Practice examination questions for integration

1. Jan 2005

4 (a) The function f is defined for all values of x by $f(x) = x^3 - 3x^2 - 6x + 8$.

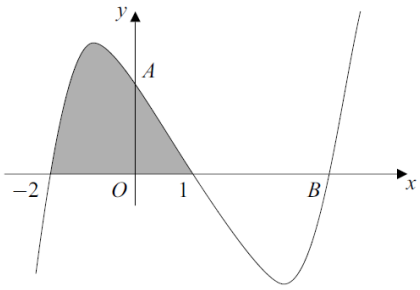
(i) Find the remainder when $f(x)$ is divided by $x + 1$. *(2 marks)*

(ii) Given that $f(1) = 0$ and $f(-2) = 0$, write down two linear factors of $f(x)$. *(2 marks)*

(iii) Hence express $x^3 - 3x^2 - 6x + 8$ as the product of three linear factors. *(2 marks)*

(b) The curve with equation $y = x^3 - 3x^2 - 6x + 8$ is sketched below.

(b) The curve with equation $y = x^3 - 3x^2 - 6x + 8$ is sketched below.



(i) The curve intersects the y-axis at the point A. Find the y-coordinate of A. *(1 mark)*

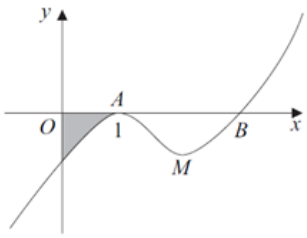
(ii) The curve crosses the x-axis when $x = -2$, when $x = 1$ and also at the point B. Use the results from part (a) to find the x-coordinate of B. *(1 mark)*

(c) (i) Find $\int (x^3 - 3x^2 - 6x + 8)\,dx$. *(4 marks)*

(ii) Hence find the area of the shaded region bounded by the curve and the x-axis. *(3 marks)*

2. June2005

4 The curve with equation $y = x^3 - 5x^2 + 7x - 3$ is sketched below.



The curve touches the x-axis at the point A (1, 0) and cuts the x-axis at the point B.

(a) (i) Use the factor theorem to show that $x-3$ is a factor of

$$p(x) = x^3 - 5x^2 + 7x - 3$$ *(2 marks)*

(ii) Hence find the coordinates of B. *(1 mark)*

(b) The point M, shown on the diagram, is a minimum point of the curve with equation $y = x^3 - 5x^2 + 7x - 3$.

(i) Find $\frac{dy}{dx}$. *(2 marks)*

(ii) Hence determine the x-coordinate of M. *(3 marks)*

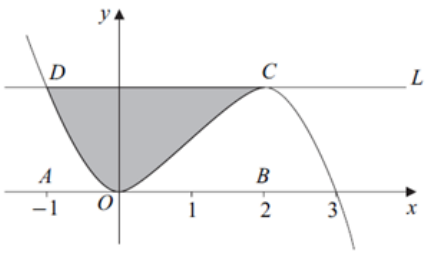
(c) Find the value of $\frac{d^2y}{dx^2}$ when $x = 1$. *(2 marks)*

(d) (i) Find $\int \left(x^3 - 5x^2 + 7x - 3\right) dx$. *(4 marks)*

(ii) Hence determine the area of the shaded region bounded by the curve and the coordinate axes. *(4 marks)*

3.Jan2006

8 The diagram shows the curve with equation $y = 3x^2 - x^3$ and the line L.



The points A and B have coordinates $(-1, 0)$ and $(2, 0)$ respectively. The curve touches the x-axis at the origin O and crosses the x-axis at the point $(3, 0)$. The line L cuts the curve at the point D where $x = -1$ and touches the curve at C where $x = 2$.

(a) Find the area of the rectangle $ABCD$. *(2 marks)*

(b) (i) Find $\int (3x^2 - x^3) dx$. *(3 marks)*

(c) For the curve above with equation $y = 3x^2 - x^3$:

(i) find $\frac{dy}{dx}$; *(2 marks)*

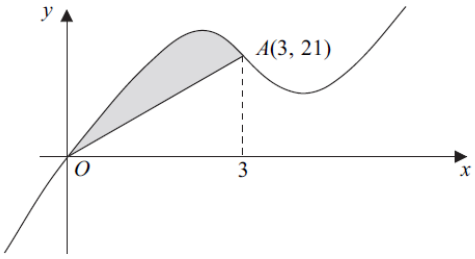
(ii) hence find an equation of the tangent at the point on the curve where $x = 1$; *(3 marks)*

(iii) show that y is decreasing when $x^2 - 2x > 0$. *(2 marks)*

(d) Solve the inequality $x^2 - 2x > 0$. *(2 marks)*

4.June2006

5 The curve with equation $y = x^3 - 10x^2 + 28x$ is sketched below.



The curve crosses the x-axis at the origin O and the point $A(3, 21)$ lies on the curve.

(a) (i) Find $\frac{dy}{dx}$. *(3 marks)*

(ii) Hence show that the curve has a stationary point when $x = 2$ and find the x-coordinate of the other stationary point. *(4 marks)*

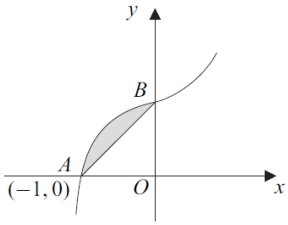
(b) (i) Find $\int (x^3 - 10x^2 + 28x)\,dx$. *(3 marks)*

(ii) Hence show that $\int_0^3 (x^3 - 10x^2 + 28x)\,dx = 56\frac{1}{4}$. *(2 marks)*

(iii) Hence determine the area of the shaded region bounded by the curve and the line OA. *(3 marks)*

5.Jan2007

6 The curve with equation $y = 3x^5 + 2x + 5$ is sketched below.



The curve cuts the x-axis at the point $A(-1, 0)$ and cuts the y-axis at the point B.

