# Practice examination questions for circles

## 1.Jan 2005

**3** A circle has equation $x^2 + y^2 - 12x - 6y + 20 = 0$.

(a) By completing the square, express the equation in the form

$$(x-a)^2 + (y-b)^2 = r^2$$ *(3 marks)*

(b) Write down:

(i) the coordinates of the centre of the circle; *(1 mark)*

(ii) the radius of the circle. *(1 mark)*

(c) The line with equation $y = x + 4$ intersects the circle at the points $P$ and $Q$.

(i) Show that the $x$-coordinates of $P$ and $Q$ satisfy the equation

$$x^2 - 5x + 6 = 0$$ *(2 marks)*

(ii) Find the coordinates of $P$ and $Q$. *(4 marks)*

## 2.June 2005

**3** A circle has centre $C(2, -1)$ and radius 5. The point $P$ has coordinates $(6, 2)$.

(a) Write down an equation of the circle. *(3 marks)*

(b) Verify that the point $P$ lies on the circle. *(2 marks)*

(c) Find the gradient of the line $CP$. *(2 marks)*

(d) (i) Find the gradient of a line which is perpendicular to $CP$. *(2 marks)*

(ii) Hence find an equation for the tangent to the circle at the point $P$. *(1 mark)*

## 3.Jan 2006

**5** A circle with centre $C$ has equation $x^2 + y^2 - 8x + 6y = 11$.

(a) By completing the square, express this equation in the form

$$(x-a)^2 + (y-b)^2 = r^2$$ *(3 marks)*

(b) Write down:

(i) the coordinates of $C$; *(1 mark)*

(ii) the radius of the circle. *(1 mark)*

(c) The point $O$ has coordinates $(0, 0)$.

(i) Find the length of $CO$. *(2 marks)*

(ii) Hence determine whether the point $O$ lies inside or outside the circle, giving a reason for your answer. *(2 marks)*

## 4.June 2006

**7** A circle has equation $x^2 + y^2 - 4x - 14 = 0$.

(a) Find:

(i) the coordinates of the centre of the circle; *(3 marks)*

(ii) the radius of the circle in the form $p\sqrt{2}$, where $p$ is an integer. *(3 marks)*

(b) A chord of the circle has length 8. Find the perpendicular distance from the centre of the circle to this chord. *(3 marks)*

(c) A line has equation $y = 2k - x$, where $k$ is a constant.

(i) Show that the $x$-coordinate of any point of intersection of the line and the circle satisfies the equation

$$x^2 - 2(k+1)x + 2k^2 - 7 = 0$$ *(3 marks)*

(ii) Find the values of $k$ for which the equation

$$x^2 - 2(k+1)x + 2k^2 - 7 = 0$$

has equal roots. *(4 marks)*

(iii) Describe the geometrical relationship between the line and the circle when $k$ takes either of the values found in part (c)(ii). *(1 mark)*

## 5.Jan 2007

**4** A circle with centre $C$ has equation $x^2 + y^2 + 2x - 12y + 12 = 0$.

(a) By completing the square, express this equation in the form

$$(x-a)^2 + (y-b)^2 = r^2$$ *(3 marks)*

(b) Write down:

(i) the coordinates of $C$; *(1 mark)*

(ii) the radius of the circle. *(1 mark)*

(c) Show that the circle does **not** intersect the $x$-axis. *(2 marks)*

(d) The line with equation $x + y = 4$ intersects the circle at the points $P$ and $Q$.

(i) Show that the $x$-coordinates of $P$ and $Q$ satisfy the equation

$$x^2 + 3x - 10 = 0$$ *(3 marks)*

(ii) Given that $P$ has coordinates $(2, 2)$, find the coordinates of $Q$. *(2 marks)*

(iii) Hence find the coordinates of the midpoint of $PQ$. *(2 marks)*

## 6.June 2007

**5** A circle with centre $C$ has equation $(x+3)^2+(y-2)^2=25$.

(a) Write down:

(i) the coordinates of $C$; *(2 marks)*

(ii) the radius of the circle. *(1 mark)*

(b) (i) Verify that the point $N(0, -2)$ lies on the circle. *(1 mark)*

(ii) Sketch the circle. *(2 marks)*

(iii) Find an equation of the normal to the circle at the point $N$. *(3 marks)*

(c) The point $P$ has coordinates $(2, 6)$.

(i) Find the distance $PC$, leaving your answer in surd form. *(2 marks)*

(ii) Find the length of a tangent drawn from $P$ to the circle. *(3 marks)*

## 7.Jan 2008

**4** A circle with centre $C$ has equation $x^2+y^2-10y+20=0$.

(a) By completing the square, express this equation in the form

$$x^2+(y-b)^2=k$$ *(2 marks)*

(b) Write down:

(i) the coordinates of $C$; *(1 mark)*

(ii) the radius of the circle, leaving your answer in surd form. *(1 mark)*

(c) A line has equation $y=2x$.

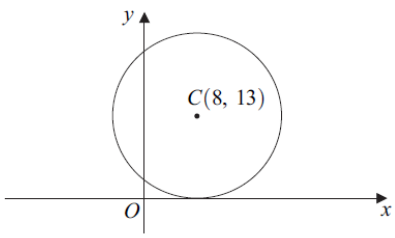
(i) Show that the $x$-coordinate of any point of intersection of the line and the circle satisfies the equation $x^2-4x+4=0$. *(2 marks)*

(ii) Hence show that the line is a tangent to the circle and find the coordinates of the point of contact, $P$. *(3 marks)*

(d) Prove that the point $Q(-1, 4)$ lies inside the circle. *(2 marks)*

## 8.June 2008

**7** The circle $S$ has centre $C(8, 13)$ and touches the $x$-axis, as shown in the diagram.




(a) Write down an equation for $S$, giving your answer in the form

$$(x-a)^2+(y-b)^2=r^2$$ *(2 marks)*

(b) The point $P$ with coordinates $(3, 1)$ lies on the circle.

