) CSO; correct in any form
(iii)	$\int \ldots dx = \frac{x^4}{4} + \frac{x^2}{2} - 10x\ (+c)$	M1 A1 A1	 3	one term correct second term correct all correct (condone no +c)
(iii)	$\int \ldots dx = \frac{x^4}{4} + \frac{x^2}{2} - 10x\ (+c)$	M1 A1 A1	 3	one term correct second term correct all correct (condone no +c)

(iv)	$[4+2-20]-[0]=-14$	M1		F(2) attempted and possibly F(0) Must have earned M1 in (b)(iii)
	Area of shaded region = 14	A1	2	CSO; separate statement following correct evaluation of limits
	Total		**15**	

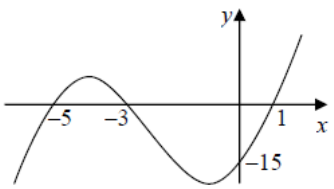
11.June 2009

Q	Solution	Marks	Total	Comments
4(a)(i)	$p(3)=27-3+6$ (Remainder) = 30 **Or** long division up to remainder Quotient= x^2+3x+8 and remainder = 30 clearly stated or indicated	M1 A1 (M1) (A1)	2	p(3) attempted
(ii)	$p(-2)=-8+2+6$ $p(-2)=0 \Rightarrow x+2$ is factor Minimum statement required "factor"	M1 A1	2	p(–2) attempted : **NOT** long division Shown = 0 plus statement May make statement *first* such as "x+2 is a factor if p(–2) = 0"
(iii)	$b=-2$ $c=3$ **or** long division/comparing coefficients $p(x)=(x+2)(x^2-2x+3)$	B1 B1 (M1) (A1)	2	No working required for B1 + B1 Try to mark first using B marks Award M1 if B0 earned and a clear method is used Must write final answer in this form if long division has been used to get A1
	$p(x)=(x+2)(x^2-2x+3)$	(A1)	2	Must write final answer in this form if long division has been used to get A1
(iv)	$b^2-4ac=(-2)^2-4\times3$ $b^2-4ac=-8$ (or <0) $\Rightarrow$ no (other) real roots **Or** $(x-1)^2+2$ $(x-1)^2+2>0$ therefore no real roots Or $(x-1)^2=-2$ has no real roots	M1 A1 (M1) (A1)	2	Discriminant correct from their quadratic M0 if $b=-1$, $c=6$ used (using cubic eqn) CSO All values must be correct plus statement Completion of square for their quadratic Shown to be positive plus statement regarding no real roots
(b)(i)	$(y_B=)\ 6$	B1	1	Condone (0, 6)
(ii)	$\frac{x^4}{4}-\frac{x^2}{2}+6x$	M1 A1 A1		One term correct Another term correct All correct (ignore + c or limits)
	$\left[\ \right]_{-2}^{0}=0-(4-2-12)$ $=10$	m1 A1	5	$F(-2)$ attempted CSO Clearly from F(0) – F(–2)
(iii)	Area of $\Delta=\frac{1}{2}\times2\times6$ $=6$ Shaded region area = 10 – 6 = 4	M1 A1 A1	3	Condone – 2 and ft their y_B value **Or** $\int_{-2}^{0}(3x+6)dx$ and attempt to integrate Must be positive allow –6 converted to +6 CSO 10 must come from correct working
	Total		**17**	

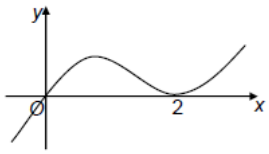
12.Jan 2010

Q	Solution	Marks	Total	Comments
1(a)	$p(-3)=(-3)^3-13(-3)-12$	M1		must attempt $p(-3)$ NOT long division
	$=-27+39-12$ $=0 \quad \Rightarrow x+3$ is factor	A1	2	shown $=0$ plus statement
(b)	$(x+3)(x^2+bx+c)$	M1		Full long division, comparing coefficients or by inspection either $b=-3$ or $c=-4$
	(x^2-3x-4) obtained	A1		or M1A1 for either $(x-4)$ or $(x+1)$ clearly found using factor theorem
	$(x+3)(x-4)(x+1)$	A1	3	CSO; must be seen as a product of 3 factors NMS full marks for correct product SC B1 for $(x+3)(x-4)(\ )$ or $(x+3)(x+1)(\ )$ or $(x+3)(x+4)(x-1)$ NMS
	Total		**5**	

