Practice examination questions for differentiation, max/min Points and optimisation

1. Jan2005

2 A curve has equation $y = x^5 - 6x^3 - 3x + 25$.

(a) Find $\frac{dy}{dx}$. *(3 marks)*

(b) The point P on the curve has coordinates $(2, 3)$.

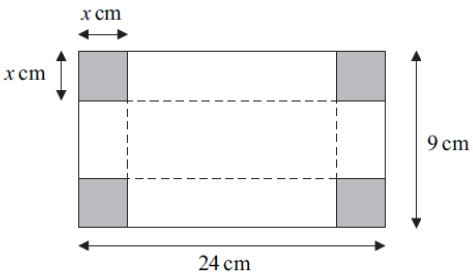
(i) Show that the gradient of the curve at P is 5. *(2 marks)*

(ii) Hence find an equation of the normal to the curve at P, expressing your answer in the form $ax + by = c$, where a, b and c are integers. *(3 marks)*

(c) Determine whether y is increasing or decreasing when $x = 1$. *(2 marks)*

2.Jan 2005

6 The diagram below shows a rectangular sheet of metal 24 cm by 9 cm.



A square of side x cm is cut from each corner and the metal is then folded along the broken lines to make an open box with a rectangular base and height x cm.

(a) Show that the volume, V cm^3, of liquid the box can hold is given by

$$V = 4x^3 - 66x^2 + 216x$$ *(3 marks)*

(b) (i) Find $\frac{dV}{dx}$. *(3 marks)*

(ii) Show that any stationary values of V must occur when $x^2 - 11x + 18 = 0$. *(2 marks)*

(iii) Solve the equation $x^2 - 11x + 18 = 0$. *(2 marks)*

(iv) Explain why there is only one value of x for which V is stationary. *(1 mark)*

(c) (i) Find $\frac{d^2V}{dx^2}$. *(2 marks)*

(ii) Hence determine whether the stationary value is a maximum or minimum. *(2 marks)*

3. Jan 2006

7 The volume, $V\,\text{m}^3$, of water in a tank at time t seconds is given by

$$V = \tfrac{1}{3}t^6 - 2t^4 + 3t^2, \quad \text{for } t \geqslant 0$$

(a) Find:

(i) $\dfrac{dV}{dt}$; *(3 marks)*

(ii) $\dfrac{d^2V}{dt^2}$. *(2 marks)*

(b) Find the rate of change of the volume of water in the tank, in $\text{m}^3\,\text{s}^{-1}$, when $t = 2$. *(2 marks)*

(c) (i) Verify that V has a stationary value when $t = 1$. *(2 marks)*

(ii) Determine whether this is a maximum or minimum value. *(2 marks)*

4.June2006

3 A curve has equation $y = 7 - 2x^5$.

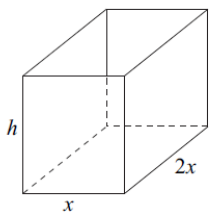
(a) Find $\dfrac{dy}{dx}$. *(2 marks)*

(b) Find an equation for the tangent to the curve at the point where $x = 1$. *(3 marks)*

(c) Determine whether y is increasing or decreasing when $x = -2$. *(2 marks)*

5.Jan 2007

5 The diagram shows an **open-topped** water tank with a horizontal rectangular base and four vertical faces. The base has width x metres and length $2x$ metres, and the height of the tank is h metres.



The combined internal surface area of the base and four vertical faces is $54\,\text{m}^2$.

(a) (i) Show that $x^2 + 3xh = 27$. *(2 marks)*

(ii) Hence express h in terms of x. *(1 mark)*

(iii) Hence show that the volume of water, $V\ \text{m}^3$, that the tank can hold when full is given by

$$V = 18x - \frac{2x^3}{3}$$ *(1 mark)*

(b) (i) Find $\frac{dV}{dx}$. *(2 marks)*

(ii) Verify that V has a stationary value when $x = 3$. *(2 marks)*

(c) Find $\frac{d^2V}{dx^2}$ and hence determine whether V has a maximum value or a minimum value when $x = 3$. *(2 marks)*

6.June 2007

4 A model helicopter takes off from a point O at time $t = 0$ and moves vertically so that its height, y cm, above O after time t seconds is given by

$$y = \frac{1}{4}t^4 - 26t^2 + 96t, \qquad 0 \leqslant t \leqslant 4$$

(a) Find:

(i) $\frac{dy}{dt}$; *(3 marks)*

(ii) $\frac{d^2y}{dt^2}$. *(2 marks)*

(b) Verify that y has a stationary value when $t = 2$ and determine whether this stationary value is a maximum value or a minimum value. *(4 marks)*

(c) Find the rate of change of y with respect to t when $t = 1$. *(2 marks)*

(d) Determine whether the height of the helicopter above O is increasing or decreasing at the instant when $t = 3$. *(2 marks)*

7.Jan 2008

2 The curve with equation $y = x^4 - 32x + 5$ has a single stationary point, M.

(a) Find $\frac{dy}{dx}$. *(3 marks)*

(b) Hence find the x-coordinate of M. *(3 marks)*

(c) (i) Find $\frac{d^2y}{dx^2}$. *(1 mark)*

(ii) Hence, or otherwise, determine whether M is a maximum or a minimum point. *(2 marks)*

(d) Determine whether the curve is increasing or decreasing at the point on the curve where $x = 0$. *(2 marks)*

8.June 2008

3 Two numbers, x and y, are such that $3x + y = 9$, where $x \geqslant 0$ and $y \geqslant 0$.

It is given that $V = xy^2$.