(i) Find the gradient of the straight line passing through $P$ and $C$. *(1 mark)*

(ii) Hence find an equation of the tangent to the circle $S$ at the point $P$, giving your answer in the form $ax + by = c$, where $a$, $b$ and $c$ are integers. *(4 marks)*

(iii) The point $Q$ also lies on the circle $S$, and the length of $PQ$ is 10. Calculate the shortest distance from $C$ to the chord $PQ$. *(3 marks)*

## 9.Jan 2009

**7** A circle with centre $C$ has equation $x^2 + y^2 - 6x + 10y + 9 = 0$.

(a) Express this equation in the form

$$(x - a)^2 + (y - b)^2 = r^2$$ *(3 marks)*

(b) Write down:

(i) the coordinates of $C$;

(ii) the radius of the circle. *(2 marks)*

(c) The point $D$ has coordinates $(7, -2)$.

(i) Verify that the point $D$ lies on the circle. *(1 mark)*

(ii) Find an equation of the normal to the circle at the point $D$, giving your answer in the form $mx + ny = p$, where $m$, $n$ and $p$ are integers. *(3 marks)*

(d) (i) A line has equation $y = kx$. Show that the $x$-coordinates of any points of intersection of the line and the circle satisfy the equation

$$(k^2 + 1)x^2 + 2(5k - 3)x + 9 = 0$$ *(2 marks)*

(ii) Find the values of $k$ for which the equation

$$(k^2 + 1)x^2 + 2(5k - 3)x + 9 = 0$$

has equal roots. *(5 marks)*

(iii) Describe the geometrical relationship between the line and the circle when $k$ takes either of the values found in part (d)(ii). *(1 mark)*

## 10.June 2009

**5** A circle with centre $C$ has equation

$$(x - 5)^2 + (y + 12)^2 = 169$$

(a) Write down:

(i) the coordinates of $C$; *(1 mark)*

(ii) the radius of the circle. *(1 mark)*

(b) (i) Verify that the circle passes through the origin $O$. *(1 mark)*

(ii) Given that the circle also passes through the points $(10, 0)$ and $(0, p)$, sketch the circle and find the value of $p$. *(3 marks)*

(c) The point $A(-7, -7)$ lies on the circle.

(i) Find the gradient of $AC$. *(2 marks)*

(ii) Hence find an equation of the tangent to the circle at the point $A$, giving your answer in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers. *(3 marks)*

## 11.Jan 2010

7 A circle with centre $C$ has equation $x^2 + y^2 - 4x + 12y + 15 = 0$.

(a) Find:

(i) the coordinates of $C$; *(2 marks)*

(ii) the radius of the circle. *(2 marks)*

(b) Explain why the circle lies entirely below the $x$-axis. *(2 marks)*

(c) The point $P$ with coordinates $(5, k)$ lies outside the circle.

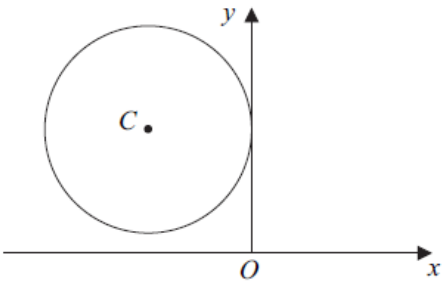
(i) Show that $PC^2 = k^2 + 12k + 45$. *(2 marks)*

(ii) Hence show that $k^2 + 12k + 20 > 0$. *(1 mark)*

(iii) Find the possible values of $k$. *(4 marks)*

## 12.June 2010

**5** A circle with centre $C(-5, 6)$ touches the $y$-axis, as shown in the diagram.




**(a)** Find the equation of the circle in the form

$$(x - a)^2 + (y - b)^2 = r^2$$ *(3 marks)*

**(b) (i)** Verify that the point $P(-2, 2)$ lies on the circle. *(1 mark)*

**(ii)** Find an equation of the normal to the circle at the point $P$. *(3 marks)*

**(iii)** The mid-point of $PC$ is $M$. Determine whether the point $P$ is closer to the point $M$ or to the origin $O$. *(4 marks)*

13.Jan 2011

**6** A circle has centre $C(-3, 1)$ and radius $\sqrt{13}$.

**(a) (i)** Express the equation of the circle in the form

$$(x-a)^2 + (y-b)^2 = k$$ *(2 marks)*

**(ii)** Hence find the equation of the circle in the form

$$x^2 + y^2 + mx + ny + p = 0$$

where $m$, $n$ and $p$ are integers. *(3 marks)*

**(b)** The circle cuts the $y$-axis at the points $A$ and $B$. Find the distance $AB$. *(3 marks)*

**(c) (i)** Verify that the point $D(-5, -2)$ lies on the circle. *(1 mark)*

**(ii)** Find the gradient of $CD$. *(2 marks)*

**(iii)** Hence find an equation of the tangent to the circle at the point $D$. *(2 marks)*

14.June 2011

**8** A circle has centre $C(3, -8)$ and radius $10$.

**(a)** Express the equation of the circle in the form

$$(x-a)^2 + (y-b)^2 = k$$ *(2 marks)*

**(b)** Find the $x$-coordinates of the points where the circle crosses the $x$-axis. *(3 marks)*

**(c)** The tangent to the circle at the point $A$ has gradient $\frac{5}{2}$. Find an equation of the line $CA$, giving your answer in the form $rx + sy + t = 0$, where $r$, $s$ and $t$ are integers. *(3 marks)*

**(d)** The line with equation $y = 2x + 1$ intersects the circle.

**(i)** Show that the $x$-coordinates of the points of intersection satisfy the equation

$$x^2 + 6x - 2 = 0$$ *(3 marks)*

**(ii)** Hence show that the $x$-coordinates of the points of intersection are of the form $m \pm \sqrt{n}$, where $m$ and $n$ are integers. *(2 marks)*

15.Jan 2012

**7** A circle with centre $C$ has equation $x^2 + y^2 + 14x - 10y + 49 = 0$.

**(a)** Express this equation in the form

$$(x-a)^2 + (y-b)^2 = r^2$$ *(3 marks)*

**(b)** Write down:

**(i)** the coordinates of $C$;

**(ii)** the radius of the circle. *(2 marks)*

**(c)** Sketch the circle. *(2 marks)*

**(d)** A line has equation $y = kx + 6$, where $k$ is a constant.

**(i)** Show that the $x$-coordinates of any points of intersection of the line and the circle satisfy the equation $(k^2+1)x^2 + 2(k+7)x + 25 = 0$. *(2 marks)*

