Formulae and Statistical Tables for GCE Mathematics and GCE Statistics

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For the new specifications for first teaching from September 2004

GCE Mathematics

ADVANCED SUBSIDIARY MATHEMATICS (5361)
ADVANCED SUBSIDIARY PURE MATHEMATICS (5366)
ADVANCED SUBSIDIARY FURTHER MATHEMATICS (5371)

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General Certificate of Education

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Statistical Tables

PURE MATHEMATICS

Mensuration

Surface area of sphere $= 4\pi r^2$

Area of curved surface of cone $= \pi r \times$ slant height

Arithmetic series

$u_n = a + (n-1)d$

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n[2a + (n-1)d]$

Geometric series

$u_n = ar^{n-1}$

$S_n = \dfrac{a(1-r^n)}{1-r}$

$S_\infty = \dfrac{a}{1-r}$ for $|r| < 1$

Summations

$$\sum_{r=1}^{n} r = \tfrac{1}{2}n(n+1)$$

$$\sum_{r=1}^{n} r^2 = \tfrac{1}{6}n(n+1)(2n+1)$$

$$\sum_{r=1}^{n} r^3 = \tfrac{1}{4}n^2(n+1)^2$$

Trigonometry – the Cosine rule

$a^2 = b^2 + c^2 - 2bc\cos A$

Binomial Series

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$$

where $\binom{n}{r} = {}^n\mathrm{C}_r = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1.2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1.2\ldots r}x^r + \ldots \quad (|x| < 1, n \in \mathbb{R})$$

Logarithms and exponentials

$a^x = \mathrm{e}^{x\ln a}$

Complex numbers

$\{r(\cos\theta + \mathrm{i}\sin\theta)\}^n = r^n(\cos n\theta + \mathrm{i}\sin n\theta)$

$\mathrm{e}^{\mathrm{i}\theta} = \cos\theta + \mathrm{i}\sin\theta$

The roots of $z^n = 1$ are given by $z = \mathrm{e}^{\frac{2\pi k\mathrm{i}}{n}}$, for $k = 0, 1, 2, \ldots, n-1$

Maclaurin's series

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \ldots + \frac{x^r}{r!} f^{(r)}(0) + \ldots$$

$$e^x = \exp(x) = 1 + x + \frac{x^2}{2!} + \ldots + \frac{x^r}{r!} + \ldots \quad \text{for all } x$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots + (-1)^{r+1} \frac{x^r}{r} + \ldots \quad (-1 < x \leqslant 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + (-1)^r \frac{x^{2r+1}}{(2r+1)!} + \ldots \quad \text{for all } x$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots + (-1)^r \frac{x^{2r}}{(2r)!} + \ldots \quad \text{for all } x$$

Hyperbolic functions

$$\cosh^2 x - \sinh^2 x = 1$$

$$\sinh 2x = 2 \sinh x \cosh x$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

$$\cosh^{-1} x = \ln\left\{x + \sqrt{x^2 - 1}\right\} \quad (x \geqslant 1)$$

$$\sinh^{-1} x = \ln\left\{x + \sqrt{x^2 + 1}\right\}$$

$$\tanh^{-1} x = \tfrac{1}{2} \ln\left(\frac{1+x}{1-x}\right) \quad (|x| < 1)$$

Conics

	Ellipse	Parabola	Hyperbola	Rectangular hyperbola
Standard form	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$y^2 = 4ax$	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$xy = c^2$
Asymptotes	none	none	$\frac{x}{a} = \pm\frac{y}{b}$	$x = 0, y = 0$

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad \left(A \pm B \neq (k + \tfrac{1}{2})\pi\right)$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

Vectors

The resolved part of $\mathbf{a}$ in the direction of $\mathbf{b}$ is $\dfrac{\mathbf{a.b}}{|\mathbf{b}|}$

The position vector of the point dividing AB in the ratio $\lambda : \mu$ is $\dfrac{\mu\mathbf{a} + \lambda\mathbf{b}}{\lambda + \mu}$

Vector product: $\mathbf{a} \times \mathbf{b} = |\mathbf{a}||\mathbf{b}| \sin\theta\, \hat{\mathbf{n}} = \begin{vmatrix} \mathbf{i} & a_1 & b_1 \\ \mathbf{j} & a_2 & b_2 \\ \mathbf{k} & a_3 & b_3 \end{vmatrix} = \begin{bmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{bmatrix}$

If A is the point with position vector $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and the direction vector $\mathbf{b}$ is given by $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$, then the straight line through A with direction vector $\mathbf{b}$ has cartesian equation

$$\frac{x - a_1}{b_1} = \frac{y - a_2}{b_2} = \frac{z - a_3}{b_3} = \lambda$$

The plane through A with normal vector $\mathbf{n} = n_1\mathbf{i} + n_2\mathbf{j} + n_3\mathbf{k}$ has cartesian equation

$n_1x + n_2y + n_3z = d$ where $d = \mathbf{a.n}$

The plane through non-collinear points A, B and C has vector equation

$\mathbf{r} = \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a}) + \mu(\mathbf{c} - \mathbf{a}) = (1 - \lambda - \mu)\mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$

The plane through the point with position vector $\mathbf{a}$ and parallel to $\mathbf{b}$ and $\mathbf{c}$ has equation

$\mathbf{r} = \mathbf{a} + s\mathbf{b} + t\mathbf{c}$

Matrix transformations

Anticlockwise rotation through θ about O: $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

Reflection in the line $y = (\tan\theta)x$: $\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$

The matrices for rotations (in three dimensions) through an angle θ about one of the axes are

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$ for the x-axis

$\begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$ for the y-axis

$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ for the z-axis

Differentiation

$\mathbf{f}(x)$	$\mathbf{f}'(x)$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$-\frac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\frac{1}{1+x^2}$
$\tan kx$	$k\sec^2 kx$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\operatorname{sech}^2 x$
$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\cosh^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$
$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\frac{\mathrm{f}(x)}{\mathrm{g}(x)}$	$\frac{\mathrm{f}'(x)\,\mathrm{g}(x)-\mathrm{f}(x)\,\mathrm{g}'(x)}{(\mathrm{g}(x))^2}$

Integration

(+ constant; $a > 0$ where relevant)