(a) Show that $V = 81x - 54x^2 + 9x^3$. *(2 marks)*

(b) (i) Show that $\frac{dV}{dx} = k(x^2 - 4x + 3)$, and state the value of the integer k. *(4 marks)*

(ii) Hence find the two values of x for which $\frac{dV}{dx} = 0$. *(2 marks)*

(c) Find $\frac{d^2V}{dx^2}$. *(2 marks)*

(d) (i) Find the value of $\frac{d^2V}{dx^2}$ for each of the two values of x found in part (b)(ii). *(1 mark)*

(ii) Hence determine the value of x for which V has a maximum value. *(1 mark)*

(iii) Find the maximum value of V. *(1 mark)*

9.Jan 2009

5 A model car moves so that its distance, x centimetres, from a fixed point O after time t seconds is given by

$$x = \frac{1}{2}t^4 - 20t^2 + 66t, \qquad 0 \leqslant t \leqslant 4$$

(a) Find:

(i) $\frac{dx}{dt}$; *(3 marks)*

(ii) $\frac{d^2x}{dt^2}$. *(2 marks)*

(b) Verify that x has a stationary value when $t = 3$, and determine whether this stationary value is a maximum value or a minimum value. *(4 marks)*

(c) Find the rate of change of x with respect to t when $t = 1$. *(2 marks)*

(d) Determine whether the distance of the car from O is increasing or decreasing at the instant when $t = 2$. *(2 marks)*

10.June 2009

3 The curve with equation $y = x^5 + 20x^2 - 8$ passes through the point P, where $x = -2$.

(a) Find $\frac{dy}{dx}$. *(3 marks)*

(b) Verify that the point P is a stationary point of the curve. *(2 marks)*

(c) (i) Find the value of $\frac{d^2y}{dx^2}$ at the point P. *(3 marks)*

(ii) Hence, or otherwise, determine whether P is a maximum point or a minimum point. *(1 mark)*

(d) Find an equation of the tangent to the curve at the point where $x = 1$. *(4 marks)*

11.Jan 2010

3 The depth of water, y metres, in a tank after time t hours is given by

$$y = \frac{1}{8}t^4 - 2t^2 + 4t, \qquad 0 \leqslant t \leqslant 4$$

(a) Find:

(i) $\frac{dy}{dt}$; *(3 marks)*

(ii) $\frac{d^2y}{dt^2}$. *(2 marks)*

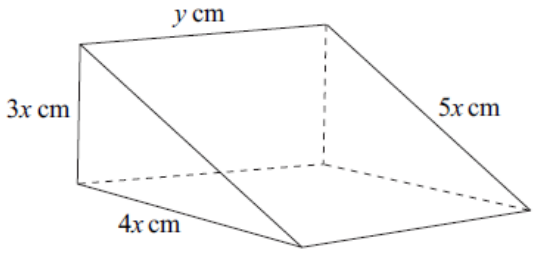
(b) Verify that y has a stationary value when $t = 2$ and determine whether it is a maximum value or a minimum value. *(4 marks)*

(c) (i) Find the rate of change of the depth of water, in metres per hour, when $t = 1$. *(2 marks)*

(ii) Hence determine, with a reason, whether the depth of water is increasing or decreasing when $t = 1$. *(1 mark)*

12.June 2010

6 The diagram shows a block of wood in the shape of a prism with triangular cross-section. The end faces are right-angled triangles with sides of lengths $3x$ cm, $4x$ cm and $5x$ cm, and the length of the prism is y cm, as shown in the diagram.



The total surface area of the five faces is $144\,\text{cm}^2$.

(a) (i) Show that $xy + x^2 = 12$. *(3 marks)*

(ii) Hence show that the volume of the block, $V\,\text{cm}^3$, is given by

$$V = 72x - 6x^3$$ *(2 marks)*

(b) (i) Find $\frac{dV}{dx}$. *(2 marks)*

(ii) Show that V has a stationary value when $x = 2$. *(2 marks)*

(c) Find $\frac{d^2V}{dx^2}$ and hence determine whether V has a maximum value or a minimum value when $x = 2$. *(2 marks)*

13.Jan 2011

1 The curve with equation $y = 13 + 18x + 3x^2 - 4x^3$ passes through the point P where $x = -1$.

(a) Find $\frac{dy}{dx}$. *(3 marks)*

(b) Show that the point P is a stationary point of the curve and find the other value of x where the curve has a stationary point. *(3 marks)*