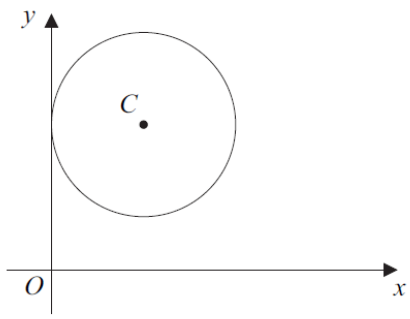
**(ii)** The equation $(k^2+1)x^2 + 2(k+7)x + 25 = 0$ has equal roots. Show that

$$12k^2 - 7k - 12 = 0$$ *(3 marks)*

**(iii)** Hence find the values of $k$ for which the line is a tangent to the circle. *(2 marks)*

16.June 2012

**6** The circle with centre $C(5, 8)$ touches the $y$-axis, as shown in the diagram.




**(a)** Express the equation of the circle in the form

$$(x-a)^2 + (y-b)^2 = k$$ *(2 marks)*

**(ii)** Find an equation of the tangent to the circle at the point $A$, giving your answer in the form $sx + ty + u = 0$, where $s$, $t$ and $u$ are integers. *(5 marks)*

**(c)** The points $P$ and $Q$ lie on the circle, and the mid-point of $PQ$ is $M(7, 12)$.

**(i)** Show that the length of $CM$ is $n\sqrt{5}$, where $n$ is an integer. *(2 marks)*

**(ii)** Hence find the area of triangle $PCQ$. *(3 marks)*

17.Jan 2013

**7** A circle with centre $C(-3, 2)$ has equation

$$x^2 + y^2 + 6x - 4y = 12$$

**(a)** Find the $y$-coordinates of the points where the circle crosses the $y$-axis. *(3 marks)*

**(b)** Find the radius of the circle. *(3 marks)*

**(c)** The point $P(2, 5)$ lies outside the circle.

**(i)** Find the length of $CP$, giving your answer in the form $\sqrt{n}$, where $n$ is an integer. *(2 marks)*

**(ii)** The point $Q$ lies on the circle so that $PQ$ is a tangent to the circle. Find the length of $PQ$. *(2 marks)*

Answers

## 1.Jan 2005

| | | | | |
|---|---|---|---|---|
| **3(a)** | $(x-6)^2+(y-3)^2$ | B1 | | |
| | $=36+9-20$ | M1 | | Generous with sign errors |
| | $=5^2$ | A1 | 3 | Condone 25 |
| **(b)** | (i) Centre (6,3) | B1√ | | ft their $a$ and $b$ |
| | (ii) Radius = 5 | B1√ | 2 | Correct or ft $\sqrt{RHS}$ if $RHS>0$ |
| **(c)(i)** | $x^2+(x+4)^2-12x-6(x+4)+20=0$ | M1 | | Or their $(x-a)^2+(x+4-b)^2=r^2$ |
| | $(2x^2-10x+12=0)\ \Rightarrow x^2-5x+6=0$ | A1 | 2 | AG (be convinced) |
| **(ii)** | $(x-3)(x-2)=0$ | M1 | | Attempt at factors or use of formula |
| | $x=2,\ \ x=3$ | A1 | | Both correct |
| | | m1 | | Substituting for one $y$ value |
| | $P$, $Q$ are (2,6) and (3,7) | A1 | 4 | Both points correct |
| | **Total** | | **11** | |

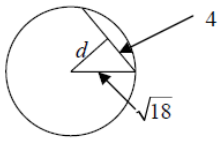
## 2.June 2005

| | | | | |
|---|---|---|---|---|
| **3(a)** | $(x-2)^2+(y+1)^2$ | M1 | | $(x\pm a)^2+(y\pm b)^2$ |
| | $=5^2$ or 25 | B1 | | |
| | | A1 | 3 | Correct equation for circle |
| **(b)** | Sub $x=6,\ y=2$ into *their* circle equation | M1 | | Or distance $PC^2=4^2+3^2$ |
| | $(6-2)^2+(2+1)^2=16+9=25$ | A1 | 2 | Shown to equal radius$^2$ |
| **(c)** | Gradient $CP=(2-\ -1)/(6-2)$ | M1 | | Must be $y$ on top and subtraction $(6-2)$ |
| | $=\frac{3}{4}\ =\ 0.75$ | A1 | 2 | Any correct equivalent |
| **(d)(i)** | Grad of perp $=-1$ / *their* gradient $CP$ | M1 | | Or $m_1m_2=-1$ used or stated |
| | $=-4/3$ | A1 | 2 | |
| **(ii)** | $y-2=$ their $(d)(i)$gradient$(x-6)$ | B1√ | 1 | **OE** such as $3y+4x=30$ |
| | **Total** | | **10** | |

## 3.Jan 2006

| | | | | |
|---|---|---|---|---|
| **5(a)** | $(x-4)^2+(y+3)^2$ | B2 | | B1 for one term correct |
| | $(11+16+9=36)$ RHS $=6^2$ | B1 | 3 | Condone 36 |
| **(b)(i)** | Centre $(4,-3)$ | B1√ | 1 | Ft their $a$ and $b$ from part (a) |
| **(ii)** | Radius = 6 | B1√ | 1 | Ft their $r$ from part (a) |
| **(c)(i)** | $CO^2=(-4)^2+3^2$ | M1 | | Accept + or – with numbers but must add |
| | $CO=5$ | A1√ | 2 | Full marks for answer only |
| **(ii)** | Considering $CO$ and radius | M1 | | |
| | $CO<r\ \Rightarrow\ O$ is **inside** the circle | A1√ | 2 | Ft outside circle when 'their $CO'>r$ or on the circle when 'their $CO'=r$ **SC** B1√ *if no explanation given* |
| | **Total** | | **9** | |

