Practice examination questions for inequalities and simultaneous equations

1. Jun 2005

7 Solve each of the following inequalities:

(a) $3(x-1) > 3 - 5(x+6)$; *(3 marks)*

(b) $x^2 - x - 6 < 0$. *(4 marks)*

2. Jun 2007

7 The quadratic equation

$$(2k-3)x^2 + 2x + (k-1) = 0$$

where k is a constant, has real roots.

(a) Show that $2k^2 - 5k + 2 \leqslant 0$. *(3 marks)*

(b) (i) Factorise $2k^2 - 5k + 2$. *(1 mark)*

(ii) Hence, or otherwise, solve the quadratic inequality

$$2k^2 - 5k + 2 \leqslant 0$$ *(3 marks)*

3. Jan 2008

7 The curve C has equation $y = x^2 + 7$. The line L has equation $y = k(3x+1)$, where k is a constant.

(a) Show that the x-coordinates of any points of intersection of the line L with the curve C satisfy the equation

$$x^2 - 3kx + 7 - k = 0$$ *(1 mark)*

(b) The curve C and the line L intersect in two distinct points. Show that

$$9k^2 + 4k - 28 > 0$$ *(3 marks)*

(c) Solve the inequality $9k^2 + 4k - 28 > 0$. *(4 marks)*

4.June 2008

1 The straight line L has equation $y = 3x - 1$ and the curve C has equation

$$y = (x+3)(x-1)$$

(a) Sketch on the same axes the line L and the curve C, showing the values of the intercepts on the x-axis and the y-axis. *(5 marks)*

(b) Show that the x-coordinates of the points of intersection of L and C satisfy the equation $x^2 - x - 2 = 0$. *(2 marks)*

(c) Hence find the coordinates of the points of intersection of L and C. *(4 marks)*

5.Jan 2009

2 (a) Factorise $2x^2 - 5x + 3$. *(1 mark)*

(b) Hence, or otherwise, solve the inequality $2x^2 - 5x + 3 < 0$. *(3 marks)*

6.June 2009

7 The curve C has equation $y = k(x^2 + 3)$, where k is a constant.

The line L has equation $y = 2x + 2$.

(a) Show that the x-coordinates of any points of intersection of the curve C with the line L satisfy the equation

$$kx^2 - 2x + 3k - 2 = 0$$ *(1 mark)*

(b) The curve C and the line L intersect in two distinct points.

(i) Show that

$$3k^2 - 2k - 1 < 0$$ *(4 marks)*

(ii) Hence find the possible values of k. *(4 marks)*

7.June 2010

7 (a) (i) Express $2x^2 - 20x + 53$ in the form $2(x - p)^2 + q$, where p and q are integers. *(2 marks)*

(ii) Use your result from part **(a)(i)** to explain why the equation $2x^2 - 20x + 53 = 0$ has no real roots. *(2 marks)*

(b) The quadratic equation $(2k - 1)x^2 + (k + 1)x + k = 0$ has real roots.

(i) Show that $7k^2 - 6k - 1 \leqslant 0$. *(4 marks)*

(ii) Hence find the possible values of k. *(4 marks)*

8.June 2011

7 Solve each of the following inequalities:

(a) $2(4 - 3x) > 5 - 4(x + 2)$; *(2 marks)*

(b) $2x^2 + 5x \geqslant 12$. *(4 marks)*

9.Jan 2012

6 A rectangular garden is to have width x metres and length $(x+4)$ metres.

(a) The perimeter of the garden needs to be greater than 30 metres.

Show that $2x > 11$. *(1 mark)*

(b) The area of the garden needs to be less than 96 square metres.

Show that $x^2 + 4x - 96 < 0$. *(1 mark)*

(c) Solve the inequality $x^2 + 4x - 96 < 0$. *(4 marks)*

(d) Hence determine the possible values of the width of the garden. *(1 mark)*

Answers

1.June 2005

7(a)	$3x-3>3-5x-30$	M1		Multiplying out (condone one slip)
	$\Rightarrow 8x>-24$	A1		Or correct equivalent eg $-8x<24$
	$\Rightarrow x>-3$	A1	3	(Penalise $\leqslant$, $\geqslant$ once only in (a) and (b))
(b)	$x^2-x-6=(x-3)(x+2)$	M1		Attempt to use quad formula or factorise
	(critical points are) 3 and –2	A1		May be seen in diagram or solution
	Sketch or sign diagram	M1		+ | – | + ; –2, 3
	$2<x<3$	A1	4	
	Total		**7**	

