Over The Moon

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The Moon is currently up for grabs. There might be a few problems (broadband connection, scenic picnic spots), but there is enough space on the surface of the moon to hold around a billion people (based on the population density of the UK).

The population of the UK is currently around 60 million. 50 years ago it was around 50 million.

1. Assuming the UK colonizes the moon, and our population continues to grow at the same rate it has during the last half century, when would the total UK(+Moon) population reach a billion?

2. On the other hand, when America was first colonized, the population doubled roughly every 30 years. If this happened on the Moon, when would we reach the limit of 1 billion?

Over The Moon SOLUTIONS

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1. Assuming the UK colonizes the moon, and our population continues to grow at the same rate it has during the last half century, when would the total UK(+Moon) population reach a billion?

$$P = P_0 e^{kt} \quad \Rightarrow \quad 60 = 50e^{50k} \quad \Rightarrow \quad k = \frac{\ln 1.2}{50} \quad \Rightarrow \quad P = 50e^{\frac{\ln 1.2}{50}t}$$

$$1000 = 50e^{\frac{\ln 1.2}{50}t} \quad \Rightarrow \quad \ln 20 = \frac{\ln 1.2}{50}t \quad \Rightarrow \quad t = \frac{50 \ln 20}{\ln 1.2} = 822 \; years$$

$$822 \; years \; from \; 50 \; years \; before \; 2013 \Rightarrow \quad \boldsymbol{The \; year \; 2785}$$

2. On the other hand, when America was first colonized, the population doubled roughly every 30 years. If this happened on the Moon, when would we reach the limit of 1 billion?

$$P = P_0 e^{kt} \quad \Rightarrow \quad 2P_0 = P_0 e^{30k} \quad \Rightarrow \quad k = \frac{\ln 2}{30} \quad \Rightarrow \quad P = 50e^{\frac{\ln 2}{30}t}$$

$$1000 = 50e^{\frac{\ln 2}{30}t} \quad \Rightarrow \quad \ln 20 = \frac{\ln 2}{30}t \quad \Rightarrow \quad t = \frac{30 \ln 20}{\ln 2} = 130 \; years$$

$$130 \; years \; from \; 50 \; years \; before \; 2013 \Rightarrow \quad \boldsymbol{The \; year \; 2093}$$