## 4.June 2006

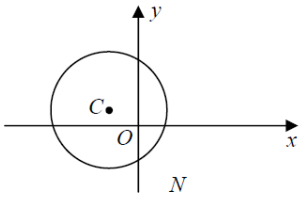
| | | | | |
|---|---|---|---|---|
| **7(a)(i)** | $(x-2)^2$<br>centre has $x$-coordinate = 2<br>and $y$-coordinate = 0 | M1<br>A1<br>B1 | <br><br>3 | Attempt to complete square for $x$<br>M1 implied if value correct or –2<br>Centre (2,0) |
| **(ii)** | RHS $= 18$<br>Radius = $\sqrt{18}$<br>Radius = $3\sqrt{2}$ | B1<br>M1<br>A1 | <br><br>3 | Withhold if circle equation RHS incorrect<br>Square root of RHS of equation (if $> 0$) |
| **(b)** | Perpendicular bisects chord so need to use<br>Length of 4<br>$d^2 = (\text{radius})^2 - 4^2$<br>$d^2 = 18 - 16$<br>so perpendicular distance = $\sqrt{2}$ | <br>B1<br>M1<br><br>A1 | <br><br><br><br>3 |  |
| **(c)(i)** | $x^2 + (2k-x)^2 - 4x - 14 = 0$<br>$(2k-x)^2 = 4k^2 - 4kx + x^2$<br>$\Rightarrow 2x^2 + 4k^2 - 4kx - 4x - 14 = 0$<br>$(\Rightarrow x^2 + 2k^2 - 2kx - 2x - 7 = 0)$<br>$\Rightarrow x^2 - 2(k+1)x + 2k^2 - 7 = 0$ | M1<br>B1<br><br><br>A1 | <br><br><br><br>3 | <br><br><br><br>AG (be convinced about algebra and = 0) |

| | | | | |
|---|---|---|---|---|
| **(ii)** | $4(k+1)^2 - 4(2k^2 - 7)$<br>$4k^2 - 8k - 32 = 0$ or $k^2 - 2k - 8 = 0$<br>$(k-4)(k+2) = 0$<br>$k = -2$ , $k = 4$ | M1<br>A1<br>m1<br>A1 | <br><br><br>4 | "$b^2 - 4ac$" in terms of $k$ (either term correct)<br>$b^2 - 4ac = 0$ correct quadratic equation in $k$<br>Attempt to factorise, solve equation<br>SC B1, B1 for –2 , 4 (if M0 scored) |
| **(iii)** | Line is a tangent to the circle | E1 | 1 | Line touches circle at one point etc |
| | **Total** | | **17** | |

## 5.Jan 2007

| | | | | |
|---|---|---|---|---|
| **4(a)** | $(x+1)^2 + (y-6)^2$<br>$(1 + 36 - 12 = 25)$ RHS $= 5^2$ | B2<br>B1 | <br>3 | B1 for one term correct or missing + sign<br>Condone 25 |
| **(b)(i)**<br>**(ii)** | Centre $(-1 , 6)$<br>Radius = 5 | B1√<br>B1√ | 1<br>1 | **FT** their $a$ and $b$ from part (a) or correct<br>**FT** their $r$ from part (a) RHS must be $> 0$ |
| **(c)** | Attempt to solve "their" $x^2 + 2x + 12 = 0$<br>(all working correct) so no real roots<br>or statement that does not intersect | M1<br><br>A1 | <br><br>2 | Or comparing "their" $y_c = 6$ and their $r = 5$<br>may use a diagram with values shown<br>$r < y_c$ so does not intersect<br>condone $\pm 1$ or $\pm 6$ in centre for A1 |
| **(d)(i)** | $(4-x)^2 = 16 - 8x + x^2$<br>$x^2 + (4-x)^2 + 2x - 12(4-x) + 12 = 0$<br>or $(x+1)^2 + (-2-x)^2 = 25$<br>$\Rightarrow 2x^2 + 6x - 20 = 0 \quad \Rightarrow x^2 + 3x - 10 = 0$ | B1<br>M1<br><br>A1 | <br><br><br>3 | Or $(-2-x)^2 = 4 + 4x + x^2$<br>Sub $y = 4 - x$ in circle eqn (condone slip)<br>or "their" circle equation<br>**AG CSO** (must have = 0) |
| **(ii)** | $(x+5)(x-2) = 0 \quad \Rightarrow x = -5, x = 2$<br>$Q$ has coordinates $(-5, 9)$ | M1<br>A1 | <br>2 | Correct factors or unsimplified solution to quadratic<br>(give credit if factorised in part (i))<br>SC2 if $Q$ correct. Allow x = –5 $y = 9$ |
| **(iii)** | Mid point of 'their' $(-5, 9)$ and (2,2)<br>$\left(-1\frac{1}{2}, 5\frac{1}{2}\right)$ | M1<br>A1 | <br>2 | Arithmetic mean of **either** $x$ or $y$ coords<br>Must follow from correct value in (ii) |
| | **Total** | | **14** | |

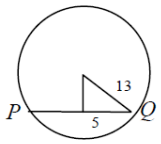
## 6.June 2007

| | | | | |
|---|---|---|---|---|
| **5(a)(i)** | Centre $(-3, 2)$ | M1<br>A1 | <br>2 | $\pm 3$ or $\pm 2$<br>correct |
| **(ii)** | Radius = 5 | B1 | 1 | accept $\sqrt{25}$ but not $\pm\sqrt{25}$ |
| **(b)(i)** | $3^2+(-4)^2=9+16=25$<br>$\Rightarrow N$ lies on circle | <br>B1 | <br>1 | <br>must have $9+16=25$ or a statement |
| **(ii)** |  | M1<br><br>A1 | <br><br>2 | must draw axes;<br>ft their centre in correct quadrant<br>correct (reasonable freehand circle enclosing origin) |
| **(iii)** | Attempt at gradient of $CN$<br>grad $CN=-\frac{4}{3}$<br>$y=-\frac{4}{3}x-2$ (or equivalent) | M1<br>A1<br>A1√ | <br><br>3 | withhold if subsequently finds tangent<br>CSO<br>ft their grad $CN$ |
| **(c)(i)** | $P(2,6)$ Hence $PC^2=5^2+4^2$<br>$\Rightarrow PC=\sqrt{41}$ | M1<br>A1 | <br>2 | "their" $PC^2$ |
| **(ii)** | Use of Pythagoras correctly<br>$PT^2=PC^2-r^2=41-25$,<br>where $T$ is a point of contact of tangent<br>$\Rightarrow PT=4$ | M1<br>A1√<br>A1 | <br><br>3 | <br>ft their $PC^2$ and $r^2$<br>**Alternative**<br>sketch with vertical tangent M1<br>showing that tangent touches circle at point (2, 2) A1<br>hence $PT=4$ A1 |
| | **Total** | | **14** | |