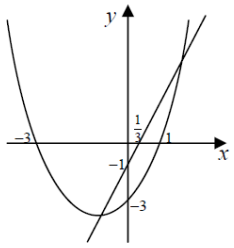
2.June 2007

7(a)	$b^2-4ac=4-4(k-1)(2k-3)$	M1		(or seen in formula) condone one slip
	Real roots when $b^2-4ac\geqslant 0$	E1		must involve $f(k)\geqslant 0$ (usually M1 must be earned)
	$4-4(2k^2-5k+3)\geqslant 0$			
	$\Rightarrow -2k^2+5k-3+1\geqslant 0$			at least one step of working justifying $\leqslant 0$
	$\Rightarrow 2k^2-5k+2\leqslant 0$	A1	3	AG
(b)(i)	$(2k-1)(k-2)$	B1	1	
(ii)	(Critical values) $\frac{1}{2}$ and 2	B1√		ft their factors or correct values seen on diagram, sketch or inequality or stated
	+ | – | + ; $\frac{1}{2}$, 2	M1		use of sketch / sign diagram
	$\Rightarrow 0.5\leqslant k\leqslant 2$	A1	3	M1A0 for $0.5<k<2$ or $k\geqslant 0.5$, $k\leqslant 2$
	Total		**7**	

3.Jan 2008

7(a)	$x^2+7=k(3x+1)\Rightarrow x^2-3kx+7-k=0$	B1	1	AG
(b)	$b^2-4ac=(-3k)^2-4(7-k)$	M1		Clear attempt at b^2-4ac Condone slip in one term of expression
	(2 distinct roots when) $b^2-4ac>0$	B1		Must involve k
	$9k^2+4k-28>0$	A1	3	CSO; AG
(c)	$(9k-14)(k+2)$	M1		Factors or formula correct unsimplified
	Critical points –2 and $\frac{14}{9}$	A1		
	Sketch ∪ or sign diagram correct	M1		**+ve** | **–ve** | **+ve** ; –2, $\frac{14}{9}$
	$k<-2,\ k>\frac{14}{9}$	A1	4	
	Total		**8**	

4.June 2008

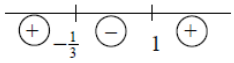
1(a)	L: straight line with positive gradient and negative intercept on y-axis	B1		Line must cross both axes but need not reach the curve
	cutting at $\left(\frac{1}{3},0\right)$ and $(0,-1)$	B1		Condone 0.33 or better for $\frac{1}{3}$
	(intercepts **stated** or marked on sketch)			
	C: **attempt at parabola** $\cup$ or $\cap$ through $(-3,0)$ and $(1,0)$ or values -3 and 1 stated as intercepts on x-axis	B1		
	$\cup$ shaped graph – vertex below x-axis and cutting x-axis twice	M1		
	through $(0,-3)$ and minimum point to left of y-axis	A1	5	(y-intercept or coordinates marked)
(b)	$(x+3)(x-1)=3x-1$	M1		
	$x^2+3x-x-3-3x+1=0$			
	$\Rightarrow x^2-x-2=0$	A1	2	AG; must have "= 0" and no errors
(c)	$(x-2)(x+1)=0$	M1		$(x\pm1)(x\pm2)$ or use of formula (one slip)
	$\Rightarrow x=2,\ -1$	A1		correct values imply M1A1
	Substitute one value of x to find y	m1		
	Points of intersection (2, 5) and (–1,–4)	A1	4	May say $x=2, y=5$ etc SC: (2, 5) $\Rightarrow$ B2 $(-1,-4)\Rightarrow$ B2 without working
	Total		**11**	

5.Jan 2009

2(a)	$(x-1)(2x-3)$	B1	1	$(1-x)(3-2x)$ or $2(x-1)(x-1.5)$ etc
(b)	Critical values are 1, $1\frac{1}{2}$	B1√		Correct or ft their factors from (a)
	Sign diagram or sketch	M1		+ | − | + 1 $1\frac{1}{2}$
	$\Rightarrow 1<x<1\frac{1}{2}$	A1	3	*Full marks for correct inequality without working*
	Total		**4**	

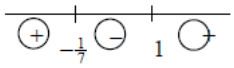
6.June 2009

7(a)	$k\left(x^2+3\right)=2x+2$			
	$\Rightarrow kx^2-2x+3k-2=0$	B1	1	AG OE all terms on one side and = 0
(b)(i)	Discriminant $=(-2)^2-4k(3k-2)$	M1		Condone one slip (including x is one slip) Condone 2^2 or 4 as first term
	$=4-12k^2+8k$	A1		condone recovery from missing brackets
	Two distinct real roots $\Rightarrow b^2-4ac>0$	B1√		"their discriminant in terms of k" >0
	$4-12k^2+8k>0$			Not simply the statement $b^2-4ac>0$
	$\Rightarrow 12k^2-8k-4<0$			Change from >0 to <0 and divide by 4
	$\Rightarrow 3k^2-2k-1<0$	A1	4	AG CSO