$\mathbf{f}(x)$	$\int \mathbf{f}(x)\,\mathbf{d}x$
$\tan x$	$\ln\lvert\sec x\rvert$
$\cot x$	$\ln\lvert\sin x\rvert$
$\operatorname{cosec} x$	$-\ln\lvert\operatorname{cosec} x+\cot x\rvert = \ln\lvert\tan(\frac{1}{2}x)\rvert$
$\sec x$	$\ln\lvert\sec x+\tan x\rvert = \ln\lvert\tan(\frac{1}{2}x+\frac{1}{4}\pi)\rvert$
$\sec^2 kx$	$\frac{1}{k}\tan kx$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\ln\cosh x$

INTEGRATION FORMULAE CONTINUE OVER THE PAGE

$\frac{1}{\sqrt{a^2 - x^2}}$	$\sin^{-1}\left(\frac{x}{a}\right) \quad (\lvert x\rvert < a)$
$\frac{1}{a^2 + x^2}$	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$
$\frac{1}{\sqrt{x^2 - a^2}}$	$\cosh^{-1}\left(\frac{x}{a}\right)$ or $\ln\left\{x + \sqrt{x^2 - a^2}\right\} \quad (x > a)$
$\frac{1}{\sqrt{a^2 + x^2}}$	$\sinh^{-1}\left(\frac{x}{a}\right)$ or $\ln\left\{x + \sqrt{x^2 + a^2}\right\}$
$\frac{1}{a^2 - x^2}$	$\frac{1}{2a}\ln\left\lvert\frac{a + x}{a - x}\right\rvert = \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right) \quad (\lvert x\rvert < a)$
$\frac{1}{x^2 - a^2}$	$\frac{1}{2a}\ln\left\lvert\frac{x - a}{x + a}\right\rvert$

$$\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$$

Area of a sector

$A = \frac{1}{2}\int r^2\,\mathrm{d}\theta$ (polar coordinates)

Arc length

$s = \int\sqrt{1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\,\mathrm{d}x$ (cartesian coordinates)

$s = \int\sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2}\,\mathrm{d}t$ (parametric form)

Surface area of revolution

$S_x = 2\pi\int y\sqrt{1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\,\mathrm{d}x$ (cartesian coordinates)

$S_x = 2\pi\int y\sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2}\,\mathrm{d}t$ (parametric form)

Numerical integration

The trapezium rule: $\int_a^b y\,\mathrm{d}x \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \frac{b-a}{n}$

The mid-ordinate rule: $\int_a^b y\,\mathrm{d}x \approx h(y_{\frac{1}{2}} + y_{\frac{3}{2}} + \ldots + y_{n-\frac{3}{2}} + y_{n-\frac{1}{2}})$, where $h = \frac{b-a}{n}$

Simpson's rule: $\int_a^b y\,\mathrm{d}x \approx \frac{1}{3}h\left\{(y_0 + y_n) + 4(y_1 + y_3 + \ldots + y_{n-1}) + 2(y_2 + y_4 + \ldots + y_{n-2})\right\}$

where $h = \frac{b-a}{n}$ and n is even

Numerical solution of differential equations

For $\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x)$ and small h, recurrence relations are:

Euler's method: $y_{n+1} = y_n + h\,\mathrm{f}(x_n); \quad x_{n+1} = x_n + h$

For $\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x, y)$:

Euler's method: $y_{r+1} = y_r + h\,\mathrm{f}(x_r, y_r)$

Improved Euler method: $y_{r+1} = y_r + \frac{1}{2}(k_1 + k_2)$, where $k_1 = h\,\mathrm{f}(x_r, y_r), k_2 = h\,\mathrm{f}(x_r + h, y_r + k_1)$

Numerical solution of equations

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \frac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

MECHANICS

Motion in a circle

Transverse velocity: $v = r\dot{\theta}$

Transverse acceleration: $\dot{v} = r\ddot{\theta}$

Radial acceleration: $-r\dot{\theta}^2 = -\frac{v^2}{r}$

Centres of mass

For uniform bodies

Triangular lamina: $\frac{2}{3}$ along median from vertex

Solid hemisphere, radius r: $\frac{3}{8}r$ from centre

Hemispherical shell, radius r: $\frac{1}{2}r$ from centre

Circular arc, radius r, angle at centre 2α: $\frac{r\sin\alpha}{\alpha}$ from centre

Sector of circle, radius r, angle at centre 2α: $\frac{2r\sin\alpha}{3\alpha}$ from centre

Solid cone or pyramid of height h: $\frac{1}{4}h$ above the base on the line from centre of base to vertex

Conical shell of height h: $\frac{1}{3}h$ above the base on the line from centre of base to vertex

Moments of inertia

For uniform bodies of mass m

Thin rod, length $2l$, about perpendicular axis through centre: $\frac{1}{3}ml^2$

Rectangular lamina about axis in plane bisecting edges of length $2l$: $\frac{1}{3}ml^2$

Thin rod, length $2l$, about perpendicular axis through end: $\frac{4}{3}ml^2$

Rectangular lamina about edge perpendicular to edges of length $2l$: $\frac{4}{3}ml^2$

Rectangular lamina, sides $2a$ and $2b$, about perpendicular axis through centre: $\frac{1}{3}m(a^2 + b^2)$

MOMENTS OF INERTIA FORMULAE CONTINUE OVER THE PAGE

Hoop or cylindrical shell of radius r about axis: mr^2

Hoop of radius r about a diameter: $\frac{1}{2}mr^2$

Disc or solid cylinder of radius r about axis: $\frac{1}{2}mr^2$

Disc of radius r about a diameter: $\frac{1}{4}mr^2$

Solid sphere, radius r, about diameter: $\frac{2}{5}mr^2$

Spherical shell of radius r about a diameter: $\frac{2}{3}mr^2$

Parallel axes theorem: $I_A = I_G + m(AG)^2$

Perpendicular axes theorem: $I_z = I_x + I_y$ (for a lamina in the x-y plane)

General motion in two dimensions

Radial velocity $\dot{r}$

Transverse velocity $r\dot{\theta}$

Radial acceleration $\ddot{r} - r\dot{\theta}^2$

Transverse acceleration $r\ddot{\theta} + 2\dot{r}\dot{\theta} = \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}t}(r^2\dot{\theta})$

Moments as vectors

The moment about O of **F** acting through the point with position vector **r** is $\mathbf{r}\times\mathbf{F}$

Universal law of gravitation

$$\text{Force} = \frac{Gm_1m_2}{d^2}$$

PROBABILITY and STATISTICS

Probability

$$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$$

$$\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B \mid A)$$

$$\mathrm{P}\left(A_j \middle| B\right) = \frac{\mathrm{P}\left(A_j\right) \times \mathrm{P}\left(B \middle| A_j\right)}{\sum_{i=1}^{n} \mathrm{P}\left(A_i\right) \times \mathrm{P}\left(B \middle| A_i\right)}$$