## 7.Jan 2008

| | | | | |
|---|---|---|---|---|
| **4(a)** | $x^2+(y-5)^2$<br>RHS = 5 | B1<br>B1 | <br>2 | $b=5$<br>$k=5$ |
| **(b)(i)** | Centre (0, 5) | B1√ | 1 | FT their $b$ from part (a) |
| **(ii)** | Radius = $\sqrt{5}$ | B1√ | 1 | FT their $k$ from part (a); RHS must be > 0 |
| **(c)(i)** | $x^2+4x^2-20x+20=0$<br>$\Rightarrow x^2-4x+4=0$ | M1<br>A1 | <br>2 | May substitute into original or "their (a)"<br>CSO; AG |
| **(ii)** | $(x-2)^2=0$ or $x=2$<br>Repeated root implies tangent<br>Point of contact is $P(2, 4)$ | M1<br>E1<br>A1 | <br><br>3 | <br>Or $b^2-4ac$ shown = 0 plus statement |
| **(d)** | $(CQ^2=)\,1^2+1^2$<br>$\sqrt{2}<\sqrt{5}\Rightarrow Q$ lies inside circle | M1<br>A1<br>CSO | <br><br>2 | FT their $C$<br>$CQ$ or $CQ^2$ OE must appear for A1 |
| | **Total** | | **11** | |

## 8.June 2008

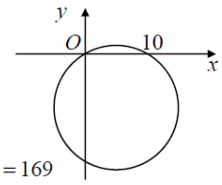
| | | | | |
|---|---|---|---|---|
| **7(a)** | $(x-8)^2+(y-13)^2$ | B1 | | **Exactly this** with + and squares |
| | $=13^2$ | B1 | 2 | Condone 169 |
| **(b)(i)** | grad $PC=\dfrac{12}{5}$ | B1 | 1 | Must simplify $\dfrac{-12}{-5}$ |
| **(ii)** | grad of tangent $=\dfrac{-1}{\text{grad } PC}=-\dfrac{5}{12}$ | B1√ | | Condone $-\dfrac{1}{2.4}$ etc |
| | tangent has equation $y-1=-\dfrac{5}{12}(x-3)$ | M1<br>A1 | | ft gradient but M0 if using grad $PC$<br>Correct – but not in required final form |
| | $5x+12y=27$ OE | A1 | 4 | MUST have integer coefficients |
| **(iii)** |  half chord $=5$ | B1 | | Seen or stated |
| | $d^2=(\text{their } r)^2-5^2$ (provided $r>5$) | M1 | | Pythagoras used correctly $d^2=13^2-5^2$ |
| | Distance $=12$ | A1 | 3 | CSO |
| | **Total** | | **10** | |

## 9.Jan 2009

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **7(a)(i)** | $(x-3)^2+(y+5)^2$ | B1 | | One term correct |
| | | B1 | | LHS correct with + and squares |
| | $=25-9+9=25\quad(=5^2)$ | B1 | 3 | Condone RHS = 25 |
| **(b)(i)** | $C\ (3,-5)$ | B1√ | | |
| **(ii)** | Radius $=5$ | B1√ | 2 | Correct or ft their RHS provided > 0 |
| **(c)(i)** | $(7-3)^2+(-2+5)^2=16+9=25$<br>$\Rightarrow D$ lies on circle<br>*Must see statement* | B1 | 1 | Or sub'n of $(7,-2)$ in original equation<br>$7^2+(-2)^2-42-20+9=0$<br>Or sub $x$=7 into eqn & showing $y=-2$ etc |
| **(ii)** | Attempt at gradient of $CD$ as normal<br>grad $CD=\dfrac{-2-(-5)}{7-3}=\dfrac{3}{4}$ | M1 | | withhold if subsequently uses $m_1m_2=-1$<br>$\dfrac{\Delta y}{\Delta x}$ (condone one slip) FT their centre $C$ |
| | $y+2=\dfrac{3}{4}(x-7)$ or $y+5=\dfrac{3}{4}(x-3)$ | A1 | | Correct equation in any form $y=\dfrac{3}{4}x-\dfrac{29}{4}$ |
| | $\Rightarrow 3x-4y=29$ | A1 | 3 | CSO ***Integer*** coefficients<br>Condone $4y-3x+29=0$ etc |
| **(d)(i)** | $y=kx$ sub'd into original circle equation<br>$x^2+(kx)^2-6x+10kx+9=0$ | M1 | | or using their completed square form and multiplying out |
| | $\Rightarrow (k^2+1)x^2+2(5k-3)x+9=0$ **AG** | A1 | 2 | CSO<br>must see at least previous line for A1<br>any error such as $kx^2=\ldots=k^2x^2$ gets A0 |

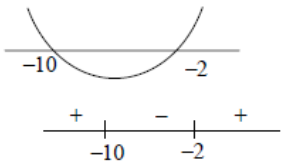
| | | | | |
|---|---|---|---|---|
| **(ii)** | $4(5k-3)^2-36(k^2+1)$ | M1 | | Discriminant in $k$ (can be seen in quad formula)<br>Condone one slip |
| | $=64k^2-120k$ | A1 | | or $8k^2-15k=0$ OE |
| | Equal roots: $4(5k-3)^2-36(k^2+1)=0$ | B1 | | $b^2-4ac=0$ clearly stated or evident by an equation in $k$ with at most 2 slips. |
| | $8k^2-15k=0$ | m1 | | Attempt to solve ***their*** quadratic or linear equation if $k$ has been cancelled |
| | $\Rightarrow k=0,\quad k=\frac{15}{8}$ | A1 | 5 | OE but must have $k$=0<br>If "=0" is not seen but correct values of $k$ are found, candidate will lose B1 mark but may earn all other marks |
| **(iii)** | (Line is a) **tangent** (to the circle) | E1 | 1 | Line **touches** circle at one point |
| | **Total** | | **17** | |