(a) (i) State the coordinates of the point B and hence find the area of the triangle AOB, where O is the origin. *(3 marks)*

(ii) Find $\int (3x^5 + 2x + 5)\,dx$. *(3 marks)*

(iii) Hence find the area of the shaded region bounded by the curve and the line AB. *(4 marks)*

(b) (i) Find the gradient of the curve with equation $y = 3x^5 + 2x + 5$ at the point $A(-1, 0)$. *(3 marks)*

(ii) Hence find an equation of the tangent to the curve at the point A. *(1 mark)*

6.June 2007

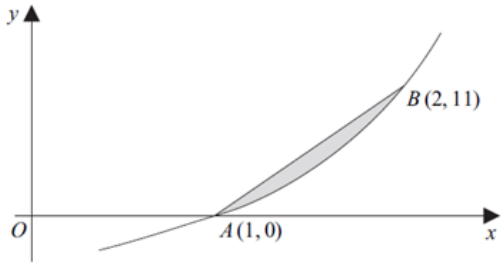
6 (a) The polynomial $f(x)$ is given by $f(x) = x^3 + 4x - 5$.

(i) Use the Factor Theorem to show that $x - 1$ is a factor of $f(x)$. *(2 marks)*

(ii) Express $f(x)$ in the form $(x-1)(x^2 + px + q)$, where p and q are integers. *(2 marks)*

(iii) Hence show that the equation $f(x) = 0$ has exactly one real root and state its value. *(3 marks)*

(b) The curve with equation $y = x^3 + 4x - 5$ is sketched below.



The curve cuts the x-axis at the point $A(1, 0)$ and the point $B(2, 11)$ lies on the curve.

(i) Find $\int (x^3 + 4x - 5)\,dx$. *(3 marks)*

(ii) Hence find the area of the shaded region bounded by the curve and the line AB. *(4 marks)*

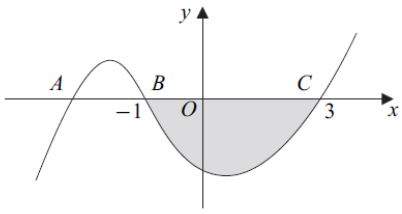
7.Jan 2008

6 (a) The polynomial $p(x)$ is given by $p(x) = x^3 - 7x - 6$.

(i) Use the Factor Theorem to show that $x + 1$ is a factor of $p(x)$. *(2 marks)*

(ii) Express $p(x) = x^3 - 7x - 6$ as the product of three linear factors. *(3 marks)*

(b) The curve with equation $y = x^3 - 7x - 6$ is sketched below.



The curve cuts the x-axis at the point A and the points $B(-1, 0)$ and $C(3, 0)$.

(i) State the coordinates of the point A. *(1 mark)*

(ii) Find $\int_{-1}^{3} (x^3 - 7x - 6)\, dx$. *(5 marks)*

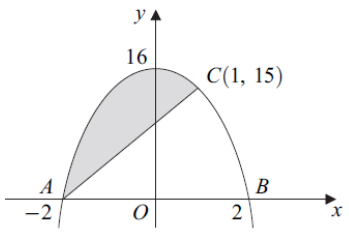
(iii) Hence find the area of the shaded region bounded by the curve $y = x^3 - 7x - 6$ and the x-axis between B and C. *(1 mark)*

(iv) Find the gradient of the curve $y = x^3 - 7x - 6$ at the point B. *(3 marks)*

(v) Hence find an equation of the normal to the curve at the point B. *(3 marks)*

8.June 2008

5 The curve with equation $y = 16 - x^4$ is sketched below.



The points $A(-2, 0)$, $B(2, 0)$ and $C(1, 15)$ lie on the curve.

(a) Find an equation of the straight line AC. *(3 marks)*

(b) (i) Find $\int_{-2}^{1} (16 - x^4)\, dx$. *(5 marks)*

(ii) Hence calculate the area of the shaded region bounded by the curve and the line AC. *(3 marks)*

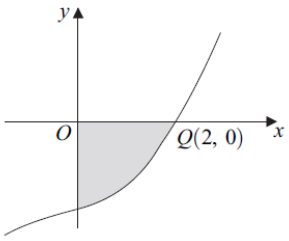
9.Jan 2009

6 (a) The polynomial $p(x)$ is given by $p(x) = x^3 + x - 10$.

(i) Use the Factor Theorem to show that $x - 2$ is a factor of $p(x)$. *(2 marks)*

(ii) Express $p(x)$ in the form $(x - 2)(x^2 + ax + b)$, where a and b are constants. *(2 marks)*

(b) The curve C with equation $y = x^3 + x - 10$, sketched below, crosses the x-axis at the point $Q(2, 0)$.



(i) Find the gradient of the curve C at the point Q. *(4 marks)*

(ii) Hence find an equation of the tangent to the curve C at the point Q. *(2 marks)*

(iii) Find $\int (x^3 + x - 10)\, dx$. *(3 marks)*

(iv) Hence find the area of the shaded region bounded by the curve C and the coordinate axes. *(2 marks)*

10.June 2009

4 (a) The polynomial $p(x)$ is given by $p(x) = x^3 - x + 6$.