(c) (i) Find the value of $\frac{d^2y}{dx^2}$ at the point P. *(3 marks)*

(ii) Hence, or otherwise, determine whether P is a maximum point or a minimum point. *(1 mark)*

14.June 2011

3 The volume, $V\,\text{m}^3$, of water in a tank after time t seconds is given by

$$V = \frac{t^3}{4} - 3t + 5$$

(a) Find $\frac{dV}{dt}$. *(2 marks)*

(b) (i) Find the rate of change of volume, in $\text{m}^3\,\text{s}^{-1}$, when $t = 1$. *(2 marks)*

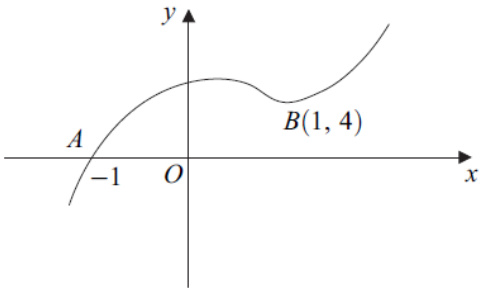
(ii) Hence determine, with a reason, whether the volume is increasing or decreasing when $t = 1$. *(1 mark)*

(c) (i) Find the positive value of t for which V has a stationary value. *(3 marks)*

(ii) Find $\frac{d^2V}{dt^2}$, and hence determine whether this stationary value is a maximum value or a minimum value. *(3 marks)*

15.Jan 2012

4 The curve with equation $y = x^5 - 3x^2 + x + 5$ is sketched below. The point O is at the origin and the curve passes through the points $A(-1, 0)$ and $B(1, 4)$.



(a) Given that $y = x^5 - 3x^2 + x + 5$, find:

(i) $\frac{dy}{dx}$; *(3 marks)*

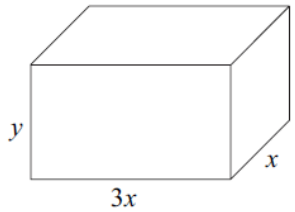
(ii) $\frac{d^2y}{dx^2}$. *(1 mark)*

(b) Find an equation of the tangent to the curve at the point $A(-1, 0)$. *(2 marks)*

(c) Verify that the point B, where $x = 1$, is a minimum point of the curve. *(3 marks)*

16.June 2012

4 The diagram shows a solid cuboid with sides of lengths x cm, $3x$ cm and y cm.



The total surface area of the cuboid is $32\,\text{cm}^2$.

(a) (i) Show that $3x^2 + 4xy = 16$. *(2 marks)*

(ii) Hence show that the volume, $V\,\text{cm}^3$, of the cuboid is given by

$$V = 12x - \frac{9x^3}{4}$$ *(2 marks)*

(b) Find $\frac{dV}{dx}$. *(2 marks)*

(c) (i) Verify that a stationary value of V occurs when $x = \frac{4}{3}$. *(2 marks)*

(ii) Find $\frac{d^2V}{dx^2}$ and hence determine whether V has a maximum value or a minimum value when $x = \frac{4}{3}$. *(2 marks)*

17.Jan 2013

2 A bird flies from a tree. At time t seconds, the bird's height, y metres, above the horizontal ground is given by

$$y = \frac{1}{8}t^4 - t^2 + 5, \quad 0 \leqslant t \leqslant 4$$

(a) Find $\frac{dy}{dt}$. *(2 marks)*

(b) (i) Find the rate of change of height of the bird in metres per second when $t = 1$. *(2 marks)*

(ii) Determine, with a reason, whether the bird's height above the horizontal ground is increasing or decreasing when $t = 1$. *(1 mark)*

(c) (i) Find the value of $\frac{d^2y}{dt^2}$ when $t = 2$. *(2 marks)*

(ii) Given that y has a stationary value when $t = 2$, state whether this is a maximum value or a minimum value. *(1 mark)*

Answers

1.Jan 2005

2(a)	$\frac{dy}{dx} = 5x^4 - 18x^2 - 3$	M1 A1 A1	 3	Decrease one power by 1 One term correct All correct
(b)(i)	Sub $x = 2$ into their $\frac{dy}{dx}$ Shown to equal 5	M1 A1	 2	$80 - 72 - 3$ AG (be convinced)
(ii)	Gradient of normal $= -\frac{1}{5}(y + \frac{1}{5}x + ...)$ $y - 3 = -\frac{1}{5}(x - 2)$ $x + 5y = 17$ (integer coefficients)	B1 M1 A1	 3	Or $m_1 m_2 = -1$ used or stated Trying normal NOT tangent or $y = mx + c$ and attempt to find c Or integer multiple of coefficients
(c)	Sub $x = 1$ into their $\frac{dy}{dx}$ $(= -16 < 0)$ Negative value $\Rightarrow$ DECREASING	M1 E1√	 2	$(5 - 18 - 3 = -16)$ (Watch $\frac{d^2y}{dx^2} = -16$!) Correct interpretation of sign of $\frac{dy}{dx}$
	Total		**10**	