## 10.June 2009

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **5(a)(i)** | $C\ (5,-12)$ | B1 | 1 | |
| **(ii)** | Radius = 13 (or $\sqrt{169}$) | B1 | 1 | $\pm\sqrt{169}$ or $\pm 13$ as final answer scores B0 |
| **(b)(i)** | $(-5)^2+12^2$ *or* $25+144$<br>$=169 \Rightarrow$ circle passes through $O$ | B1 | 1 | Correct arithmetic plus statement<br>Eg "$O$ lies on circle", "as required" etc |
| **(ii)** | Sketch  | B1 | | Freehand circle through origin and cutting positive $x$-axis with centre in 4th quadrant<br>Condone value 10 missing or incorrect |
| | $25+(p+12)^2=169$ | M1 | | Or doubling their $y_C$-coordinate |
| | $(p+12)=\pm 12 \qquad p=-24$ | A1 | 3 | Condone use of $y$ instead of $p$<br>SC B2 for correct value of $p$ stated or marked on diagram |
| **(c)(i)** | grad $AC=\frac{-12+7}{5+7}$ | M1 | | correct expression, but ft their $C$ |
| | $=-\frac{5}{12}$ | A1 | 2 | Condone $\frac{5}{-12}$ |
| **(ii)** | grad tangent $=\frac{12}{5}$ | B1 √ | | $\frac{-1}{\text{their grad } AC}$ |
| | $y+7=\frac{12}{5}(x+7)$ | M1 | | ft "their $\frac{12}{5}$" must be **tangent** and not $AC$ |
| | $\Rightarrow 12x-5y+49=0$ | A1 | 3 | OE with **integer** coefficients with all terms on one side of the equation |
| | **Total** | | **11** | |

## 11.Jan 2010

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **7(a)(i)** | $x=\pm 2$ or $y=\pm 6$ or $(x-2)^2+(y+6)^2$ | M1 | | |
| | $C(2,-6)$ | A1 | 2 | correct |
| **(ii)** | $(r^2=)4+36-15$ | M1 | | (RHS = ) *their* $(-2)^2$ + *their* $(6)^2-15$ |
| | $\Rightarrow r=5$ | A1 | 2 | Not $\pm 5$ *or* $\sqrt{25}$ |

| | | | | |
|---|---|---|---|---|
| **(b)** | explaining why $\lvert y_C \rvert > r$ ; $6 > 5$ | E1 | | Comparison of $y_C$ and $r$ , eg $-6+5=-1$ or indicated on diagram |
| | full convincing argument, but must have correct $y_C$ and $r$ | E1 | 2 | Eg "highest point is at $y=-1$" scores E2<br>E1: showing no real solutions when $y=0$<br>+E1 stating centre or any point below $x$-axis |
| **(c)(i)** | $(PC^2=)\quad (5-2)^2+(k+6)^2$ | | | ft their $C$ coords |
| | $=9+k^2+12k+36$ | M1 | | and attempt to multiply out |
| | $PC^2=k^2+12k+45$ | A1 | 2 | **AG** CSO (must see $PC^2=$ at least once) |
| **(ii)** | $\left.\begin{array}{l} PC>r \Rightarrow PC^2>25 \\ \Rightarrow k^2+12k+20>0 \end{array}\right\}$ | B1 | 1 | **AG** Condone $\left.\begin{array}{l} k^2+12k+45>25 \\ \Rightarrow k^2+12k+20>0 \end{array}\right\}$ |
| **(iii)** | $(k+2)(k+10)$ | M1 | | **Correct** factors or **correct** use of formula |
| | $k=-2,\ k=-10$ are critical values | A1 | | May score M1, A1 for correct critical values seen as part of incorrect final answer with or without working. |
| | Use of sketch or sign diagram:<br>  <br> | M1 | | If previous A1 earned, sign diagram or sketch must be correct for M1, otherwise M1 may be earned for an attempt at the sketch or sign diagram using their critical values. |
| | $\Rightarrow k>-2,\ k<-10$ | A1 | 4 | $k \geqslant -2,\ k \leqslant -10$ loses final A mark |
| | Condone $k>-2$ OR $k<-10$ for full marks but not AND instead of OR<br>Take final line as their answer | | | *Answer only* of $k>-2,\ k>-10$ *etc* scores M1, A1, M0 since the critical values are evident.<br>*Answer only* of $k>2,\ k<-10$ *etc* scores M0, M0 since the critical values are not both correct. |
| | **Total** | | **13** | |

## 12.June 2010

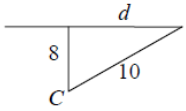
| | | | | |
|---|---|---|---|---|
| **5(a)** | $(x+5)^2+(y-6)^2=5^2$ | M1<br>A1<br>B1 | 3 | One term correct LHS<br>LHS all correct<br>RHS correct: condone = 25 |
| **(b)(i)** | sub $x=-2,\ y=2$ into circle equation<br>$3^2+(-4)^2=25$<br>$\Rightarrow$ lies on circle | B1 | 1 | Circle equation must be correct<br>Must have concluding statement |
| **(ii)** | Grad $PC = -\frac{4}{3}$ | B1 | | Condone $\frac{4}{-3}$ |
| | Normal to circle has equation<br>$y-6=$ '*their gradient PC*'$(x+5)$<br>or $y-2=$ '*their gradient PC*'$(x+2)$ | M1 | | M0 if tangent attempted or incorrect coordinates used |
| | $y-6=-\frac{4}{3}(x+5)$<br>or $y-2=-\frac{4}{3}(x+2)$ | A1cso | 3 | Any correct form eg $4x+3y+2=0$<br>$y=-\frac{4}{3}x+c,\quad c=-\frac{2}{3}$ |

