Quadratic Powers Problem

Find all real solutions to the equation:

$$(x^2 - 7x + 11)^{x^2-11x+30} = 1$$

Quadratic Powers Problem

Find all real solutions to the equation:

$$(x^2 - 7x + 11)^{x^2-11x+30} = 1$$

Quadratic Powers Problem

Find all real solutions to the equation:

$$(x^2 - 7x + 11)^{x^2-11x+30} = 1$$

Quadratic Powers Problem

Find all real solutions to the equation:

$$(x^2 - 7x + 11)^{x^2-11x+30} = 1$$

Quadratic Powers Solution

Note that the right-hand-side is 1, and that there are a limited number of ways this can be accomplished:

1. The power is zero.
Since $a^0 = 1$ for any a, solutions can be found by solving $x^2 - 11x + 30 = 0$:

$$x^2 - 11x + 30 = 0 \quad \Rightarrow \quad (x-5)(x-6) = 0 \quad \Rightarrow \quad \boldsymbol{x = 5} \ \textit{or} \ \boldsymbol{x = 6}$$

2. The base is one.
Since $1^a = 1$ for any a, solutions can be found by solving $x^2 - 7x + 11 = 1$:

$$\Rightarrow \quad x^2 - 7x + 10 = 0 \quad \Rightarrow \quad (x-2)(x-5) = 0 \quad \Rightarrow \quad \boldsymbol{x = 2} \ \textit{or} \ \boldsymbol{x = 5}$$

3. The base is negative one and the power is even.
Since $(-1)^a = 1$ for even values of a, solutions to $x^2 - 7x + 11 = -1$ will be solutions provided they also satisfy $x^2 - 11x + 30 = 2n, n \in \mathbb{N}$ (that is, $x^2 - 11x + 30$ is even):

$$\Rightarrow \quad x^2 - 7x + 12 = 0 \quad \Rightarrow \quad (x-3)(x-4) = 0 \quad \Rightarrow \quad \boldsymbol{x = 3} \ \textit{or} \ \boldsymbol{x = 4}$$

Checking: $(3)^2 - 11(3) + 30 = 6 = 2(3)$ and $(4)^2 - 11(4) + 30 = 2 = 2(1)$

Both give even values, therefore the full list of solutions is:

$$\begin{array}{ccc} \boldsymbol{x = 2} & \Leftrightarrow & 1^{12} = 1 \\ \boldsymbol{x = 3} & \Leftrightarrow & (-1)^6 = 1 \\ \boldsymbol{x = 4} & \Leftrightarrow & (-1)^2 = 1 \\ \boldsymbol{x = 5} & \Leftrightarrow & 1^0 = 1 \\ \boldsymbol{x = 6} & \Leftrightarrow & 4^0 = 1 \end{array}$$