Pedal Power

The humble bicycle is an incredibly energy-efficient piece of machinery.

Used all over the world for moving people and cargo, they are the best solution we have for a self-powered means of transport.

1. Wikipedia claims that a cyclist with a power output of $60W$ (the same power as someone walking at $5\ kmph$ ($1.4\ ms^{-1}$)) can travel at around $15\ kmph$ ($4.2\ ms^{-1}$). Assuming that the cyclist is travelling along a straight, horizontal road, and that the resistance forces acting are proportional to its speed, find an expression for the total resistive force acting on the cyclist at a speed v.

2. It is estimated that an amateur cyclist can typically sustain $3W$ of power per kg. For example, a $70kg$ rider could output around $210W$ for an extended period of time. Calculate the speed of a $70kg$ rider making this power output.

3. A professional can sustain $6W$ per kg. What would the speed be for our $70kg$ rider in this case?

4. For brief periods, professional cyclists can increase their power output to $25W$ per kg. What would the top speed of a $70kg$ cyclist be while generating this much power?

5. In reality, resistance forces are much more accurately modelled as proportional to the *square* of the speed. Answer questions 1 to 4 again, using this refined model.

*6. Our amateur cyclist (who can output $210W$ of power and weighs $70kg$) is now cycling up a steep hill, inclined at $5°$ to the horizontal (this is a grade of around 10%). Construct a force diagram, and use it to find his maximum speed up the hill.

Pedal Power SOLUTIONS

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1. Wikipedia claims that a cyclist with a power output of $60W$ (the same power as someone walking at $5\ kmph$ ($1.4\ ms^{-1}$)) can travel at around $15\ kmph$ ($4.2\ ms^{-1}$). Assuming that the cyclist is travelling along a straight, horizontal road, and that the resistance forces acting are proportional to its speed, find an expression for the total resistive force acting on the cyclist at a speed v.

$$P = F_m v \implies 60 = 4.2F_m \implies F_m = 14.28 \ldots N \quad and \quad F_R = kv = 4.2k$$

$$a = 0 \implies F_R = F_m \implies 4.2k = \frac{60}{4.2} \implies k = \frac{60}{4.2^2} = 3.40\ to\ 3\ s.f.$$

$$\boldsymbol{F_R = 3.40v}$$

2. It is estimated that an amateur cyclist can typically sustain $3W$ of power per kg. For example, a $70kg$ rider could output around $210W$ for an extended period of time. Calculate the speed of a $70kg$ rider making this power output.

$$P = F_m v \implies 210 = F_m v \quad and \quad F_R = 3.4v$$

$$Max\ speed \implies \frac{210}{v} = 3.4v \implies 210 = 3.4v^2 \implies \boldsymbol{v = 7.86ms^{-1}\ to\ 3\ s.f.}$$

3. A professional can sustain $6W$ per kg. What would the speed be for our $70kg$ rider in this case?

$$\frac{420}{v} = 3.4v \implies v^2 = \frac{420}{3.4} \implies \boldsymbol{v = 11.11ms^{-1}\ to\ 3\ s.f.}$$

4. For brief periods, professional cyclists can increase their power output to $25W$ per kg. What would the top speed of a $70kg$ cyclist be while generating this much power?

$$P = 25 \times 70 = 1750 = F_m v \quad and \quad F_R = 3.4v$$

$$Max\ speed \implies \frac{1750}{v} = 3.4v \implies v^2 = \frac{1750}{3.4} \implies \boldsymbol{v = 22.7ms^{-1}\ to\ 3\ s.f.}$$

5. In reality, resistance forces are much more accurately modelled as proportional to the *square* of the speed. Answer questions 1 to 4 again, using this refined model.