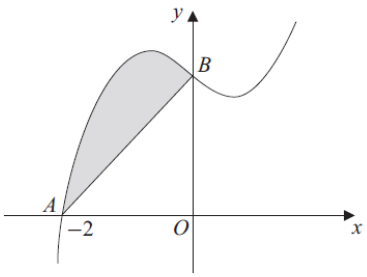
(i) Find the remainder when $p(x)$ is divided by $x - 3$. *(2 marks)*

(ii) Use the Factor Theorem to show that $x + 2$ is a factor of $p(x)$. *(2 marks)*

(iii) Express $p(x) = x^3 - x + 6$ in the form $(x + 2)(x^2 + bx + c)$, where b and c are integers. *(2 marks)*

(iv) The equation $p(x) = 0$ has one root equal to -2. Show that the equation has no other real roots. *(2 marks)*

(b) The curve with equation $y = x^3 - x + 6$ is sketched below.



The curve cuts the x-axis at the point $A\,(-2, 0)$ and the y-axis at the point B.

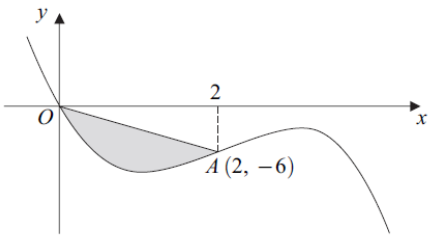
(i) State the y-coordinate of the point B. *(1 mark)*

(ii) Find $\int_{-2}^{0} (x^3 - x + 6)\, dx$. *(5 marks)*

(iii) Hence find the area of the shaded region bounded by the curve $y = x^3 - x + 6$ and the line AB. *(3 marks)*

11.Jan 2010

6 The curve with equation $y = 12x^2 - 19x - 2x^3$ is sketched below.



The curve crosses the x-axis at the origin O, and the point $A\,(2, -6)$ lies on the curve.

(a) (i) Find the gradient of the curve with equation $y = 12x^2 - 19x - 2x^3$ at the point A. *(4 marks)*

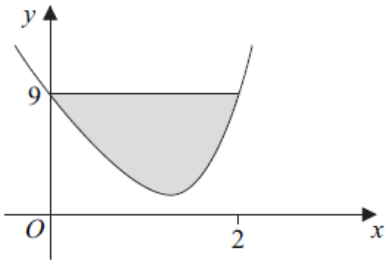
(ii) Hence find the equation of the normal to the curve at the point A, giving your answer in the form $x + py + q = 0$, where p and q are integers. *(3 marks)*

(b) (i) Find the value of $\int_0^2 (12x^2 - 19x - 2x^3)\,dx$. *(5 marks)*

(ii) Hence determine the area of the shaded region bounded by the curve and the line OA. *(3 marks)*

12.June 2010

4 The curve with equation $y = x^4 - 8x + 9$ is sketched below.



The point $(2, 9)$ lies on the curve.

(a) (i) Find $\int_0^2 (x^4 - 8x + 9)\,dx$. *(5 marks)*

(ii) Hence find the area of the shaded region bounded by the curve and the line $y = 9$. *(2 marks)*

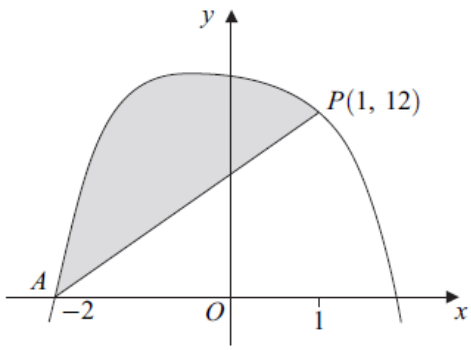
(b) The point $A(1, 2)$ lies on the curve with equation $y = x^4 - 8x + 9$.

(i) Find the gradient of the curve at the point A. *(4 marks)*

(ii) Hence find an equation of the tangent to the curve at the point A. *(1 mark)*

13.Jan 2011

4 The curve sketched below passes through the point $A(-2, 0)$.



The curve has equation $y = 14 - x - x^4$ and the point $P(1, 12)$ lies on the curve.

(a) (i) Find the gradient of the curve at the point P. *(3 marks)*

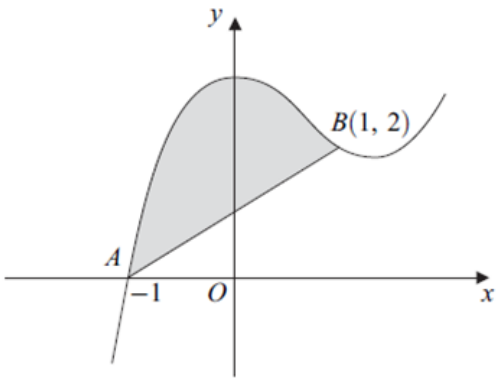
(ii) Hence find the equation of the tangent to the curve at the point P, giving your answer in the form $y = mx + c$. *(2 marks)*

(b) (i) Find $\int_{-2}^{1} (14 - x - x^4)\,dx$. *(5 marks)*

(ii) Hence find the area of the shaded region bounded by the curve $y = 14 - x - x^4$ and the line AP. *(2 marks)*

14.June 2011

6 The curve with equation $y = x^3 - 2x^2 + 3$ is sketched below.



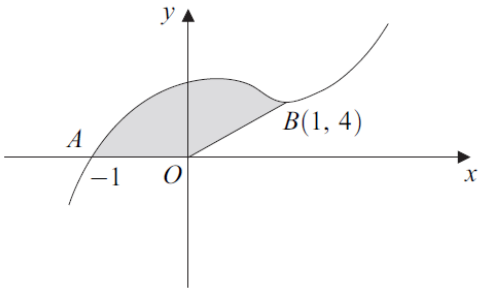
The curve cuts the x-axis at the point $A(-1, 0)$ and passes through the point $B(1, 2)$.