| | | | | |
|---|---|---|---|---|
| | | | | **Alternative 1** |
| **(iii)** | $PM = \frac{1}{2} \times \text{radius}$ | M1 | | Attempt at $M\left(-\frac{7}{2}, 4\right)$ with at least one correct coordinate **and** $PM^2$ attempted |
| | $= 2.5$ | A1cso | | $PM^2 = \frac{9}{4} + 4 = \frac{25}{4}$ |
| | $PO = \sqrt{8}$ | B1 | | $PO^2 = 4 + 4 = 8$ |
| | $P$ is closer to the point $M$ | E1cso | 4 | Statement following correct values |
| | | | | **Alternative 2** |
| | | (M1) | | Attempt at $M\left(-\frac{7}{2}, 4\right)$ with at least one correct coordinate **and** attempt at vectors or difference of coordinates |
| | | (A1cso) | | $\overrightarrow{PM} = \begin{pmatrix} -1.5 \\ 2 \end{pmatrix}$ OE |
| | | (E1cso) | | $P$ is closer to the point $M$ |
| | | (E1) | (4) | Components of their $\overrightarrow{PM}$ and $\overrightarrow{OP}$ considered – ***totally independent*** of M1 |
| | **Total** | | **11** | |

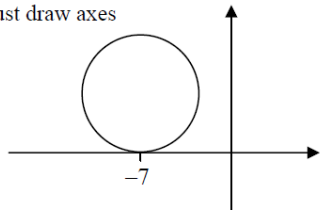
## 13.Jan 2011

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **6(a)(i)** | $(x+3)^2 + (y-1)^2$ | B1 | | condone $(x - -3)^2$ |
| | $= 13$ | B1 | 2 | condone $\left(\sqrt{13}\right)^2$ |
| **(ii)** | $x^2 + 6x + 9 + y^2 - 2y + 1$ | M1 | | attempt to multiply out both of 'their' brackets; must have $x$ and $y$ terms |
| | $x^2 + y^2 + 6x - 2y$ | A1 | | both $m = 6$ and $n = -2$ |
| | $-3 = 0$ | A1 | 3 | All correct, $p = -3$ and $\ldots = 0$ |
| **(b)** | $x = 0 \Rightarrow y^2 - 2y - 3 = 0$ | | | putting $x = 0$ PI |
| | $\Rightarrow (y-3)(y+1) = 0$ | M1 | | and attempt to solve or factorise |
| | $y = 3,\ y = -1$ | A1 | | |
| | $\Rightarrow$ Distance $AB = 3 + 1 = 4$ | A1cso | 3 | **OR** Pythagoras $d^2 = 13 - 3^2$ M1<br>$d = 2$ A1<br>distance $= 2 \times 2 = 4$ A1 |
| **(c)(i)** | $(-5+3)^2 + (-2-1)^2 = 4 + 9$<br>$= 13$ | | | Substitution $x = -5$, $y = -2$ into any correct circle equation |
| | $\Rightarrow D$ lies on circle | B1 | 1 | convincing verification **plus statement** |
| **(ii)** | grad $CD = \frac{1+2}{-3+5}$ | M1 | | condone one sign slip |
| | $= \frac{3}{2}$ (or 1.5) | A1 | 2 | not $\frac{-3}{-2}$ |
| **(iii)** | Perpendicular gradient $= -\frac{2}{3}$ | M1 | | ft their grad $CD$ or $m_1 m_2 = -1$ stated |
| | Tangent has equation $y + 2 = -\frac{2}{3}(x+5)$ | A1 | 2 | any form of correct equation eg $2x + 3y + 16 = 0$<br>$y = -\frac{2}{3}x + c,\ c = -\frac{16}{3}$ |
| | **Total** | | **13** | |

## 14.June 2011

| | | | | |
|---|---|---|---|---|
| **8(a)** | $(x-3)^2+(y+8)^2$ | B1 | | accept $(y--8)^2$ |
| | $=100$ | B1 | 2 | condone RHS $=10^2$ or $k=10^2$ |
| **(b)** | $y=0 \Rightarrow$ 'their' $(x-a)^2+b^2=k$ | M1 | | **Alternative**   |
| | $(x-3)^2=36$ or $x^2-6x-27$ $(=0)$ (PI) | A1 | | |
| | $\Rightarrow x=-3,\ 9$ | A1 | 3 | $(d^2=)\ 10^2-8^2$ M1<br>$d^2=36$ A1 or $d=6$<br>$\Rightarrow x=-3,\ 9$ A1 |
| **(c)** | Line $CA$ has gradient $-\frac{2}{5}$ | M1 | | |
| | $CA$ has equation $(y+8)=-\frac{2}{5}(x-3)$ | A1 | | any form of correct equation<br>eg $y=-\frac{2}{5}x+c,\quad c=-\frac{34}{5}$ |
| | $2x+5y+34=0$ | A1cso | 3 | integer coefficients - all terms on 1 side |
| **(d)(i)** | their $(x-3)^2+(2x+1+8)^2$ *or*<br>$x^2+(2x+1)^2-6x+16(2x+1)$ $(+73)$ | M1 | | substituting $y=2x+1$ correctly into LHS of "their" circle equation and attempt to expand in terms of $x$ only |
| | $x^2-6x+9+4x^2+36x+81=100$<br>*or* $x^2+4x^2+4x+1-6x+32x+16+73=100$ | A1 | | any correct equation (with brackets expanded) |
| | $\Rightarrow 5x^2+30x-10=0$ | | | must see this line or equivalent |
| | $\Rightarrow x^2+6x-2=0$ | A1cso | 3 | AG; all algebra must be correct |
| **(ii)** | $(x+3)^2=11$ | M1 | | or correct use of formula<br>must get as far as $x=\frac{-6\pm\sqrt{44}}{2}$ |
| | $x=-3\pm\sqrt{11}$ | A1cso | 2 | exactly this |
| | **Total** | | **13** | |