13.June 2010

Q	Solution	Marks	Total	Comments
3(a)(i)	$p(-3)=(-3)^3+7(-3)^2+7(-3)-15$ $=-27+63-21-15$	M1		$p(-3)$ attempted; NOT long division This line alone implies M1
	$p(-3)=0 \Rightarrow (x+3$ is) factor	A1	2	$p(-3)$ shown $=0$ plus statement
(ii)	$p(x)=(x+3)(x^2+px+q)$	M1		Full long division, comparing coefficients or by inspection either $p=4$ or $q=-5$
	(Quadratic factor) (x^2+4x-5)	A1		or M1 A1 for either $x-1$ or $x+5$ *clearly* found using Factor Theorem
	$(p(x)=)\quad (x+3)(x-1)(x+5)$	A1	3	Must be seen as a product of 3 factors NMS full marks for correct product SC B2 for 3 correct factors listed NMS SC B1 for $(x+3)(x-1)(\ )$ or $(x+3)(x+5)(\ )$ or $(x+3)(x+1)(x-5)$
(b)	$p(2)=2^3+7\times2^2+7\times2-15$ or $(2+3)(2-1)(2+5)$ (Remainder) $=35$	M1 A1cso	 2	NOT long division; must be $p(2)$ May use "their" product of factors
(c)(i)	$p(-1)=-16$; $p(0)=-15$ $\Rightarrow p(-1)<p(0)$	B1	1	Values must be evaluated correctly
(ii)	 Cannot score M1A0A1 but can score B0M1A1A1	B1 M1 A1 A1	 4	y- intercept -15 marked or $(0,-15)$ stated Cubic graph – 1 max, 1 min $\wedge\!\!\!\sim$ shape with -5, -3, 1 marked Graph correct with minimum point to left of y-axis and going beyond both -5 and 1 Previous A1 must be scored
	Total		**12**	

14.Jan 2011

Q	Solution	Marks	Total	Comments
5(a)(i)		M1 A1 A1	 3	cubic curve with one max and one min (either way up) curve touching positive x-axis (either way up) correct graph passing through O and touching x-axis at 2
(ii)	$x(x^2-4x+4)=3$ $\Rightarrow x^3-4x^2+4x-3=0$	 B1	 1	 AG (must have = 0)
(b)(i)	$p(-1)=(-1)^3-4(-1)^2+4(-1)-3$ $(=-1-4-4-3)$ $=-12$	M1 A1	 2	p(–1) attempted (condone one slip) or full long division to remainder must indicate remainder $=-12$ if long division used
(ii)	$p(3)=3^3-4\times3^2+4\times3-3$ $p(3)=27-36+12-3$ $p(3)=0\Rightarrow x-3$ is factor	M1 A1	 2	p(3) attempted (condone one slip) NOT long division shown = 0 **plus statement**
(iii)	Either $b=-1$ (coefficient of x correct) or $c=1$ (constant term correct) $p(x)=(x-3)(x^2-x+1)$	M1 A1	 2	allow M1 for full attempt at long division or comparing coefficients if neither b nor c is correct
(c)	Discriminant of 'their quadratic' $=(-1)^2-4$ Discriminant $=-3$ (or <0) $\Rightarrow$ no real roots (Only real root is $x=$) 3	M1 A1cso B1	 3	numerical expression must be seen must have correct quadratic and statement and all working correct
	Total		**13**	

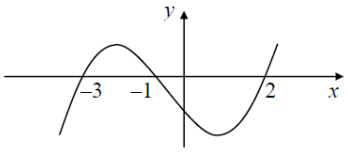
15.June 2011

Q	Solution	Marks	Total	Comments
5(a)	$p(3)=3^3-2\times3^2+3\ (=27-18+3)$ $=12$	M1 A1	 2	p(3) attempted; not long division
(b)	$p(-1)=(-1)^3-2(-1)^2+3$ $p(-1)=-1-2+3=0\ \Rightarrow\ x+1$ is a factor	M1 A1cso	 2	p(–1) attempted; not long division correctly shown = 0 plus statement
(c)(i)	Quadratic factor (x^2-3x+3) $(p(x)=)\ \ (x+1)(x^2-3x+3)$	M1 A1	 2	$b=-3$ or $c=3$ by inspection or full long division attempt or comparing coefficients must see correct product
(ii)	Discriminant of quadratic $b^2-4ac=(-3)^2-4\times3$ $b^2-4ac<0\Rightarrow$ no real roots from quadratic $\Rightarrow$ only one real root	M1 A1cso	 2	'their' discriminant considered possibly within quadratic equation formula
	Total		**8**	