(a) Find $\int_{-1}^{1} (x^3 - 2x^2 + 3)\,dx$. *(5 marks)*

(b) Hence find the area of the shaded region bounded by the curve $y = x^3 - 2x^2 + 3$ and the line AB. *(3 marks)*

15.Jan 2012

4 (d) The curve with equation $y = x^5 - 3x^2 + x + 5$ is sketched below. The point O is at the origin and the curve passes through the points $A(-1, 0)$ and $B(1, 4)$.



(i) Find $\int_{-1}^{1} (x^5 - 3x^2 + x + 5)\,dx$. *(5 marks)*

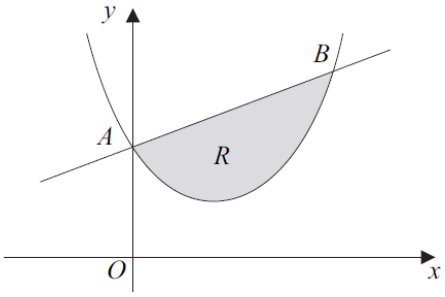
(ii) Hence find the area of the shaded region bounded by the curve between A and B and the line segments AO and OB. *(2 marks)*

16.June2012

5 (a) (i) Express $x^2 - 3x + 5$ in the form $(x - p)^2 + q$. *(2 marks)*

(ii) Hence write down the equation of the line of symmetry of the curve with equation $y = x^2 - 3x + 5$. *(1 mark)*

(b) The curve C with equation $y = x^2 - 3x + 5$ and the straight line $y = x + 5$ intersect at the point $A(0, 5)$ and at the point B, as shown in the diagram below.



(i) Find the coordinates of the point B. *(3 marks)*

(ii) Find $\int (x^2 - 3x + 5)\,dx$. *(3 marks)*

(iii) Find the area of the shaded region R bounded by the curve C and the line segment AB. *(4 marks)*

17.Jan 2013

6 The gradient, $\frac{dy}{dx}$, of a curve at the point (x, y) is given by

$$\frac{dy}{dx} = 10x^4 - 6x^2 + 5$$

The curve passes through the point $P(1, 4)$.

(a) Find the equation of the tangent to the curve at the point P, giving your answer in the form $y = mx + c$. *(3 marks)*

(b) Find the equation of the curve. *(5 marks)*

Answers

1.Jan 2005

4(a)(i)	f(–1) = –1 – 3 + 6 + 8 (Remainder) = 10	M1 A1	 2	Or long division up to remainder term
(ii)	$x-1$ is a factor $x+2$ is a factor	B1 B1	 2	May be earned retrospectively From part (iii)
(iii)	Attempt at third factor f(x) = $(x-1)(x+2)(x-4)$	M1 A1	 2	Multiplying/ dividing/factor theorem $(x+4) \Rightarrow$ M1, A0
(b)(i)	At A , $y = 8$	B1	1	Or (0,8)
(ii)	At B , $x = 4$	B1	1	Or (4,0) NO ft of wrong factor
(c)(i)	$\frac{x^4}{4} - x^3 - 3x^2 + 8x \quad (+c)$	M1 A1 A1 A1	 4	Increase one power by 1 One term correct (unsimplified) Two other terms correct (unsimplified) All correct (unsimplified) (condone missing + c)
(ii)	Realisation that limits are –2 and 1 Area = $\left[\frac{1}{4} - 1 - 3 + 8\right] - [4 + 8 - 12 - 16]$ $= 20\frac{1}{4}$	B1 M1 A1	 3	Condone wrong way round Attempt to sub their limits into their (c)(i) CSO. Must use F(1) – F(–2) correctly
	Total		**15**	

2.June 2005

4(a)(i)	p(3) = 27 – 45 + 21 – 3 p(3) = 0 $\Rightarrow$ x–3 is a factor	M1 A1	 2	Finding p(3) Shown = 0 plus a statement Or $(x-3)(x^2-2x+1)$
(ii)	B is point (3,0)	B1	1	Must have coordinates
(b)(i)	$\frac{dy}{dx} = 3x^2 - 10x + 7$	M1 A1	 2	One term correct All correct with NO +c etc
(ii)	$3x^2 - 10x + 7 = 0$ $\Rightarrow (x-1)(3x-7) = 0$ $\Rightarrow$ at M, $x = \frac{7}{3}$	M1 m1 A1	 3	Putting their $\frac{dy}{dx} = 0$ Attempt to use quad formula or factorise **CSO** factors correct etc
(c)	$\frac{d^2y}{dx^2} = 6x - 10$ sub $x = 1$, $\Rightarrow \frac{d^2y}{dx^2} = -4$	B1√ B1√	 2	ft their $\frac{dy}{dx}$ ft their $\frac{d^2y}{dx^2}$
(d)(i)	$\frac{x^4}{4} - \frac{5x^3}{3} + \frac{7x^2}{2} - 3x \quad (+c)$	M1 A1 A1 A1	 4	Increase one power by 1 One term correct Two other terms correct All correct (condone missing + c)

(ii)	Realisation that limits are 0 and 1	B1		Condone wrong way round
	$\left[\frac{1}{4}-\frac{5}{3}+\frac{7}{2}-3\right]-0$	M1		Attempt to sub their limits into their (d)(i)
	$=-\frac{11}{12}$	A1		**CSO**. Must use F(1) – F(0) correctly
	Area $=\frac{11}{12}$	E1	4	**CSO**. Convincing argument
	Total		**18**	