2.Jan 2005

6(a)	Sides $24 - 2x$, $9 - 2x$ $V = x(24 - 2x)(9 - 2x)$ $= 4x^3 - 66x^2 + 216x$	B1 M1 A1	 3	Either correct 3 sides involving x multiplied together AG (be convinced)
(b)(i)	$\frac{dV}{dx} = 12x^2 - 132x + 216$	M1 A1 A1	 3	Power decreased by 1 One term correct All correct (no $+C$ etc)
(ii)	Putting their $\frac{dV}{dx} = 0$ (must see this first) $\Rightarrow x^2 - 11x + 18 = 0$	M1 A1	 2	Or their $12x^2 - 132x + 216 = 0$ Or $12(x^2 - 11x + 18) = 0$ or statement AG (be convinced)
(iii)	$(x - 2)(x - 9) = 0$ $\Rightarrow x = 2, \quad x = 9$	M1 A1	 2	Factors, comp sq or formulae used (1 slip)
(iv)	Reject $x = 9$, since $9 - 2x < 0$	E1	1	$x = 2$ is only possible value
(c)(i)	$\frac{d^2V}{dx^2} = 24x - 132$	M1 A1	 2	Differentiating their $\frac{dV}{dx}$ (eg $2x - 11$) Correct
(ii)	$x = 2$ only $\Rightarrow \frac{d^2V}{dx^2} = -84$ (or < 0) Maximum value	B1 E1√	 2	Correct $\frac{d^2V}{dx^2}$ value OE full test. ft if their test implies minimum
	Total		**15**	

3.Jan 2006

Q	Solution	Marks	Total	Comments
7(a)(i)	$\frac{dV}{dt} = 2t^5 - 8t^3 + 6t$	M1 A1 A1	3	One term correct unsimplified Further term correct unsimplified All correct unsimplified (no + c etc)
(ii)	$\frac{d^2V}{dt^2} = 10t^4 - 24t^2 + 6$	M1 A1	2	One term FT correct unsimplified **CSO**. All correct simplified
(b)	Substitute $t = 2$ into their $\frac{dV}{dt}$ $(= 64 - 64 + 12) = 12$	M1 A1	2	**CSO**. Rate of change of volume is $12\text{m}^3\ \text{s}^{-1}$
(c)(i)	$t = 1 \Rightarrow \frac{dV}{dt} = 2 - 8 + 6$ $= 0 \Rightarrow$ Stationary value	M1 A1	2	Or putting their $\frac{dV}{dt} = 0$ **CSO**. Shown to = 0 **AND** statement (If solving equation must obtain $t = 1$)
(ii)	$t = 1 \Rightarrow \frac{d^2V}{dt^2} = -8$ Maximum value	M1 A1√	2	Sub $t = 1$ into their second derivative or equivalent full test. Ft if their test implies minimum
	Total		**11**	

4.June 2006

3(a)	$\frac{dy}{dx} = -10x^4$	M1 A1	2	kx^4 condone extra term Correct derivative unsimplified
(b)	When $x = 1$, gradient $= -10$	B1√		FT their gradient when $x = 1$
	Tangent is $y - 5 = -10(x - 1)$ *or* $y + 10x = 15$ etc	M1 A1	3	Attempt at y & tangent (**not** normal) CSO Any correct form
(c)	When $x = -2$ $\frac{dy}{dx} = -160$ (or < 0)	B1√		Value of their $\frac{dy}{dx}$ when $x = -2$
	($\frac{dy}{dx} < 0$ hence) y is **decreasing**	E1√	2	ft Increasing if their $\frac{dy}{dx} > 0$
	Total		**7**	

5.Jan 2007

5(a)(i)	$2x^2 + 2xh + 4xh \quad (= 54)$ $\Rightarrow x^2 + 3xh = 27$	M1 A1	2	Attempt at surface area (one slip) **AG CSO**
(ii)	$h = \frac{27 - x^2}{3x}$ or $h = \frac{9}{x} - \frac{x}{3}$ etc	B1	1	Any correct form
(iii)	$V = 2x^2h = 18x - \frac{2x^3}{3}$	B1	1	**AG** (watch fudging) condone omission of brackets
(b)(i)	$\frac{dV}{dx} = 18 - 2x^2$	M1 A1	2	One term correct "their" V All correct unsimplified $18 - 6x^2/3$
(ii)	Sub $x = 3$ into their $\frac{dV}{dx}$	M1		Or attempt to solve their $\frac{dV}{dx} = 0$
	Shown to equal 0 plus **statement** that this implies a stationary point if verifying	A1	2	**CSO** Condone $x = \pm 3$ or $x = 3$ if solving