Expectation algebra

Covariance: $\mathrm{Cov}(X, Y) = \mathrm{E}((X - \mu_X)(Y - \mu_Y)) = \mathrm{E}(XY) - \mu_X\mu_Y$

$$\mathrm{Var}(aX \pm bY) = a^2\,\mathrm{Var}(X) + b^2\,\mathrm{Var}(Y) \pm 2ab\,\mathrm{Cov}(X, Y)$$

Product moment correlation coefficient: $\rho = \frac{\mathrm{Cov}(X, Y)}{\sigma_X\sigma_Y}$

For independent random variables X and Y

$$\mathrm{E}(XY) = \mathrm{E}(X)\,\mathrm{E}(Y)$$
$$\mathrm{Var}(aX \pm bY) = a^2\,\mathrm{Var}(X) + b^2\,\mathrm{Var}(Y)$$

Discrete distributions

For a discrete random variable X taking values x_i with probabilities p_i

Expectation (mean): $\mathrm{E}(X) = \mu = \sum x_i p_i$

Variance: $\mathrm{Var}(X) = \sigma^2 = \sum (x_i - \mu)^2 p_i = \sum x_i^2 p_i - \mu^2 = \mathrm{E}(X^2) - \mu^2$

For a function $\mathrm{g}(X)$: $\mathrm{E}(\mathrm{g}(X)) = \sum \mathrm{g}(x_i) p_i$

Standard discrete distributions:

Distribution of X	$\mathrm{P}(X = x)$	Mean	Variance
Binomial $\mathrm{B}(n, p)$	$\binom{n}{x} p^x (1-p)^{n-x}$	np	$np(1-p)$
Poisson $\mathrm{Po}(\lambda)$	$\mathrm{e}^{-\lambda} \dfrac{\lambda^x}{x!}$	λ	λ
Geometric $\mathrm{Geo}(p)$ on 1, 2, …	$p(1-p)^{x-1}$	$\dfrac{1}{p}$	$\dfrac{1-p}{p^2}$

Continuous distributions

For a continuous random variable X having probability density function $\mathrm{f}(x)$

Expectation (mean): $\mathrm{E}(X) = \mu = \int x\, \mathrm{f}(x)\, \mathrm{d}x$

Variance: $\mathrm{Var}(X) = \sigma^2 = \int (x-\mu)^2\, \mathrm{f}(x)\, \mathrm{d}x = \int x^2\, \mathrm{f}(x)\, \mathrm{d}x - \mu^2 = \mathrm{E}(X^2) - \mu^2$

For a function $\mathrm{g}(X)$: $\mathrm{E}(\mathrm{g}(X)) = \int \mathrm{g}(x)\, \mathrm{f}(x)\, \mathrm{d}x$

Cumulative distribution function: $\mathrm{F}(x) = \mathrm{P}(X \leqslant x) = \int_{-\infty}^{x} \mathrm{f}(t)\, \mathrm{d}t$

Standard continuous distributions:

Distribution of X	Probability density function	Mean	Variance
Uniform (Rectangular) on $[a, b]$	$\dfrac{1}{b-a}$	$\frac{1}{2}(a+b)$	$\frac{1}{12}(b-a)^2$
Normal $\mathrm{N}(\mu, \sigma^2)$	$\dfrac{1}{\sigma\sqrt{2\pi}} \mathrm{e}^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$	μ	σ^2
Exponential	$\lambda \mathrm{e}^{-\lambda x}$	$\dfrac{1}{\lambda}$	$\dfrac{1}{\lambda^2}$

Sampling distributions

For a random sample $X_1, X_2, \ldots, X_n$ of n independent observations from a distribution having mean μ and variance σ^2

$\overline{X}$ is an unbiased estimator of μ, with $\text{Var}(\overline{X}) = \dfrac{\sigma^2}{n}$

S^2 is an unbiased estimator of σ^2, where $S^2 = \dfrac{\sum (X_i - \overline{X})^2}{n-1}$

For a random sample of n observations from $\text{N}(\mu, \sigma^2)$

$$\frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \sim \text{N}(0,1)$$

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$$

$$\frac{\overline{X} - \mu}{\frac{S}{\sqrt{n}}} \sim t_{n-1}$$ (also valid in matched-pairs situations)

If X is the observed number of successes in n independent Bernoulli trials in each of which the probability of success is p, and $Y = \dfrac{X}{n}$, then

$\text{E}(Y) = p$ and $\text{Var}(Y) = \dfrac{p(1-p)}{n}$

For a random sample of n_x observations from $\text{N}(\mu_x, \sigma_x^2)$ and, independently, a random sample of n_y observations from $\text{N}(\mu_y, \sigma_y^2)$

$$\frac{(\overline{X} - \overline{Y}) - (\mu_x - \mu_y)}{\sqrt{\frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}}} \sim \text{N}(0,1)$$

$$\frac{S_x^2 / \sigma_x^2}{S_y^2 / \sigma_y^2} \sim F_{n_x - 1, n_y - 1}$$

If $\sigma_x^2 = \sigma_y^2 = \sigma^2$ (unknown),

then $\dfrac{(\overline{X} - \overline{Y}) - (\mu_x - \mu_y)}{\sqrt{S_p^2\left(\frac{1}{n_x} + \frac{1}{n_y}\right)}} \sim t_{n_x + n_y - 2}$ where $S_p^2 = \dfrac{(n_x - 1)S_x^2 + (n_y - 1)S_y^2}{n_x + n_y - 2}$

Correlation and regression

For a set of n pairs of values (x_i, y_i)

$$S_{xx} = \sum (x_i - \bar{x})^2 = \sum x_i^2 - \frac{\left(\sum x_i\right)^2}{n}$$

$$S_{yy} = \sum (y_i - \bar{y})^2 = \sum y_i^2 - \frac{\left(\sum y_i\right)^2}{n}$$

$$S_{xy} = \sum (x_i - \bar{x})(y_i - \bar{y}) = \sum x_i y_i - \frac{\left(\sum x_i\right)\left(\sum y_i\right)}{n}$$