3.Jan 2006

8(a)	$y_D=3+1=4$ or $y_C=12-8=4$	M1		Attempt at either y coordinate
	Area $ABCD$ $=3\times4=12$	A1	2	
(b)(i)	$x^3-\frac{x^4}{4}$ $(+C)$	M1		Increase one power by 1
		A1		One term correct unsimplified
		A1	3	All correct unsimplified (condone no +C)
(ii)	Sub limits –1 and 2 into their (b) (i) ans	M1		May use both –1, 0 and 0, 2 instead
	$[8-4]-\left[-1-\frac{1}{4}\right]$ $=5\frac{1}{4}$	A1		
	Shaded area = "their" (rectangle– integral)	M1		Alt method: difference of two integrals
	$=12-5\frac{1}{4}=6\frac{3}{4}$	A1	4	**CSO**. Attempted M2, A2
(c)(i)	$\frac{dy}{dx}=6x-3x^2$	M1		One term correct
		A1	2	All correct (no +C etc)
(ii)	When $x=1$, $y=2$ when $x=1$,	B1		May be implied by correct tgt equation
	$\frac{dy}{dx}=3$ as 'their' grad of tgt	M1√		Ft their derivative when $x=1$
	Tangent is $y-2=3(x-1)$	A1	3	Any correct form $y=3x-1$ etc
(iii)	Decreasing when $\frac{dy}{dx}=6x-3x^2<0$	M1		Watch no fudging here!! May work backwards in proof.
	$3(2x-x^2)<0$ $\Rightarrow x^2-2x>0$	A1	2	**AG** (be convinced no step incorrect)
(d)	Two critical points 0 and 2	M1		Marked on diagram or in solution
	$x>2$, $x<0$ ONLY	A1	2	or M1 A0 for $0<x<2$ or $0>x>2$
				SC B1 for $x>2$ (or $x<0$)
	Total		**18**	

4.June 2006

5(a)(i)	$\frac{dy}{dx}=3x^2-20x+28$	M1		One term correct
		A1		Another term correct
		A1	3	All correct (no + c etc)
(ii)	Their $\frac{dy}{dx}=0$ for stationary point	M1		Or realising condition for stationary pt
	$(x-2)(3x-14)=0$	m1		Attempt to solve using formula/ factorise
	$\Rightarrow x=2$	A1		Award M1, A1 for verification that
	or $x=\frac{14}{3}$	A1	4	$x=2\Rightarrow\frac{dy}{dx}=0$ then may earn m1 later
(b)(i)	$\frac{x^4}{4}-\frac{10x^3}{3}+14x^2$ $(+c)$	M1		One term correct unsimplified
		A1		Another term correct unsimplified
		A1	3	All correct unsimplified (condone missing + c)

(ii)	$\left[\frac{81}{4}-90+126\right]$ (-0)	M1		Attempt to sub limit 3 into their (b)(i)
	$= 56\frac{1}{4}$	A1	2	AG Integration, limit sub'n all correct
(iii)	Area of triangle = $31\frac{1}{2}$	B1		Correct unsimplified $\frac{1}{2}\times 21\times 3$
	Shaded Area = $56\frac{1}{4}-$ *triangle area*	M1		
	$= 24\frac{3}{4}$	A1	3	Or equivalent such as $\frac{99}{4}$
	Total		**15**	

5.Jan 2007

6(a)(i)	B (0,5)	B1		
	Area $AOB = \frac{1}{2}\times 1\times 5$	M1		Condone slip in number or a minus sign
	$= 2\frac{1}{2}$	A1	3	
(ii)	$\frac{3x^6}{6}+\frac{2x^2}{2}+5x$ or $\frac{x^6}{2}+x^2+5x$	M1		Raise one power by 1
		A1		One term correct
	(may have $+c$ or not)	A1	3	All correct unsimplified
(iii)	Area under curve = $\int_{-1}^{0} f(x)\,dx$	B1		Correctly written or F(0) – F(–1) correct
	$[0]-\left[\frac{1}{2}+1-5\right]$	M1		Attempt to sub limit(s) of –1 (and 0) Must have integrated
	Area under curve = $3\frac{1}{2}$	A1		**CSO** (no fudging)
	Area of shaded region = $3\frac{1}{2}-2\frac{1}{2}=1$	B1√	4	**FT** their integral and triangle (very generous)

(b)(i)	$\frac{dy}{dx}=15x^4+2$	M1		One term correct
		A1		All correct (no +c etc)
	when $x=-1$, gradient = 17	A1	3	**cso**
(ii)	$y=$ "their gradient"$(x+1)$	B1√	1	Must be finding **tangent** – not normal any form e.g. $y=17x+17$
	Total		**14**	

6.June 2007

Q	Solution	Marks	Total	Comments
6(a)(i)	$f(1)=1+4-5$	M1		must find $f(1)$ NOT long division
	$\Rightarrow f(1)=0\Rightarrow (x-1)$ is factor	A1	2	shown $=0$ plus a statement
(ii)	Attempt at x^2+x+5	M1		long division leading to $x^2\pm x+\ldots$ or equating coefficients
	$f(x)=(x-1)(x^2+x+5)$	A1	2	$p=1,\ q=5$ by inspection scores B1, B1
(iii)	$(x=)\,1$ is real root	B1		
	Consider b^2-4ac for their x^2+x+5	M1		not the cubic!
	$b^2-4ac=1^2-4\times 5=-19<0$			
	Hence no real roots (or only real root is 1)	A1	3	CSO; all values correct plus a statement

(b)(i)	$\int \ldots \, dx = \frac{x^4}{4} + 2x^2 - 5x \ (+c)$	M1 A1 A1	 3	one term correct unsimplified second term correct unsimplified all correct unsimplified
(ii)	$\left[4+8-10\right] - \left[\frac{1}{4}+2-5\right]$	M1		correct use of limits 1 and 2; $F(2) - F(1)$ attempted
	$= 4\frac{3}{4}$	A1		
	Area of $\Delta = \frac{1}{2} \times 11 = 5\frac{1}{2}$	B1		correct unsimplified
	$\Rightarrow$ shaded area $= 5\frac{1}{2} - 4\frac{3}{4}$			combined integral of $7x - 6 - x^3$ scores M1 for limits correctly used then
	$= \frac{3}{4}$	A1	4	A3 correct answer with all working correct
	Total		**14**	