## 15.Jan 2012

| | | | | |
|---|---|---|---|---|
| **7(a)** | $(x+7)^2+(y-5)^2$ | M1 | | one term correct ; condone $(x--7)^2$ |
| | | A1 | | both terms correct with squares and plus sign between terms |
| | $(x+7)^2+(y-5)^2=5^2$ | A1cao | 3 | condone 25 for $5^2$ |
| **(b)(i)** | $C(-7,\ 5)$ | B1√ | | correct or FT 'their' circle equation |
| **(ii)** | $r=5$ | B1√ | 2 | correct or FT 'their' $r^2>0$<br>condone $\sqrt{25}$ etc but not $\pm\sqrt{25}$ |
| **(c)** | must draw axes   | M1 | | freehand circle with $C$ correct or FT 'their $C$' for quadrant of centre |
| | | A1 | 2 | circle touching $x$-axis at $-7$ with $-7$ marked (need not show 5 on $y$-axis ) but circle must not touch $y$-axis |

| | | | | |
|---|---|---|---|---|
| **(d)(i)** | $x^2+(kx+6)^2+14x-10(kx+6)+49=0$ | | | clear attempt to sub $y=kx+6$ into original or 'their' circle equation … |
| | $x^2+k^2x^2+12kx+36+14x$ $-10kx-60+49=0$ | M1 | | …***and*** attempt to multiply out |
| | $(1+k^2)x^2+2kx+14x+25=0$ $\Rightarrow (k^2+1)x^2+2(k+7)x+25=0$ | A1cso | 2 | **AG** condone $x^2(1+k^2)+2x(7+k)+...$ etc |
| **(ii)** | Equal roots '$b^2-4ac=0$' | B1 | | allow statement alone if discriminant in terms of $k$ attempted |
| | $\left[2(k+7)\right]^2-4\times 25\left(k^2+1\right)$ $4\left\{k^2+14k+49-25k^2-25\right\}=0$ $-24k^2+14k+24=0$ | M1 | | discriminant (condone one slip) |
| | $\Rightarrow 12k^2-7k-12=0$ | A1 | 3 | **AG** all working correct but = 0 must appear before last line |
| **(iii)** | $(4k+3)(3k-4)$ | M1 | | correct factors or correct use of formula as far as $k=\dfrac{7\pm\sqrt{49+576}}{24}$ |
| | $\Rightarrow k=-\dfrac{3}{4},\ k=\dfrac{4}{3}$ OE are values of $k$ for which line is a tangent | A1 | 2 | |
| | **Total** | | **14** | |

# 16.June 2012

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **6(a)** | $(x-5)^2+(y-8)^2$ | B1 | | |
| | $=25$ | B1 | 2 | condone $5^2$ |
| **(b)(i)** | $(2-5)^2+(12-8)^2$ $=9+16\quad=25$ $\Rightarrow A$ lies on circle | B1 | 1 | or $AC^2=3^2+4^2$ hence $AC=5$ ; ( also radius = 5) CSO $(\Rightarrow$ radius $=AC)$ $\Rightarrow A$ lies on circle |
| | (must have concluding statement and circle equation correct if using equation) | | | (must have concluding statement & RHS of circle equation correct or $r=5$ stated if Pythagoras is used) |
| **(ii)** | grad $AC=-\dfrac{4}{3}$ | B1 | | |
| | Gradient of tangent is $\dfrac{3}{4}$ | B1√ | | FT their $-1/$ grad $AC$ |
| | $y-12='their\ tangent\ grad'\ (x-2)$ | M1 | | **or** $y=$ *'their tangent grad'* $x+c$ & attempt to find $c$ using $x=2$ , $y=12$ |
| | $y-12=\dfrac{3}{4}(x-2)$ ***or*** $y=\dfrac{3}{4}x+\dfrac{21}{2}$ etc | A1 | | correct equation in any form |
| | $3x-4y+42=0$ | A1 | 5 | CSO; must have integer coefficients with all terms on one side of equation accept $0=8y-6x-84$ etc |

| | | | | |
|---|---|---|---|---|
| **(c)(i)** | $(CM^2 =)\quad (7-5)^2+(12-8)^2$<br>$(\Rightarrow CM = \sqrt{20})\quad \Rightarrow (CM =)\quad 2\sqrt{5}$ | M1<br>A1 | <br>2 | **or** $(CM^2 =)\quad 20$ |
| **(ii)** | $PM^2 = PC^2 - CM^2 \quad = 25 - 20$<br>$\Rightarrow PM = \sqrt{5}$<br>Area $\Delta PCQ = \sqrt{5} \times 2\sqrt{5}$<br>$= 10$ | M1<br>A1<br><br>A1 | <br><br><br>3 | Pythagoras used correctly<br>eg $d^2 + (2\sqrt{5})^2 = 5^2$<br><br>CSO |
| | **Total** | | **13** | |

17.Jan 2013

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **7(a)** | $x=0 \Rightarrow y^2-4y-12\ (=0)$<br>$(y-6)(y+2)\quad (=0)$<br>$\Rightarrow y = -2,\ 6$ | M1<br>A1<br>A1 | <br><br>3 | sub $x = 0$ & correct quadratic in $y$<br>*or* $(y-2)^2 = 16$ *or* $(y-2)^2 - 16 = 0$<br>correct factors<br>*or* formula as far as $\frac{4 \pm \sqrt{64}}{2}$<br>*or* $y - 2 = \pm\sqrt{16}$<br>condone $(0, -2)$ & $(0, 6)$ |
| **(b)** | $(x+3)^2 - 9 + (y-2)^2 - 4\ (=12)$<br>$(r^2 =)\quad 9+4+12$<br>$(\Rightarrow r =)\ 5$ | M1<br>A1<br>A1 | <br><br>3 | ***correct*** sum of square terms **and** attempt to complete squares ( or multiply out) PI by next line<br>$(r^2 =)25$ seen on RHS<br>$r = \sqrt{25}$ or $r = \pm 5$ scores A0 |
| **(c)(i)** | $(CP^2 =)\quad (2--3)^2 + (5-2)^2$<br>$\Rightarrow (CP =)\quad \sqrt{34}$ | M1<br>A1 | <br>2 | condone one sign slip within one bracket<br>$n = 34$ |
| **(ii)** | $PQ^2 = CP^2 - r^2 = 34 - 25$<br>$(\Rightarrow PQ =)\quad 3$ | M1<br>A1 | <br>2 | Pythagoras used correctly with values FT "their" $r$ and $CP$ |
| | **Total** | | **10** | |