The product moment correlation coefficient is

$$r = \frac{S_{xy}}{\sqrt{S_{xx} \times S_{yy}}} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\left\{\sum (x_i - \bar{x})^2\right\}\left\{\sum (y_i - \bar{y})^2\right\}}} = \frac{\sum x_i y_i - \frac{\left(\sum x_i\right)\left(\sum y_i\right)}{n}}{\sqrt{\left(\sum x_i^2 - \frac{\left(\sum x_i\right)^2}{n}\right)\left(\sum y_i^2 - \frac{\left(\sum y_i\right)^2}{n}\right)}}$$

Spearman's rank correlation coefficient is the product moment correlation coefficient between ranks

When there are no tied ranks it may be calculated using $r_s = 1 - \frac{6\sum d_i^{\,2}}{n(n^2 - 1)}$

The regression coefficient of y on x is $b = \frac{S_{xy}}{S_{xx}} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2}$

Least squares regression line of y on x is $y = a + bx$, where $a = \bar{y} - b\bar{x}$

Analysis of variance

One-factor model: $x_{ij} = \mu + \alpha_i + \varepsilon_{ij}$, where $\varepsilon_{ij} \sim \mathrm{N}(0, \sigma^2)$

Total sum of squares $SS_T = \sum_i \sum_j x_{ij}^2 - \frac{T^2}{n}$

Between groups sum of squares $SS_B = \sum_i \frac{T_i^2}{n_i} - \frac{T^2}{n}$

Two-factor model (with m rows and n columns): $x_{ij} = \mu + \alpha_i + \beta_j + \varepsilon_{ij}$, where $\varepsilon_{ij} \sim \mathrm{N}(0, \sigma^2)$

Total sum of squares, $SS_T = \sum_i \sum_j x_{ij}^2 - \frac{T^2}{mn}$

Between rows sum of squares, $SS_R = \sum_i \frac{R_i^2}{n} - \frac{T^2}{mn}$

Between columns sum of squares, $SS_C = \sum_j \frac{C_j^2}{m} - \frac{T^2}{mn}$

Distribution-free (non-parametric) tests

Goodness-of-fit tests and contingency tables: $\sum \frac{(O_i - E_i)^2}{E_i}$ is approximately distributed as χ^2

Wilcoxon signed rank test

T is the sum of the ranks of observations with the same sign

Mann-Whitney test

$U = T - \frac{n(n+1)}{2}$ where T is the sum of the ranks of the sample of size n

Kruskal-Wallis test

$$H = \frac{12}{N(N+1)} \sum_i \frac{T_i^2}{n_i} - 3(N+1)$$

where T_i is the sum of the ranks of a sample of size n_i and $N = \sum_i n_i$

H is approximately distributed as χ^2 with $k-1$ degrees of freedom
where k is the number of samples

TABLE 1 CUMULATIVE BINOMIAL DISTRIBUTION FUNCTION

The tabulated value is $P(X \leqslant x)$, where X has a binomial distribution with parameters n and p.