7.Jan 2008

6(a)(i)	$p(-1) = -1 + 7 - 6$ $= 0$ therefore $x + 1$ is a factor	M1 A1	 2	Finding p(–1) Shown to $= 0$ **plus statement**
(ii)	$p(x) = (x+1)(x^2 - x - 6)$	M1 A1		Long division/inspection (2 terms correct) Quadratic factor correct
	$p(x) = (x+1)(x+2)(x-3)$	A1	3	May earn M1,A1 for correct second factor then A1 for $(x+1)(x+2)(x-3)$
(b)(i)	$A(-2, 0)$	B1	1	Condone $x = -2$
(ii)	$\frac{x^4}{4} - \frac{7x^2}{2} - 6x \quad (+c)$ (may have + c or not)	M1 A1 A1		One term correct Another term correct All correct unsimplified
	$\left[\frac{81}{4} - \frac{63}{2} - 18\right] - \left[\frac{1}{4} - \frac{7}{2} + 6\right]$	m1		F(3) – F(–1) attempted in correct order
	$= -32$	A1	5	CSO; OE
(iii)	Area of shaded region = 32	B1√	1	FT their (b)(ii) but positive value needed
(iv)	$\frac{dy}{dx} = 3x^2 - 7$ When $x = -1$, gradient $= -4$	M1 A1 A1	 3	One term correct All correct (no + c etc) CSO
(v)	Gradient of normal $= \frac{1}{4}$	B1√		
	$y =$ "their gradient" $(x \pm 1)$	M1		Must be finding **normal**, not tangent
	$y = \frac{1}{4}(x+1)$	A1	3	CSO; any correct form eg $4y - x = 1$
	Total		**18**	

8.June 2008

5(a)	Grad $AC = \frac{15}{3} = 5$	B1		OE
	Equation of AC: $y = m(x+2)$ or $(y - 15) = m(x - 1)$	M1		Or use of $y = mx + c$ with (–2, 0) or (1, 15) correctly substituted for x and y
	$y = 5x + 10$	A1	3	OE eg $y - 15 = 5(x-1)$, $y = 5(x+2)$

(b)(i)	$\left[16x-\frac{x^5}{5}\right]$	M1 A1 A1		Raise one power by 1 One term correct All correct
	$\left(16-\frac{1}{5}\right)-\left(-32+\frac{32}{5}\right)$	m1		F(1) – F(–2) attempted
	$=41\frac{2}{5}$ (or 41.4 , $\frac{207}{5}$ etc)	A1	5	CSO; withhold if + c added
(ii)	Area $\Delta = \frac{1}{2}\times 3\times 15$ or $22\frac{1}{2}$ or 22.5	B1		Or $\int_{-2}^{1}(5x+10)\,dx = 22.5$
	Shaded area = "their (b)(i) answer" – correct triangle	M1		Condone "difference" if $\Delta > \int$
	$\Rightarrow$ shaded area $=18\frac{9}{10}$	A1	3	CSO; OE (18.9 etc)
	Total		**11**	

9.Jan 2009

6(a)(i)	$p(2)=8+2-10$	M1		Must find p(2) NOT long division
	$\Rightarrow p(2)=0 \Rightarrow (x-2)$ is factor	A1	2	Shown = 0 plus a statement
(ii)	Attempt at long division (generous)	M1		Obtaining a quotient x^2+cx+d or equating coefficients (full method)
	$p(x)=(x-2)(x^2+2x+5)$	A1	2	$a=2,\ b=5$ by inspection B1, B1
(b)(i)	$\frac{dy}{dx}=3x^2+1$	M1 A1		One term correct All correct – no +c etc
	When $x=2$ $\frac{dy}{dx}=3\times 4+1$	m1		Sub $x=2$ into their $\frac{dy}{dx}$
	Therefore gradient at Q is 13	A1	4	CSO
(ii)	$y=13(x-2)$	M1		Tangent (NOT normal) attempted ft their gradient answer from (b)(i)
		A1	2	CSO; correct in any form
(iii)	$\int \ldots dx=\frac{x^4}{4}+\frac{x^2}{2}-10x\ (+c)$	M1 A1 A1	3	one term correct second term correct all correct (condone no +c)
(iv)	$[4+2-20]-[0]=-14$	M1		F(2) attempted and possibly F(0) Must have earned M1 in (b)(iii)
	Area of shaded region = 14	A1	2	CSO; separate statement following correct evaluation of limits
	Total		**15**	

10.June 2009

4(a)(i)	$p(3)=27-3+6$ (Remainder) = 30 **Or** long division up to remainder Quotient= x^2+3x+8 and remainder = 30 clearly stated or indicated	M1 A1 (M1) (A1)	2	p(3) attempted
(ii)	$p(-2)=-8+2+6$	M1		p(–2) attempted : **NOT** long division
	$p(-2)=0 \Rightarrow x+2$ is factor Minimum statement required "factor"	A1	2	Shown = 0 plus statement May make statement *first* such as "x+2 is a factor if p(–2) = 0"