(c)	$\frac{d^2V}{dx^2} = -4x$ $(=-12)$	B1√		**FT** their $\frac{dV}{dx}$
	$\frac{d^2V}{dx^2} < 0$ at stationary point $\Rightarrow$ maximum	E1√	2	**FT** their second derivative conclusion If "their" $\frac{d^2y}{dx^2} > 0 \Rightarrow$ minimum etc
	Total		**10**	

6.June 2007

4(a)(i)	$t^3 - 52t + 96$	M1 A1 A1	 3	one term correct another term correct all correct (no + c etc)
(ii)	$3t^2 - 52$	M1 A1√	 2	ft one term correct ft all "correct"
(b)	$\frac{dy}{dt} = 8 - 104 + 96$ $= 0 \Rightarrow$ stationary value Substitute $t = 2$ into $\frac{d^2y}{dt^2}$ $(=-40)$ $\frac{d^2y}{dt^2} < 0 \Rightarrow$ max value	M1 A1 M1 A1	 4	substitute $t = 2$ into their $\frac{dy}{dt}$ CSO; shown = 0 + statement any appropriate test, e.g. $y'(1)$ and $y'(3)$ all values (if stated) must be correct
(c)	Substitute $t = 1$ into their $\frac{dy}{dt}$ Rate of change = 45 (cms^{-1})	M1 A1√	 2	must be their $\frac{dy}{dt}$ NOT $\frac{d^2y}{dt^2}$ ft their $y'(1)$
(d)	Substitute $t = 3$ into their $\frac{dy}{dt}$ $(27 - 156 + 96 = -33 < 0)$ $\Rightarrow$ decreasing when $t = 3$	M1 E1√	 2	interpreting their value of $\frac{dy}{dt}$ allow increasing if their $\frac{dy}{dt} > 0$
	Total		**13**	

7.Jan 2008

2(a)	$\frac{dy}{dx} = 4x^3 - 32$	M1 A1 A1	 3	Reduce one power by 1 One term correct All correct (no + c etc)
(b)	Stationary point $\Rightarrow \frac{dy}{dx} = 0$ $\Rightarrow x^3 = 8$ $\Rightarrow x = 2$	M1 A1√ A1	 3	 $x^n = k$ following from their $\frac{dy}{dx}$ CSO
(c)(i)	$\frac{d^2y}{dx^2} = 12x^2$	B1√	1	FT their $\frac{dy}{dx}$
(ii)	When $x = 2$, $\frac{d^2y}{dx^2}$ considered $\Rightarrow$ minimum point	M1 E1√	 2	Or complete test with $2 \pm \varepsilon$ using $\frac{dy}{dx}$

(d)	Putting $x = 0$ into their $\frac{dy}{dx}$ $(= -32)$	M1		
	$\frac{dy}{dx} < 0 \Rightarrow$ decreasing	A1√	2	Allow "increasing" if their $\frac{dy}{dx} > 0$
	Total		**11**	

8.June 2008

3(a)	$V = x(9-3x)^2$	M1		Attempt at V in terms of x (condone slip when rearranging formula for $y = 9 - 3x$) or $(9-3x)^2 = 81 - 54x + 9x^2$
	$V = x(81 - 54x + 9x^2)$ $= 81x - 54x^2 + 9x^3$	A1	2	AG; no errors in algebra
(b)(i)	$\frac{dV}{dx} = 81 - 108x + 27x^2$	M1 A1 A1		One term correct Another correct All correct (no + c etc)
	$= 27(x^2 - 4x + 3)$	A1	4	CSO; all algebra and differentiation correct
(ii)	$(x-3)(x-1)$ or $(27x-81)(x-1)$ etc $\Rightarrow x = 1, 3$	M1 A1	2	"Correct" factors or correct use of formula SC: B1,B1 for $x = 1$, $x = 3$ found by inspection (provided no other values)
(c)	$\frac{d^2V}{dx^2} = -108 + 54x$ (condone one slip)	M1 A1	2	ft their $\frac{dV}{dx}$ (may have cancelled 27 etc) CSO; all differentiation correct
(d)(i)	$x = 3 \Rightarrow \frac{d^2V}{dx^2} = 54$; $x = 1 \Rightarrow \frac{d^2V}{dx^2} = -54$	B1√	1	ft their $\frac{d^2V}{dx^2}$ and their **two** x-values
(ii)	$(x =)$ 1 (gives maximum value)	E1	1	Provided their $\frac{d^2V}{dx^2} < 0$
(iii)	$V_{max} = 36$	B1	1	CAO
	Total		**13**	

9.Jan 2009

5(a)(i)	$\frac{dx}{dt} = 2t^3 - 40t + 66$	M1 A1 A1	3	one term correct another term correct all correct unsimplified (no + c etc)
(ii)	$\frac{d^2x}{dt^2} = 6t^2 - 40$	M1 A1√	2	ft one term correct ft all "correct", 2 terms equivalent
(b)	$\frac{dx}{dt} = 54 - 120 + 66$ $= 0 \Rightarrow$ stationary value	M1 A1		substitute $t = 3$ into their $\frac{dx}{dt}$ CSO shown = 0 (54 or 2×27 seen) and statement