p	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**	**0.10**	**0.15**	**0.20**	**0.25**	**0.30**	**0.35**	**0.40**	**0.45**	**0.50**	p
x	$n=2$																		x
0	0.9801	0.9604	0.9409	0.9216	0.9025	0.8836	0.8649	0.8464	0.8281	0.8100	0.7225	0.6400	0.5625	0.4900	0.4225	0.3600	0.3025	0.2500	**0**
1	0.9999	0.9996	0.9991	0.9984	0.9975	0.9964	0.9951	0.9936	0.9919	0.9900	0.9775	0.9600	0.9375	0.9100	0.8775	0.8400	0.7975	0.7500	**1**
2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	**2**
x	$n=3$																		x
0	0.9703	0.9412	0.9127	0.8847	0.8574	0.8306	0.8044	0.7787	0.7536	0.7290	0.6141	0.5120	0.4219	0.3430	0.2746	0.2160	0.1664	0.1250	**0**
1	0.9997	0.9988	0.9974	0.9953	0.9928	0.9896	0.9860	0.9818	0.9772	0.9720	0.9393	0.8960	0.8438	0.7840	0.7183	0.6480	0.5748	0.5000	**1**
2	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998	0.9997	0.9995	0.9993	0.9990	0.9966	0.9920	0.9844	0.9730	0.9571	0.9360	0.9089	0.8750	**2**
3				1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	**3**
x	$n=4$																		x
0	0.9606	0.9224	0.8853	0.8493	0.8145	0.7807	0.7481	0.7164	0.6857	0.6561	0.5220	0.4096	0.3164	0.2401	0.1785	0.1296	0.0915	0.0625	**0**
1	0.9994	0.9977	0.9948	0.9909	0.9860	0.9801	0.9733	0.9656	0.9570	0.9477	0.8905	0.8192	0.7383	0.6517	0.5630	0.4752	0.3910	0.3125	**1**
2	1.0000	1.0000	0.9999	0.9998	0.9995	0.9992	0.9987	0.9981	0.9973	0.9963	0.9880	0.9728	0.9492	0.9163	0.8735	0.8208	0.7585	0.6875	**2**
3			1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9995	0.9984	0.9961	0.9919	0.9850	0.9744	0.9590	0.9375	**3**
4									1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	**4**
x	$n=5$																		x
0	0.9510	0.9039	0.8587	0.8154	0.7738	0.7339	0.6957	0.6591	0.6240	0.5905	0.4437	0.3277	0.2373	0.1681	0.1160	0.0778	0.0503	0.0313	**0**
1	0.9990	0.9962	0.9915	0.9852	0.9774	0.9681	0.9575	0.9456	0.9326	0.9185	0.8352	0.7373	0.6328	0.5282	0.4284	0.3370	0.2562	0.1875	**1**
2	1.0000	0.9999	0.9997	0.9994	0.9988	0.9980	0.9969	0.9955	0.9937	0.9914	0.9734	0.9421	0.8965	0.8369	0.7648	0.6826	0.5931	0.5000	**2**
3		1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998	0.9997	0.9995	0.9978	0.9933	0.9844	0.9692	0.9460	0.9130	0.8688	0.8125	**3**
4						1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9990	0.9976	0.9947	0.9898	0.9815	0.9688	**4**
5											1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	**5**
x	$n=6$																		x
0	0.9415	0.8858	0.8330	0.7828	0.7351	0.6899	0.6470	0.6064	0.5679	0.5314	0.3771	0.2621	0.1780	0.1176	0.0754	0.0467	0.0277	0.0156	**0**
1	0.9985	0.9943	0.9875	0.9784	0.9672	0.9541	0.9392	0.9227	0.9048	0.8857	0.7765	0.6554	0.5339	0.4202	0.3191	0.2333	0.1636	0.1094	**1**
2	1.0000	0.9998	0.9995	0.9988	0.9978	0.9962	0.9942	0.9915	0.9882	0.9842	0.9527	0.9011	0.8306	0.7443	0.6471	0.5443	0.4415	0.3438	**2**
3		1.0000	1.0000	1.0000	0.9999	0.9998	0.9997	0.9995	0.9992	0.9987	0.9941	0.9830	0.9624	0.9295	0.8826	0.8208	0.7447	0.6563	**3**
4					1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9984	0.9954	0.9891	0.9777	0.9590	0.9308	0.8906	**4**
5										1.0000	1.0000	0.9999	0.9998	0.9993	0.9982	0.9959	0.9917	0.9844	**5**
6												1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	**6**
x	$n=7$																		x
0	0.9321	0.8681	0.8080	0.7514	0.6983	0.6485	0.6017	0.5578	0.5168	0.4783	0.3206	0.2097	0.1335	0.0824	0.0490	0.0280	0.0152	0.0078	**0**
1	0.9980	0.9921	0.9829	0.9706	0.9556	0.9382	0.9187	0.8974	0.8745	0.8503	0.7166	0.5767	0.4449	0.3294	0.2338	0.1586	0.1024	0.0625	**1**
2	1.0000	0.9997	0.9991	0.9980	0.9962	0.9937	0.9903	0.9860	0.9807	0.9743	0.9262	0.8520	0.7564	0.6471	0.5323	0.4199	0.3164	0.2266	**2**
3		1.0000	1.0000	0.9999	0.9998	0.9996	0.9993	0.9988	0.9982	0.9973	0.9879	0.9667	0.9294	0.8740	0.8002	0.7102	0.6083	0.5000	**3**
4				1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998	0.9988	0.9953	0.9871	0.9712	0.9444	0.9037	0.8471	0.7734	**4**
5								1.0000	1.0000	1.0000	0.9999	0.9996	0.9987	0.9962	0.9910	0.9812	0.9643	0.9375	**5**
6											1.0000	1.0000	0.9999	0.9998	0.9994	0.9984	0.9963	0.9922	**6**
7													1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	**7**
x	$n=8$																		x
0	0.9227	0.8508	0.7837	0.7214	0.6634	0.6096	0.5596	0.5132	0.4703	0.4305	0.2725	0.1678	0.1001	0.0576	0.0319	0.0168	0.0084	0.0039	**0**
1	0.9973	0.9897	0.9777	0.9619	0.9428	0.9208	0.8965	0.8702	0.8423	0.8131	0.6572	0.5033	0.3671	0.2553	0.1691	0.1064	0.0632	0.0352	**1**
2	0.9999	0.9996	0.9987	0.9969	0.9942	0.9904	0.9853	0.9789	0.9711	0.9619	0.8948	0.7969	0.6785	0.5518	0.4278	0.3154	0.2201	0.1445	**2**
3	1.0000	1.0000	0.9999	0.9998	0.9996	0.9993	0.9987	0.9978	0.9966	0.9950	0.9786	0.9437	0.8862	0.8059	0.7064	0.5941	0.4770	0.3633	**3**
4			1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9997	0.9996	0.9971	0.9896	0.9727	0.9420	0.8939	0.8263	0.7396	0.6367	**4**
5							1.0000	1.0000	1.0000	1.0000	0.9998	0.9988	0.9958	0.9887	0.9747	0.9502	0.9115	0.8555	**5**
6											1.0000	0.9999	0.9996	0.9987	0.9964	0.9915	0.9819	0.9648	**6**
7												1.0000	1.0000	0.9999	0.9998	0.9993	0.9983	0.9961	**7**
8														1.0000	1.0000	1.0000	1.0000	1.0000	**8**