(iii)	$b=-2$ $c=3$	B1 B1		No working required for B1 + B1 Try to mark first using B marks
	or long division/comparing coefficients	(M1)		Award M1 if B0 earned and a clear method is used
	$p(x)=(x+2)(x^2-2x+3)$	(A1)	2	Must write final answer in this form if long division has been used to get A1
(iv)	$b^2-4ac=(-2)^2-4\times 3$	M1		Discriminant correct from their quadratic M0 if $b=-1$, $c=6$ used (using cubic eqn)
	$b^2-4ac=-8$ (or <0) $\Rightarrow$ no (other) real roots	A1		CSO All values must be correct plus statement
	Or $(x-1)^2+2$	(M1)		Completion of square for their quadratic
	$(x-1)^2+2>0$ therefore no real roots Or $(x-1)^2=-2$ has no real roots	(A1)	2	Shown to be positive plus statement regarding no real roots

(b)(i)	$(y_B=)\ 6$	B1	1	Condone (0, 6)
(ii)	$\frac{x^4}{4}-\frac{x^2}{2}+6x$	M1 A1 A1		One term correct Another term correct All correct (ignore + c or limits)
	$\left[\ \right]_{-2}^{0}=0-(4-2-12)$	m1		$F(-2)$ attempted
	$=10$	A1	5	CSO Clearly from F(0) – F(–2)
(iii)	Area of $\Delta=\frac{1}{2}\times 2\times 6$	M1		Condone – 2 and ft their y_B value **Or** $\int_{-2}^{0}(3x+6)\mathrm{d}x$ and attempt to integrate
	$=6$	A1		Must be positive allow –6 converted to +6
	Shaded region area = 10 – 6 = 4	A1	3	CSO 10 must come from correct working
	Total		**17**	

11.Jan 2010

6(a)(i)	$\frac{dy}{dx}=24x-19-6x^2$	M1 A1		2 terms correct all correct (no + c etc)
	when $x=2$, $\frac{dy}{dx}=48-19-24$	m1		
	$\Rightarrow$ gradient $=5$	A1	4	CSO
(ii)	grad of normal $=-\frac{1}{5}$	B1√		ft their answer from (a)(i)
	$y+6=\left(their-\frac{1}{5}\right)(x-2)$ or $y=\left(their-\frac{1}{5}\right)x+c$ and c evaluated using $x=2$ and $y=-6$	M1		ft grad of their normal using **correct** coordinates BUT must not be tangent condone omission of brackets
	$x+5y+28=0$	A1	3	CSO; condone all on one side in different order

				order
(b)(i)	$\frac{12}{3}x^3-\frac{19}{2}x^2-\frac{2}{4}x^4$	M1 A1 A1		one term correct another term correct all correct (ignore $+c$ or limits)
	$=32-38-8$	m1		$F(2)$ attempted
	$=-14$	A1	5	CSO; withhold A1 if changed to +14 here
(ii)	Area $\Delta=\frac{1}{2}\times2\times6=6$	B1		condone -6
	Shaded region area $=14-6$	M1		difference of $\pm\lvert\int\rvert\pm\lvert\Delta\rvert$
	$=8$	A1	3	CSO
	Total		**15**	

12.June 2010

Q	Solution	Marks	Total	Comments
4(a)(i)	$\frac{x^5}{5}-\frac{8}{2}x^2+9x$	M1 A1 A1		One term correct Another term correct All correct (may have $+c$)
	$\frac{32}{5}-16+18$	m1		$F(2)$ attempted
	$=8\frac{2}{5}$	A1	5	$\frac{42}{5}$, 8.4
(ii)	Shaded area $=18-$ '*their integral*'	M1		PI by 18 – (a)(i) NMS
	$=9\frac{3}{5}$	A1	2	$\frac{48}{5}$, 9.6 NMS full marks
(b)(i)	$\frac{dy}{dx}=4x^3-8$	M1 A1		One term correct All correct (no $+c$ etc)
	$x=1\Rightarrow\frac{dy}{dx}=4-8$	m1		sub $x=1$ into *their* $\frac{dy}{dx}$
	(Gradient of curve $)=-4$	A1cso	4	No ISW
(ii)	$y-2=-4(x-1)$; $y=-4x+c$, $c=6$	B1√	1	any correct form ; FT *their* answer from (b)(i) but must use $x=1$ and $y=2$
	Total		**12**	

13.Jan 2011

4(a)(i)	$\frac{dy}{dx}=-1-4x^3$	M1 A1		one of these terms correct all correct (no $+c$)
	(When $x=1$, grad $=$) -5	A1cso	3	(Check that $\frac{dy}{dx}$ is actually correct!)
(ii)	$y-12=$ 'their grad'$(x-1)$	M1		any form of equation through (1, 12) and attempt at c if using $y=mx+c$
	$y=-5x+17$ (or $y=17-5x$)	A1√	2	FT their gradient Condone $y=-5x+c$, $c=17$ etc
(b)(i)	$14x-\frac{x^2}{2}-\frac{x^5}{5}$	M1 A1 A1		one of these terms correct another term correct all correct (may have $+c$)
	$[\ ]_{-2}^{1}=$ $\left(14-\frac{1}{2}-\frac{1}{5}\right)-\left(-28-2+\frac{32}{5}\right)$	m1		F(1) and F(–2) attempted
	$=36.9$ OE	A1	5	Condone recovery to this value

(ii)	Area $\Delta = \frac{1}{2} \times 3 \times 12$ $= 18$ $\Rightarrow$ shaded area = 18.9	M1 A1cso	 2	Correct area of triangle unsimplified
	Total		**12**	