	Substitute $t = 3$ into $\frac{d^2x}{dt^2}$ $(= 14)$	M1		
	$\frac{d^2x}{dt^2} > 0 \Rightarrow$ minimum value	A1	4	CSO; all values (if stated) must be correct
(c)	Substitute $t = 1$ into their $\frac{dx}{dt}$	M1		must be their $\frac{dx}{dt}$ NOT $\frac{d^2x}{dt^2}$ etc
	$\frac{dx}{dt} = 28$	A1√	2	ft their $\frac{dx}{dt}$ when $t = 1$
(d)	Substitute $t = 2$ into their $\frac{dx}{dt}$	M1		must be their $\frac{dx}{dt}$ NOT $\frac{d^2x}{dt^2}$ or x
	$= 16 - 80 + 66 = 2 \quad (> 0)$			Interpreting their value of $\frac{dx}{dt}$
	$\Rightarrow$ increasing when $t = 2$	E1√	2	Allow decreasing if their $\frac{dx}{dt} < 0$
	Total		**13**	

10.June 2009

3(a)	$\frac{dy}{dx} = 5x^4 + 40x$	M1 A1 A1	3	One of these powers correct One of these terms correct All correct (no + c etc)
(b)	$x = -2 \quad \frac{dy}{dx} = 5\times(-2)^4 + (40\times -2)$	M1		Substitute $x = -2$ into their $\frac{dy}{dx}$
	$\frac{dy}{dx} = 5\times 16 + (40\times -2) = 0$			
	$\Rightarrow P$ is stationary point	A1		CSO Shown = 0 plus statement eg "st pt", "as required", "grad = 0"etc
	Or *their* $\frac{dy}{dx} = 0 \Rightarrow x^n = k$	(M1)		
	$x^3 = -8 \Rightarrow x = -2$	(A1)	2	CSO $x = 0$ need not be considered
(c)(i)	$\frac{d^2y}{dx^2} = 20x^3 + 40$	B1√		Correct ft their $\frac{dy}{dx}$
	$= 20\times(-2)^3 + 40$	M1		Subst $x = -2$ into their second derivative
	$(= -160 + 40) \quad = -120$	A1	3	CSO
(ii)	Maximum (value) their c(i) answer must be < 0 Other valid methods acceptable provided "maximum" is the conclusion	E1√	1	Accept minimum if their $c(i)$ *answer* > 0 and correctly interpreted Parts (i) and (ii) may be combined by candidate but –120 must be seen to award A1 in part (c)(i)
(d)	(When $x = 1$) $y = 13$	B1		
	When $x = 1$, $\frac{dy}{dx} = 5 + 40$	M1		Sub $x = 1$ into their $\frac{dy}{dx}$
	$y =$ (their 45)$x + k$ OE	m1		ft their $\frac{dy}{dx}$
	Tangent has equation $y - 13 = 45(x - 1)$	A1	4	CSO OE $y = 45x + c$, $c = -32$
	Total		**13**	

11.Jan 2010

3(a)(i)	$\frac{dy}{dt} = \frac{4t^3}{8} - 4t + 4$	M1 A1 A1	 3	one term correct another term correct all correct (no + c etc) unsimplified
(ii)	$\frac{d^2y}{dt^2} = \frac{12t^2}{8} - 4$	M1 A1	 2	ft one term "correct" correct unsimplified (penalise inclusion of $+c$ once only in question)
(b)	$t=2$; $\frac{dy}{dt} = 4-8+4$ $\frac{dy}{dt}=0 \Rightarrow$ stationary value $t=2$; $\frac{d^2y}{dt^2}=6-4=2$ $\Rightarrow y$ has MINIMUM value	M1 A1 M1 A1	 4	Substitute $t=2$ into their $\frac{dy}{dt}$ CSO; shown = 0 plus statement Sub $t=2$ into their $\frac{d^2y}{dt^2}$ CSO
(c)(i)	$t=1$; $\frac{dy}{dt} = \frac{1}{2} - 4 + 4$ $= \frac{1}{2}$	M1 A1	 2	Substitute $t=1$ into their $\frac{dy}{dt}$ OE; CSO
(ii)	$\frac{dy}{dt} > 0 \Rightarrow$ (depth is) INCREASING	E1√	1	NMS full marks if $\frac{dy}{dt}$ correct allow decreasing if states that their $\frac{dy}{dt} < 0$ **Reason must be given not just the word increasing or decreasing**
	Total		**12**	