p	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	p
x	n=9																		x
0	0.9135	0.8337	0.7602	0.6925	0.6302	0.5730	0.5204	0.4722	0.4279	0.3874	0.2316	0.1342	0.0751	0.0404	0.0207	0.0101	0.0046	0.0020	**0**
1	0.9966	0.9869	0.9718	0.9522	0.9288	0.9022	0.8729	0.8417	0.8088	0.7748	0.5995	0.4362	0.3003	0.1960	0.1211	0.0705	0.0385	0.0195	**1**
2	0.9999	0.9994	0.9980	0.9955	0.9916	0.9862	0.9791	0.9702	0.9595	0.9470	0.8591	0.7382	0.6007	0.4628	0.3373	0.2318	0.1495	0.0898	**2**
3	1.0000	1.0000	0.9999	0.9997	0.9994	0.9987	0.9977	0.9963	0.9943	0.9917	0.9661	0.9144	0.8343	0.7297	0.6089	0.4826	0.3614	0.2539	**3**
4			1.0000	1.0000	1.0000	0.9999	0.9998	0.9997	0.9995	0.9991	0.9944	0.9804	0.9511	0.9012	0.8283	0.7334	0.6214	0.5000	**4**
5						1.0000	1.0000	1.0000	1.0000	0.9999	0.9994	0.9969	0.9900	0.9747	0.9464	0.9006	0.8342	0.7461	**5**
6										1.0000	1.0000	0.9997	0.9987	0.9957	0.9888	0.9750	0.9502	0.9102	**6**
7												1.0000	0.9999	0.9996	0.9986	0.9962	0.9909	0.9805	**7**
8													1.0000	1.0000	0.9999	0.9997	0.9992	0.9980	**8**
9															1.0000	1.0000	1.0000	1.0000	**9**
x	n=10																		x
0	0.9044	0.8171	0.7374	0.6648	0.5987	0.5386	0.4840	0.4344	0.3894	0.3487	0.1969	0.1074	0.0563	0.0282	0.0135	0.0060	0.0025	0.0010	**0**
1	0.9957	0.9838	0.9655	0.9418	0.9139	0.8824	0.8483	0.8121	0.7746	0.7361	0.5443	0.3758	0.2440	0.1493	0.0860	0.0464	0.0233	0.0107	**1**
2	0.9999	0.9991	0.9972	0.9938	0.9885	0.9812	0.9717	0.9599	0.9460	0.9298	0.8202	0.6778	0.5256	0.3828	0.2616	0.1673	0.0996	0.0547	**2**
3	1.0000	1.0000	0.9999	0.9996	0.9990	0.9980	0.9964	0.9942	0.9912	0.9872	0.9500	0.8791	0.7759	0.6496	0.5138	0.3823	0.2660	0.1719	**3**
4			1.0000	1.0000	0.9999	0.9998	0.9997	0.9994	0.9990	0.9984	0.9901	0.9672	0.9219	0.8497	0.7515	0.6331	0.5044	0.3770	**4**
5					1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9986	0.9936	0.9803	0.9527	0.9051	0.8338	0.7384	0.6230	**5**
6									1.0000	1.0000	0.9999	0.9991	0.9965	0.9894	0.9740	0.9452	0.8980	0.8281	**6**
7											1.0000	0.9999	0.9996	0.9984	0.9952	0.9877	0.9726	0.9453	**7**
8												1.0000	1.0000	0.9999	0.9995	0.9983	0.9955	0.9893	**8**
9														1.0000	1.0000	0.9999	0.9997	0.9990	**9**
10																1.0000	1.0000	1.0000	**10**
x	n=11																		x
0	0.8953	0.8007	0.7153	0.6382	0.5688	0.5063	0.4501	0.3996	0.3544	0.3138	0.1673	0.0859	0.0422	0.0198	0.0088	0.0036	0.0014	0.0005	**0**
1	0.9948	0.9805	0.9587	0.9308	0.8981	0.8618	0.8228	0.7819	0.7399	0.6974	0.4922	0.3221	0.1971	0.1130	0.0606	0.0302	0.0139	0.0059	**1**
2	0.9998	0.9988	0.9963	0.9917	0.9848	0.9752	0.9630	0.9481	0.9305	0.9104	0.7788	0.6174	0.4552	0.3127	0.2001	0.1189	0.0652	0.0327	**2**
3	1.0000	1.0000	0.9998	0.9993	0.9984	0.9970	0.9947	0.9915	0.9871	0.9815	0.9306	0.8389	0.7133	0.5696	0.4256	0.2963	0.1911	0.1133	**3**
4			1.0000	1.0000	0.9999	0.9997	0.9995	0.9990	0.9983	0.9972	0.9841	0.9496	0.8854	0.7897	0.6683	0.5328	0.3971	0.2744	**4**
5					1.0000	1.0000	1.0000	0.9999	0.9998	0.9997	0.9973	0.9883	0.9657	0.9218	0.8513	0.7535	0.6331	0.5000	**5**
6								1.0000	1.0000	1.0000	0.9997	0.9980	0.9924	0.9784	0.9499	0.9006	0.8262	0.7256	**6**
7											1.0000	0.9998	0.9988	0.9957	0.9878	0.9707	0.9390	0.8867	**7**
8												1.0000	0.9999	0.9994	0.9980	0.9941	0.9852	0.9673	**8**
9													1.0000	1.0000	0.9998	0.9993	0.9978	0.9941	**9**
10															1.0000	1.0000	0.9998	0.9995	**10**
11																		1.0000	**11**
x	n=12																		x
0	0.8864	0.7847	0.6938	0.6127	0.5404	0.4759	0.4186	0.3677	0.3225	0.2824	0.1422	0.0687	0.0317	0.0138	0.0057	0.0022	0.0008	0.0002	**0**
1	0.9938	0.9769	0.9514	0.9191	0.8816	0.8405	0.7967	0.7513	0.7052	0.6590	0.4435	0.2749	0.1584	0.0850	0.0424	0.0196	0.0083	0.0032	**1**
2	0.9998	0.9985	0.9952	0.9893	0.9804	0.9684	0.9532	0.9348	0.9134	0.8891	0.7358	0.5583	0.3907	0.2528	0.1513	0.0834	0.0421	0.0193	**2**
3	1.0000	0.9999	0.9997	0.9990	0.9978	0.9957	0.9925	0.9880	0.9820	0.9744	0.9078	0.7946	0.6488	0.4925	0.3467	0.2253	0.1345	0.0730	**3**
4		1.0000	1.0000	0.9999	0.9998	0.9996	0.9991	0.9984	0.9973	0.9957	0.9761	0.9274	0.8424	0.7237	0.5833	0.4382	0.3044	0.1938	**4**
5				1.0000	1.0000	1.0000	0.9999	0.9998	0.9997	0.9995	0.9954	0.9806	0.9456	0.8822	0.7873	0.6652	0.5269	0.3872	**5**
6							1.0000	1.0000	1.0000	0.9999	0.9993	0.9961	0.9857	0.9614	0.9154	0.8418	0.7393	0.6128	**6**
7										1.0000	0.9999	0.9994	0.9972	0.9905	0.9745	0.9427	0.8883	0.8062	**7**
8											1.0000	0.9999	0.9996	0.9983	0.9944	0.9847	0.9644	0.9270	**8**
9												1.0000	1.0000	0.9998	0.9992	0.9972	0.9921	0.9807	**9**
10														1.0000	0.9999	0.9997	0.9989	0.9968	**10**
11															1.0000	1.0000	0.9999	0.9998	**11**
12																	1.0000	1.0000	**12**
x	n=13																		x
0	0.8775	0.7690	0.6730	0.5882	0.5133	0.4474	0.3893	0.3383	0.2935	0.2542	0.1209	0.0550	0.0238	0.0097	0.0037	0.0013	0.0004	0.0001	**0**
1	0.9928	0.9730	0.9436	0.9068	0.8646	0.8186	0.7702	0.7206	0.6707	0.6213	0.3983	0.2336	0.1267	0.0637	0.0296	0.0126	0.0049	0.0017	**1**
2	0.9997	0.9980	0.9938	0.9865	0.9755	0.9608	0.9422	0.9201	0.8946	0.8661	0.6920	0.5017	0.3326	0.2025	0.1132	0.0579	0.0269	0.0112	**2**
3	1.0000	0.9999	0.9995	0.9986	0.9969	0.9940	0.9897	0.9837	0.9758	0.9658	0.8820	0.7473	0.5843	0.4206	0.2783	0.1686	0.0929	0.0461	**3**
4		1.0000	1.0000	0.9999	0.9997	0.9993	0.9987	0.9976	0.9959	0.9935	0.9658	0.9009	0.7940	0.6543	0.5005	0.3530	0.2279	0.1334	**4**
5				1.0000	1.0000	0.9999	0.9999	0.9997	0.9995	0.9991	0.9925	0.9700	0.9198	0.8346	0.7159	0.5744	0.4268	0.2905	**5**
6						1.0000	1.0000	1.0000	0.9999	0.9999	0.9987	0.9930	0.9757	0.9376	0.8705	0.7712	0.6437	0.5000	**6**
7									1.0000	1.0000	0.9998	0.9988	0.9944	0.9818	0.9538	0.9023	0.8212	0.7095	**7**
8											1.0000	0.9998	0.9990	0.9960	0.9874	0.9679	0.9302	0.8666	**8**
9												1.0000	0.9999	0.9993	0.9975	0.9922	0.9797	0.9539	**9**
10													1.0000	0.9999	0.9997	0.9987	0.9959	0.9888	**10**
11														1.0000	1.0000	0.9999	0.9995	0.9983	**11**
12																1.0000	1.0000	0.9999	**12**
13																		1.0000	**13**