1.

$$P = F_m v \implies 60 = 4.2F_m \implies F_m = 14.28 \ldots N \quad and \quad F_R = kv^2 = 4.2^2k$$

$$a = 0 \implies F_R = F_m \implies 4.2^2k = \frac{60}{4.2} \implies k = \frac{60}{4.2^3} = 0.810\ to\ 3\ s.f.$$

$$\boldsymbol{F_R = 0.810v^2}$$

2.

$$P = F_m v \implies 210 = F_m v \quad and \quad F_R = 0.81v^2$$

$$Max\ speed \implies \frac{210}{v} = 0.81v^2 \implies 210 = 0.81v^3 \implies \boldsymbol{v = 6.38ms^{-1}\ to\ 3\ s.f.}$$

3.

$$\frac{420}{v} = 0.81v^2 \implies v^3 = \frac{420}{0.81} \implies \boldsymbol{v = 8.03ms^{-1}\ to\ 3\ s.f.}$$

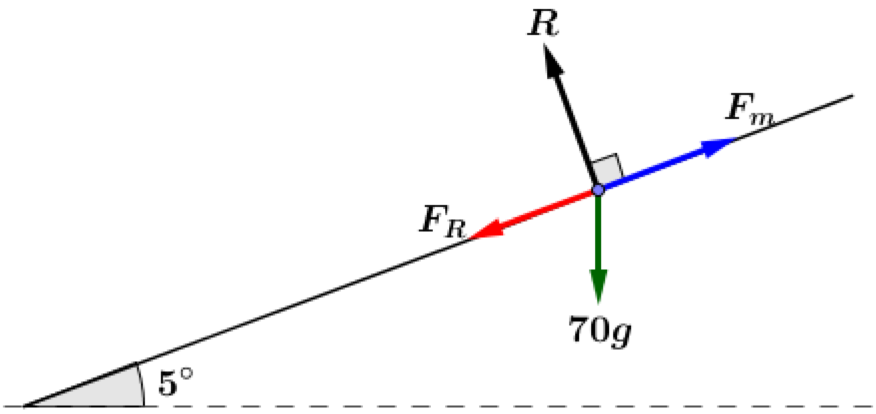
4.

$$P = 25 \times 70 = 1750 = F_m v \quad and \quad F_R = 0.81v^2$$

$$Max\ speed \implies \frac{1750}{v} = 0.81v^2 \implies v^3 = \frac{1750}{0.81} \implies \boldsymbol{v = 12.9ms^{-1}\ to\ 3\ s.f.}$$

Pedal Power SOLUTIONS continued…

*6. Our amateur cyclist (who can output $210W$ of power and weighs $70kg$) is now cycling up a steep hill, inclined at 5° to the horizontal (this is a grade of around 10%). Construct a force diagram, and use it to find his maximum speed up the hill.



$$P = F_m v \quad \Longrightarrow \quad F_m = \frac{210}{v}$$

$$F_R = 3.4v$$

$$a = 0 \quad \Longrightarrow \quad F = ma = 0 \quad \Longrightarrow \quad F_m - F_R - 70g\sin 5 = 0$$

$$\frac{210}{v} - 3.4v - 70g\sin 5 = 0$$

$$210 - 3.4v^2 - (70g\sin 5)v = 0$$

$$\Longrightarrow \quad v = 3.0004\ldots \quad or \quad v = -20.5854\ldots$$

$$v = -20.6\ ms^{-1}\ \ represents\ the\ maximum\ downslope\ speed$$

$$\boldsymbol{v = 3.00\ ms^{-1}\ to\ 3\ s.f.}$$

Note: changing the force diagram for downward motion results in a very similar quadratic:

$$210 - 3.4v^2 + (70g\sin 5)v = 0$$

Notice that equations of this form have related roots, because:

$$\frac{-(-b) \pm \sqrt{(-b)^2 - 4ac}}{2a} = -\left(\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right)$$

Which means that instead of 3 and -20.6, we would get -3 and 20.6 as our solutions. The algebraic solutions effectively allow for the possibility of a negative motive force and a negative resistive force, yielding a negative 'maximum' speed. Making these two forces negative effectively changes the direction of motion, so the results reflect a choice of 'positive', and hence are applicable, with an appropriate sign change, to motion either up or down the slope.