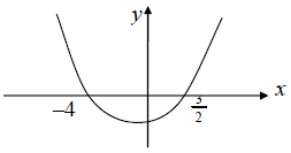
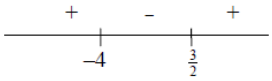
(ii)	$(3k+1)(k-1)$ Critical values 1 and $-\frac{1}{3}$ Use of sign diagram or sketch 	M1 A1 M1		**Correct** factors or **correct** use of formula May score M1, A1 for correct critical values seen as part of incorrect final answer with or without working If previous A1 earned, sign diagram or sketch must be correct for M1 Otherwise, M1 may be earned for an attempt at the sketch or sign diagram using their critical values.
	$\Rightarrow -\frac{1}{3} < k < 1$ or $1 > k > -\frac{1}{3}$ condone $-\frac{1}{3} < k$ AND $k < 1$ for full marks but not OR or "," instead of AND	A1	4	Full marks for correct final answer with or without working $\leqslant$ loses final A mark *Answer only* of $1 < k < -\frac{1}{3}$ or $k < -\frac{1}{3}; k < 1$ *etc* scores M1,A1,M0 since the correct critical values are evident *Answer only* of $\frac{1}{3} < k < 1$ etc where critical values are not both correct gets M0,M0
	Total		**9**	

7.June 2010

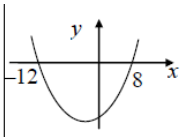
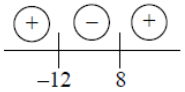
7(a)(i)	$2(x-5)^2$ $+3$	B1 B1	 2	$p=5$ $q=3$
(ii)	Stating both $(x-5)^2 \geqslant 0$ and $3>0$ $\Rightarrow 2x^2-20x+53>0$ *or* $2(x-5)^2+3>0$ $\Rightarrow 2x^2-20x+53=0$ has no real roots	M1 A1cso	 2	FT their p & q, but must have $q>0$ Must have statement and correct p & q.
(b)(i)	$b^2-4ac=(k+1)^2-4k(2k-1)$ $=-7k^2+6k+1$ real roots $\Rightarrow b^2-4ac \geqslant 0$ $-7k^2+6k+1 \geqslant 0$ $\Rightarrow 7k^2-6k-1 \leqslant 0$	M1 A1 B1√ A1cso	 4	Condone one slip (including x is one slip) Condone recovery from missing brackets Their discriminant $\geqslant 0$ (in terms of k) Need not be simplified & may earn earlier AG (must see sign change)

(ii)	$(7k+1)(k-1)$	M1		**Correct** factors or **correct** use of formula
	Critical values $k=1, -\frac{1}{7}$	A1		May score M1, A1 for correct critical values seen as part of incorrect final answer with or without working.
	Use of sign diagram or sketch 	M1		If previous A1 earned, sign diagram or sketch must be correct for M1 Otherwise M1 may be earned for an attempt at the sketch or sign diagram using ***their*** critical values.
	$-\frac{1}{7} \leqslant k \leqslant 1$	A1	4	$\left(-\frac{1}{7} < k < 1\right)$, $\left(k \geqslant -\frac{1}{7} \text{ OR } k \leqslant 1\right)$, $\left(k \geqslant -\frac{1}{7},\ k \leqslant 1\right)$ score M1A1M1A0
	Full marks for correct answer NMS			
	Condone $-\frac{2}{14}$ throughout			*Answer only of* $k < -\frac{1}{7}$, $k < 1$ etc scores M1, A1, M0 since the critical values are evident.
	Condone $k \geqslant -\frac{1}{7}$ AND $k \leqslant 1$ for full marks			*Answer only of* $\frac{1}{7} \leqslant k \leqslant 1$ etc scores M0, M0 since the critical values are not both correct.
	Take their final line as their answer.			
	Total		**12**	

8.June 2011

7(a)	$8-6x > 5-4x-8$ $11 > 2x$	M1		multiplying out correctly and $>$ sign used
	$x < 5\frac{1}{2}$ $\left(\text{or } x < \frac{11}{2}\right)$	A1cso	2	accept $5.5 > x$ OE
(b)	$2x^2+5x-12 \geqslant 0$			
	$(x+4)(2x-3)$	M1		correct factors (or roots unsimplified) $\frac{-5 \pm \sqrt{121}}{4}$
	Critical values are -4 and $\frac{3}{2}$	A1		both CVs correct; condone $\frac{6}{4}$, $-\frac{16}{4}$ etc here but must be single fractions
		M1		sketch or sign diagram including values 
	$x \leqslant -4$, $x \geqslant \frac{3}{2}$ ***take their final line as their answer***	A1	4	fractions must be simplified condone use of **OR** but not **AND**
	Total		**6**	

9.Jan 2012

6(a)	Sides are x and $x+4$ $\Rightarrow x+x+x+4+x+4>30$ *or* $2x+2x+8>30$ *or* $2(2x+4)>30$ *or* $4x+8>30$ $(\Rightarrow 4x>22)$ $\Rightarrow 2x>11$	 B1	 1	 must see this line OE **AG** (be convinced) condone $11<2x$
(b)	$x(x+4)<96$ $\Rightarrow x^2+4x-96<0$	 B1	 1	must see this line OE **AG**
(c)	$(x+12)(x-8)$ Critical values 8, -12 y ↑ or	M1 A1		correct factors or correct quadratic equation formula
	 or  $\Rightarrow -12<x<8$	M1 A1cso	 4	sketch or sign diagram accept $x<8$ **AND** $x>-12$ but **not** $x<8$ **OR** $x>-12$ **nor** $x<8$, $x>-12$
(d)	$5\frac{1}{2}<x<8$	B1	1	
	Total		7	