p	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	p
x	n=14																		x
0	0.8687	0.7536	0.6528	0.5647	0.4877	0.4205	0.3620	0.3112	0.2670	0.2288	0.1028	0.0440	0.0178	0.0068	0.0024	0.0008	0.0002	0.0001	**0**
1	0.9916	0.9690	0.9355	0.8941	0.8470	0.7963	0.7436	0.6900	0.6368	0.5846	0.3567	0.1979	0.1010	0.0475	0.0205	0.0081	0.0029	0.0009	**1**
2	0.9997	0.9975	0.9923	0.9833	0.9699	0.9522	0.9302	0.9042	0.8745	0.8416	0.6479	0.4481	0.2811	0.1608	0.0839	0.0398	0.0170	0.0065	**2**
3	1.0000	0.9999	0.9994	0.9981	0.9958	0.9920	0.9864	0.9786	0.9685	0.9559	0.8535	0.6982	0.5213	0.3552	0.2205	0.1243	0.0632	0.0287	**3**
4		1.0000	1.0000	0.9998	0.9996	0.9990	0.9980	0.9965	0.9941	0.9908	0.9533	0.8702	0.7415	0.5842	0.4227	0.2793	0.1672	0.0898	**4**
5				1.0000	1.0000	0.9999	0.9998	0.9996	0.9992	0.9985	0.9885	0.9561	0.8883	0.7805	0.6405	0.4859	0.3373	0.2120	**5**
6						1.0000	1.0000	1.0000	0.9999	0.9998	0.9978	0.9884	0.9617	0.9067	0.8164	0.6925	0.5461	0.3953	**6**
7									1.0000	1.0000	0.9997	0.9976	0.9897	0.9685	0.9247	0.8499	0.7414	0.6047	**7**
8											1.0000	0.9996	0.9978	0.9917	0.9757	0.9417	0.8811	0.7880	**8**
9												1.0000	0.9997	0.9983	0.9940	0.9825	0.9574	0.9102	**9**
10													1.0000	0.9998	0.9989	0.9961	0.9886	0.9713	**10**
11														1.0000	0.9999	0.9994	0.9978	0.9935	**11**
12															1.0000	0.9999	0.9997	0.9991	**12**
13																1.0000	1.0000	0.9999	**13**
14																		1.0000	**14**
x	n=15																		x
0	0.8601	0.7386	0.6333	0.5421	0.4633	0.3953	0.3367	0.2863	0.2430	0.2059	0.0874	0.0352	0.0134	0.0047	0.0016	0.0005	0.0001	0.0000	**0**
1	0.9904	0.9647	0.9270	0.8809	0.8290	0.7738	0.7168	0.6597	0.6035	0.5490	0.3186	0.1671	0.0802	0.0353	0.0142	0.0052	0.0017	0.0005	**1**
2	0.9996	0.9970	0.9906	0.9797	0.9638	0.9429	0.9171	0.8870	0.8531	0.8159	0.6042	0.3980	0.2361	0.1268	0.0617	0.0271	0.0107	0.0037	**2**
3	1.0000	0.9998	0.9992	0.9976	0.9945	0.9896	0.9825	0.9727	0.9601	0.9444	0.8227	0.6482	0.4613	0.2969	0.1727	0.0905	0.0424	0.0176	**3**
4		1.0000	0.9999	0.9998	0.9994	0.9986	0.9972	0.9950	0.9918	0.9873	0.9383	0.8358	0.6865	0.5155	0.3519	0.2173	0.1204	0.0592	**4**
5			1.0000	1.0000	0.9999	0.9999	0.9997	0.9993	0.9987	0.9978	0.9832	0.9389	0.8516	0.7216	0.5643	0.4032	0.2608	0.1509	**5**
6					1.0000	1.0000	1.0000	0.9999	0.9998	0.9997	0.9964	0.9819	0.9434	0.8689	0.7548	0.6098	0.4522	0.3036	**6**
7								1.0000	1.0000	1.0000	0.9994	0.9958	0.9827	0.9500	0.8868	0.7869	0.6535	0.5000	**7**
8											0.9999	0.9992	0.9958	0.9848	0.9578	0.9050	0.8182	0.6964	**8**
9											1.0000	0.9999	0.9992	0.9963	0.9876	0.9662	0.9231	0.8491	**9**
10												1.0000	0.9999	0.9993	0.9972	0.9907	0.9745	0.9408	**10**
11													1.0000	0.9999	0.9995	0.9981	0.9937	0.9824	**11**
12														1.0000	0.9999	0.9997	0.9989	0.9963	**12**
13															1.0000	1.0000	0.9999	0.9995	**13**
14																	1.0000	1.0000	**14**
x	n=20																		x
0	0.8179	0.6676	0.5438	0.4420	0.3585	0.2901	0.2342	0.1887	0.1516	0.1216	0.0388	0.0115	0.0032	0.0008	0.0002	0.0000	0.0000	0.0000	**0**
1	0.9831	0.9401	0.8802	0.8103	0.7358	0.6605	0.5869	0.5169	0.4516	0.3917	0.1756	0.0692	0.0243	0.0076	0.0021	0.0005	0.0001	0.0000	**1**
2	0.9990	0.9929	0.9790	0.9561	0.9245	0.8850	0.8390	0.7879	0.7334	0.6769	0.4049	0.2061	0.0913	0.0355	0.0121	0.0036	0.0009	0.0002	**2**
3	1.0000	0.9994	0.9973	0.9926	0.9841	0.9710	0.9529	0.9294	0.9007	0.8670	0.6477	0.4114	0.2252	0.1071	0.0444	0.0160	0.0049	0.0013	**3**
4		1.0000	0.9997	0.9990	0.9974	0.9944	0.9893	0.9817	0.9710	0.9568	0.8298	0.6296	0.4148	0.2375	0.1182	0.0510	0.0189	0.0059	**4**
5			1.0000	0.9999	0.9997	0.9991	0.9981	0.9962	0.9932	0.9887	0.9327	0.8042	0.6172	0.4164	0.2454	0.1256	0.0553	0.0207	**5**
6				1.0000	1.0000	0.9999	0.9997	0.9994	0.9987	0.9976	0.9781	0.9133	0.7858	0.6080	0.4166	0.2500	0.1299	0.0577	**6**
7						1.0000	1.0000	0.9999	0.9998	0.9996	0.9941	0.9679	0.8982	0.7723	0.6010	0.4159	0.2520	0.1316	**7**
8								1.0000	1.0000	0.9999	0.9987	0.9900	0.9591	0.8867	0.7624	0.5956	0.4143	0.2517	**8**
9										1.0000	0.9998	0.9974	0.9861	0.9520	0.8782	0.7553	0.5914	0.4119	**9**
10											1.0000	0.9994	0.9961	0.9829	0.9468	0.8725	0.7507	0.5881	**10**
11												0.9999	0.9991	0.9949	0.9804	0.9435	0.8692	0.7483	**11**
12												1.0000	0.9998	0.9987	0.9940	0.9790	0.9420	0.8684	**12**
13													1.0000	0.9997	0.9985	0.9935	0.9786	0.9423	**13**
14														1.0000	0.9997	0.9984	0.9936	0.9793	**14**
15															1.0000	0.9997	0.9985	0.9941	**15**
16																1.0000	0.9997	0.9987	**16**
17																	1.0000	0.9998	**17**
18																		1.0000	**18**