12.June 2010

6(a)(i)	S.A. $= 4xy + 5xy + 3xy + 6x^2 + 6x^2$ OE $= 12xy + 12x^2$ $144 = 12xy + 12x^2$ $\Rightarrow xy + x^2 = 12$	M1 A1 A1cso	 3	Condone one slip or omission Must see this line AG
(ii)	(Volume =) $\frac{1}{2} \times 3x \times 4x \times y$ OE $= 6x^2 \times \frac{(12-x^2)}{x}$ $(V =) 72x - 6x^3$	M1 A1	 2	 Must see $(y =)\frac{(12-x^2)}{x}$ or $xy = 12 - x^2$ for A1 AG must be convinced not working back from answer
(b)(i)	$\frac{dV}{dx} = 72 - 18x^2$	M1 A1	 2	One term correct All correct (no + c etc)
(ii)	$x=2 \Rightarrow \frac{dV}{dx} = 72 - 18 \times 2^2$ $\Rightarrow \frac{dV}{dx} = 72 - 72 = 0$ $\Rightarrow$ stationary (value when $x = 2$)	M1 A1	 2	Substitute $x=2$ into their $\frac{dV}{dx}$ Shown = 0 plus statement Statement may appear first

(c)	$\frac{d^2V}{dx^2} = -36x$	B1√		FT their $\frac{dV}{dx}$
	$\frac{d^2V}{dx^2} = -72$ *or* when $x = 2 \Rightarrow \frac{d^2V}{dx^2} < 0$			
	$\Rightarrow$ maximum	E1√	2	FT their $\frac{d^2V}{dx^2}$ value when $x = 2$ with appropriate conclusion
	Total		**11**	

13.Jan 2011

1(a)	$\frac{dy}{dx} = 18 + 6x - 12x^2$	M1 A1 A1	3	one of these terms correct another term correct all correct (no + c etc) (penalise + c once only in question)
(b)	$18 + 6x - 12x^2 = 0$	M1		putting their $\frac{dy}{dx} = 0$, PI by attempt to solve or factorise
	$6\ (3-2x)(x+1) \quad (= 0)$	m1		attempt at factors of **their quadratic** or use of quadratic equation formula
	$x = -1,\ x = \frac{3}{2}$ OE	A1	3	must see both values unless $x = -1$ is verified separately If M1 not scored, award SC B1 for verifying that $x = -1$ leads to $\frac{dy}{dx} = 0$ and a further SC B2 for finding $x = \frac{3}{2}$ as other value
(c)(i)	$\frac{d^2y}{dx^2} = 6 - 24x$	B1√		FT their $\frac{dy}{dx}$ but $\frac{d^2y}{dx^2}$ must be correct if 3 marks earned in part (a)
	When $x = -1$, $\frac{d^2y}{dx^2} = 6 - (24 \times -1)$	M1		Sub $x = -1$ into 'their' $\frac{d^2y}{dx^2}$
(c)(i)	$\frac{d^2y}{dx^2} = 6 - 24x$	B1√		FT their $\frac{dy}{dx}$ but $\frac{d^2y}{dx^2}$ must be correct if 3 marks earned in part (a)
	When $x = -1$, $\frac{d^2y}{dx^2} = 6 - (24 \times -1)$	M1		Sub $x = -1$ into 'their' $\frac{d^2y}{dx^2}$
	$\frac{d^2y}{dx^2} = 30$	A1cso	3	
(ii)	Minimum point	E1√	1	must have a value in (c)(i) FT "maximum" if their value of $\frac{d^2y}{dx^2} < 0$
	Total		**10**	

14.June 2011

3(a)	$\left(\frac{dV}{dt}=\right)\frac{3t^2}{4}-3$	M1 A1	 2	one of these terms correct all correct (no + c etc)
(b)(i)	$t=1 \Rightarrow \frac{dV}{dt}=\frac{3}{4}-3$ $=-2\frac{1}{4}$	M1 A1cso	 2	substituting $t=1$ into their $\frac{dV}{dt}$ (–2.25 OE) *BUT* must have $\frac{dV}{dt}$ correct
(ii)	Volume is decreasing when $t=1$			must have used $\frac{dV}{dt}$ in (b)(i) or starts again
(c)(i)	$\left(\frac{dV}{dt}=0\Rightarrow\right)\frac{3t^2}{4}-3=0$ $\Rightarrow t^2=4$ $t=2$	M1 A1√ A1cso	 3	PI by "correct" equation being solved obtaining $t^n=k$ correctly from their $\frac{dV}{dt}$ withhold if answer left as $t=\pm 2$
(ii)	$\left(\frac{d^2V}{dt^2}=\right)\frac{3t}{2}$ When $t=2$, $\frac{d^2V}{dt^2}=3$ or $\frac{d^2V}{dt^2}>0$ $\Rightarrow$ minimum	B1√ M1 A1cso	 3	(condone unsimplified) ft their $\frac{dV}{dt}$ ft their $\frac{d^2V}{dt^2}$ and value of t from (c)(i)
	Total		**11**	