14.June 2011

6(a)	$\int_{-1}^{1} \left(x^3 - 2x^2 + 3 \right) dx$ $= \left[\frac{x^4}{4} - \frac{2x^3}{3} + 3x \right]_{-1}^{1}$ $= \left(\frac{1}{4} - \frac{2}{3} + 3 \right) - \left(\frac{1}{4} + \frac{2}{3} - 3 \right)$ $= 4\frac{2}{3}$	 M1 A1 A1 B1√ A1cso	 5	 one term correct another term correct all correct (condone + c) 'their' F(1) – F(–1) with $(-1)^3$ etc evaluated correctly but must have earned M1 $\frac{14}{3}$, $\frac{56}{12}$ etc but combined as single fraction
(b)	Area of Δ $\left(= \frac{1}{2} \times 2 \times 2 \right)$ $= 2$ Shaded region has area $4\frac{2}{3} - 2$ $= 2\frac{2}{3}$	 B1 M1 A1cso	 PI ± their (a) ± their Δ area $\frac{8}{3}$, $\frac{32}{12}$ etc	 3	 but combined as single fraction
	Total		**8**	

15.Jan 2012

4(a)(i)	$\left(\frac{dy}{dx} = \right) \; 5x^4 - 6x + 1$	M1 A1 A1	 3	one term correct another term correct all correct (no + c etc)
(ii)	$\left(\frac{d^2y}{dx^2} = \right) \; 20x^3 - 6$	B1√	1	FT 'their' $\frac{dy}{dx}$
(b)	$x = -1 \Rightarrow \frac{dy}{dx} = 5(-1)^4 - 6(-1) + 1 \;\; (=12)$ $\Rightarrow \; y = 12(x+1)$	M1 A1cso	 2	must sub $x = -1$ into 'their' $\frac{dy}{dx}$ any correct form with $(x - -1)$ simplified condone $y = 12x + c, \quad c = 12$
(c)	$x = 1 \Rightarrow \frac{dy}{dx} = 5 - 6 + 1$ $\frac{dy}{dx} = 0 \Rightarrow$ stationary point when $x = 1$, $\frac{d^2y}{dx^2} = 14$ $\Rightarrow$ (B is a) minimum (point)	M1 A1cso E1	 3	sub $x = 1$ into their $\frac{dy}{dx}$ shown $= 0$ plus correct statement or $\frac{d^2y}{dx^2} = 20 - 6 > 0$ $\Rightarrow$ (B is a) minimum (point) must have correct $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for E1

(d)(i)	$\frac{x^6}{6}-\frac{3x^3}{3}+\frac{x^2}{2}+5x$	M1 A1 A1		one term correct another term correct all correct (may have $+c$)
	$\left[\frac{1}{6}-1+\frac{1}{2}+5\right]-\left[\frac{1}{6}+1+\frac{1}{2}-5\right]$	m1		'their' F(1) – F(–1) with powers of 1 and –1 evaluated correctly
	$= 8$	A1cso	5	
(ii)	'their answer to part (i)' – 2	M1		
	$\Rightarrow$ Area = 6	A1cso	2	
	Total		**16**	

16.June 2012

5(a)(i)	$\left(x-\frac{3}{2}\right)^2$	M1		or $p = 1.5$ stated
	$\left(x-\frac{3}{2}\right)^2+\frac{11}{4}$	A1	2	$(x-1.5)^2+2.75$
	Mark their final line as their answer			
(ii)	$x=\frac{3}{2}$	B1√	1	correct or FT their "$x=p$"
(b)(i)	$x^2-3x+5=x+5 \Rightarrow x^2=4x$	M1		eliminating x or y and collecting like terms (condone **one** slip) **or** $(y-5)^2-3(y-5)+5=y$ $\Rightarrow y^2-14y+45=0$
	$(x\neq 0)$ $\Rightarrow x=4$	A1		
	$y=9$	A1	3	
(ii)	$\frac{x^3}{3}-\frac{3x^2}{2}+5x\ (+c)$	M1 A1 A1	3	one of these terms correct another term correct all correct (need not have $+c$)
(iii)	$[\ ]_0^4=\frac{4^3}{3}-3\times\frac{4^2}{2}+5\times 4$	M1		must have earned M1 in part(b)(ii) F(their x_B) $\{-F(0)\}$ "correctly sub'd"
	$=17\frac{1}{3}$	A1		$\left(\frac{64}{3}-24+20=\right)$ $\frac{52}{3}$ or $\frac{104}{6}$ etc condone 17.3 but not $16\frac{4}{3}$ etc
	Area trapezium $=\frac{1}{2}(x_B)(5+y_B)$	B1√		FT their numerical values of x_B, y_B Area = $\frac{1}{2}\times 4\times 14$ (= 28)
	Area of shaded region $=28-17\frac{1}{3}$ $=10\frac{2}{3}$	A1	4	CSO; $\frac{32}{3}$, accept 10.7 or better
	Total		**13**	

17.Jan 2013

6(a)	(Gradient = 10 – 6 + 5) = 9	B1		correct gradient from sub $x=1$ into $\frac{dy}{dx}$
	$y-4=$ "*their* 9" $(x-1)$ *or* $y=$ "*their* 9" $x+c$ **and** attempt to find c using $x=1$ and $y=4$	M1		must attempt to use given expression for $\frac{dy}{dx}$ **and** must be attempting tangent and not normal **and** coordinates must be correct

	$y = 9x - 5$	A1	3	condone $y = 9x + c, \ldots \quad c = -5$
(b)	$(y =)\frac{10}{5}x^5 - \frac{6}{3}x^3 + 5x + C$	M1		one term correct
		A1 A1		another term correct all integration correct including + C
	$4 = 2 - 2 + 5 + C$ $\Rightarrow C = -1$	m1		substituting both $x = 1$ and $y = 4$ **and** attempting to find C
	$y = 2x^5 - 2x^3 + 5x - 1$	A1cso	5	must have $y = \ldots$ and coefficients simplified
	Total		**8**	