16.Jan 2012

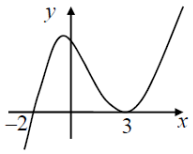
5(a)	$p(-2)=(-2)^3+(-2)^2c+(-2)d-12$	M1		p(–2) attempted *or* long division by $x+2$ as far as remainder
	'their' $-8+4c-2d-12=-150$	m1		putting expression for remainder = –150
	$\Rightarrow\ 2c-d+65=0$	A1cso	3	**AG** terms all on one side in any order (check that there are no errors in working)
(b)	$p(3)=3^3+3^2c+3d-12$	M1		p(3) attempted *or* long division by $x-3$ as far as remainder
	$9c+3d+15=0$	A1	2	any correct equation with terms collected eg $3c+d=-5$
(c)	$\left.\begin{array}{l}2c-d+65=0\\3c+d+5=0\end{array}\right\}\Rightarrow 5c=-70$	M1		Elimination of c or d
	$\Rightarrow\ c=-14\ ,\ d=37$ OE	A1 A1	 3	value of c ***or*** d correct unsimplified both c and d correct unsimplified
	Total		**8**	

17.June 2012

3(a)(i)	$p(-1)=(-1)^3+2(-1)^2-5(-1)-6$	M1		$p(-1)$ attempted **not** long division
	$p(-1)=-1+2+5-6=0\Rightarrow x+1$ is a factor	A1	2	CSO; correctly shown = 0 plus statement
(ii)	Quad factor in this form: (x^2+bx+c)	M1		long division as far as constant term ***or*** comparing coefficients, ***or*** $b=1$ ***or*** $c=-6$ by inspection
	x^2+x-6	A1		correct quadratic factor
	$[p(x)=]\ (x+1)(x+3)(x-2)$	A1	3	**must** see correct product
(b)	$p(0)=-6\ ;\ p(1)=-8$	M1		**both** $p(0)$ and $p(1)$ attempted **and** at least one value correct
	$\Rightarrow p(0)>p(1)$	A1	2	**AG** both values correct plus correct statement involving p(0) and p(1)
(c)		M1 A1 A1	3	cubic with one max and one min $\wedge\!\vee$ with $-3, -1, 2$ marked correct with minimum to right of y-axis AND going beyond –3 and 2
	Total		**10**	

18.Jan 2013

Q	Solution	Marks	Total	Comments
5(a)	$p(-1)=(-1)^3-4\times(-1)^2-3(-1)+18$ $(=-1-4+3+18)\ =16$	M1 A1	 2	p(–1) attempted **not** long division
(b)(i)	$p(3)=3^3-4\times3^2-3\times3+18$ $p(3)=27-36-9+18=0\ \Rightarrow\ x-3$ is a factor	M1 A1	 2	p(3) attempted **not** long division shown = 0 plus statement

(ii)	Quadratic factor $(x^2 - x + c)$ or $(x^2 + bx - 6)$	M1		$-x$ or -6 term by inspection *or* full long division by $x - 3$ *or* comparing coefficients *or* p(–2) attempted
	Quadratic factor $(x^2 - x - 6)$	A1		correct quadratic factor (or x+2 shown to be factor by Factor Theorem)
	$[p(x) =]\ (x-3)(x-3)(x+2)$	A1	3	or $[p(x) =]\ (x-3)^2(x+2)$ must see product of factors
(c)		M1		cubic curve with one maximum and one minimum
		A1		meeting x-axis at –2 and touching x-axis at 3
	Final A1 is dependent on previous A1 and can be withheld if curve has very poor curvature beyond $x = 3$, V shape at $x = 3$ etc	A1	3	graph as shown , going beyond $x = -2$ but condone max on or to right of y-axis
	Total		**10**	