15.Jan 2012

4(a)(i)	$\left(\frac{dy}{dx}=\right)5x^4-6x+1$	M1 A1 A1	 3	one term correct another term correct all correct (no + c etc)
(ii)	$\left(\frac{d^2y}{dx^2}=\right)20x^3-6$	B1√	1	FT 'their' $\frac{dy}{dx}$
(b)	$x=-1 \Rightarrow \frac{dy}{dx}=5(-1)^4-6(-1)+1\ (=12)$ $\Rightarrow y=12(x+1)$	M1 A1cso	 2	must sub $x=-1$ into 'their' $\frac{dy}{dx}$ any correct form with $(x--1)$ simplified condone $y=12x+c$, $c=12$
(c)	$x=1 \Rightarrow \frac{dy}{dx}=5-6+1$ $\frac{dy}{dx}=0 \Rightarrow$ stationary point when $x=1$, $\frac{d^2y}{dx^2}=14$ $\Rightarrow$ (B is a) minimum (point)	M1 A1cso E1	 3	sub $x=1$ into their $\frac{dy}{dx}$ shown $=0$ plus correct statement or $\frac{d^2y}{dx^2}=20-6>0$ $\Rightarrow$ (B is a) minimum (point)
(d)(i)	$\frac{x^6}{6}-\frac{3x^3}{3}+\frac{x^2}{2}+5x$ $\left[\frac{1}{6}-1+\frac{1}{2}+5\right]-\left[\frac{1}{6}+1+\frac{1}{2}-5\right]$ $=8$	M1 A1 A1 m1 A1cso	 5	one term correct another term correct all correct (may have $+c$) 'their' F(1) – F(–1) with powers of 1 and –1 evaluated correctly
(ii)	'their answer to part (i)' – 2 $\Rightarrow$ Area = 6	M1 A1cso	 2	
	Total		**16**	

16.June 2012

4(a)(i)	$3x^2+3x^2+xy+xy+3xy+3xy$	M1		correct expression for surface area
	$6x^2+8xy=32$			$2(3x^2+xy+3xy)=32$ etc
	$\Rightarrow 3x^2+4xy=16$	A1	2	**AG** be convinced
(ii)	$(V=)3x^2y$ OE	M1		correct volume in terms of x and y
	$=3x\left(\frac{16-3x^2}{4}\right)$ **or** $=3x^2\left(\frac{16-3x^2}{4x}\right)$			OE
	$=12x-\frac{9x^3}{4}$	A1	2	CSO **AG** be convinced that all working is correct
(b)	$\left(\frac{dV}{dx}=\right)12-\frac{27}{4}x^2$	M1 A1	2	one of these terms correct all correct with 9×3 evaluated (no + c etc)
(c)(i)	$x=\frac{4}{3}\Rightarrow\frac{dV}{dx}=12-\frac{27}{4}\times\left(\frac{4}{3}\right)^2$	M1		attempt to sub $x=\frac{4}{3}$ into '*their*' $\frac{dV}{dx}$
	$\frac{dV}{dx}=12-\frac{27}{4}\times\frac{16}{9}=12-12$			or $12-\frac{432}{36}=12-12$ or $12-\frac{48}{4}=0$ etc
	$\frac{dV}{dx}=0\Rightarrow$ stationary value	A1	2	CSO; shown $=0$ plus statement
(ii)	$\frac{d^2V}{dx^2}=-\frac{27x}{2}$ OE	B1√		FT for '*their*' $\frac{dV}{dx}=a+bx^2$
	when $x=\frac{4}{3}$, $\frac{d^2V}{dx^2}<0\Rightarrow$ maximum	E1√	2	**or** sub of $x=\frac{4}{3}$ into '*their*' $\frac{d^2V}{dx^2}$ $\Rightarrow$ maximum E0 if numerical error seen
	$\left(\text{FT "}minimum\text{" if their } \frac{d^2V}{dx^2}>0\right)$			
	Total		**10**	

17.Jan 2013

2(a)	$\left(\frac{dy}{dt}=\right)\frac{4t^3}{8}-2t$	M1 A1	2	one of these terms correct all correct (no + c etc)
(b)(i)	$t=1\Rightarrow\frac{dy}{dt}=\frac{4}{8}-2$	M1		Correctly sub $t=1$ into their $\frac{dy}{dt}$
	$=-1\frac{1}{2}$	A1cso	2	must have $\frac{dy}{dt}$ correct (watch for t^3 etc)
(ii)	$\frac{dy}{dt}<0$			must have used $\frac{dy}{dt}$ in part (b)(i)
	$\Rightarrow$ (height is) **decreasing** (when $t=1$)	E1√	1	must state that "$\frac{dy}{dt}<0$" or "$-1.5<0$" or the equivalent in words FT their value of $\frac{dy}{dt}$ with appropriate explanation and conclusion
(c)(i)	$\left(\frac{d^2y}{dt^2}=\right)\frac{4}{8}\times 3t^2-2$	M1		Correctly differentiating their $\frac{dy}{dt}$ even if wrongly simplified
	$\left(t=2,\ \frac{d^2y}{dt^2}=\right)$ 4	A1cso	2	Both derivatives correct and simplified to 4
(ii)	$\Rightarrow$ minimum	E1√	1	FT their numerical value of $\frac{d^2y}{dt^2}$ from part (c) (i)
	Total		**8**	