p	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	p
x	n=25																		x
0	0.7778	0.6035	0.4670	0.3604	0.2774	0.2129	0.1630	0.1244	0.0946	0.0718	0.0172	0.0038	0.0008	0.0001	0.0000	0.0000	0.0000	0.0000	**0**
1	0.9742	0.9114	0.8280	0.7358	0.6424	0.5527	0.4696	0.3947	0.3286	0.2712	0.0931	0.0274	0.0070	0.0016	0.0003	0.0001	0.0000	0.0000	**1**
2	0.9980	0.9868	0.9620	0.9235	0.8729	0.8129	0.7466	0.6768	0.6063	0.5371	0.2537	0.0982	0.0321	0.0090	0.0021	0.0004	0.0001	0.0000	**2**
3	0.9999	0.9986	0.9938	0.9835	0.9659	0.9402	0.9064	0.8649	0.8169	0.7636	0.4711	0.2340	0.0962	0.0332	0.0097	0.0024	0.0005	0.0001	**3**
4	1.0000	0.9999	0.9992	0.9972	0.9928	0.9850	0.9726	0.9549	0.9314	0.9020	0.6821	0.4207	0.2137	0.0905	0.0320	0.0095	0.0023	0.0005	**4**
5		1.0000	0.9999	0.9996	0.9988	0.9969	0.9935	0.9877	0.9790	0.9666	0.8385	0.6167	0.3783	0.1935	0.0826	0.0294	0.0086	0.0020	**5**
6			1.0000	1.0000	0.9998	0.9995	0.9987	0.9972	0.9946	0.9905	0.9305	0.7800	0.5611	0.3407	0.1734	0.0736	0.0258	0.0073	**6**
7					1.0000	0.9999	0.9998	0.9995	0.9989	0.9977	0.9745	0.8909	0.7265	0.5118	0.3061	0.1536	0.0639	0.0216	**7**
8						1.0000	1.0000	0.9999	0.9998	0.9995	0.9920	0.9532	0.8506	0.6769	0.4668	0.2735	0.1340	0.0539	**8**
9								1.0000	1.0000	0.9999	0.9979	0.9827	0.9287	0.8106	0.6303	0.4246	0.2424	0.1148	**9**
10										1.0000	0.9995	0.9944	0.9703	0.9022	0.7712	0.5858	0.3843	0.2122	**10**
11											0.9999	0.9985	0.9893	0.9558	0.8746	0.7323	0.5426	0.3450	**11**
12											1.0000	0.9996	0.9966	0.9825	0.9396	0.8462	0.6937	0.5000	**12**
13												0.9999	0.9991	0.9940	0.9745	0.9222	0.8173	0.6550	**13**
14												1.0000	0.9998	0.9982	0.9907	0.9656	0.9040	0.7878	**14**
15													1.0000	0.9995	0.9971	0.9868	0.9560	0.8852	**15**
16														0.9999	0.9992	0.9957	0.9826	0.9461	**16**
17														1.0000	0.9998	0.9988	0.9942	0.9784	**17**
18															1.0000	0.9997	0.9984	0.9927	**18**
19																0.9999	0.9996	0.9980	**19**
20																1.0000	0.9999	0.9995	**20**
21																	1.0000	0.9999	**21**
22																		1.0000	**22**
x	n=30																		x
0	0.7397	0.5455	0.4010	0.2939	0.2146	0.1563	0.1134	0.0820	0.0591	0.0424	0.0076	0.0012	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	**0**
1	0.9639	0.8795	0.7731	0.6612	0.5535	0.4555	0.3694	0.2958	0.2343	0.1837	0.0480	0.0105	0.0020	0.0003	0.0000	0.0000	0.0000	0.0000	**1**
2	0.9967	0.9783	0.9399	0.8831	0.8122	0.7324	0.6487	0.5654	0.4855	0.4114	0.1514	0.0442	0.0106	0.0021	0.0003	0.0000	0.0000	0.0000	**2**
3	0.9998	0.9971	0.9881	0.9694	0.9392	0.8974	0.8450	0.7842	0.7175	0.6474	0.3217	0.1227	0.0374	0.0093	0.0019	0.0003	0.0000	0.0000	**3**
4	1.0000	0.9997	0.9982	0.9937	0.9844	0.9685	0.9447	0.9126	0.8723	0.8245	0.5245	0.2552	0.0979	0.0302	0.0075	0.0015	0.0002	0.0000	**4**
5		1.0000	0.9998	0.9989	0.9967	0.9921	0.9838	0.9707	0.9519	0.9268	0.7106	0.4275	0.2026	0.0766	0.0233	0.0057	0.0011	0.0002	**5**
6			1.0000	0.9999	0.9994	0.9983	0.9960	0.9918	0.9848	0.9742	0.8474	0.6070	0.3481	0.1595	0.0586	0.0172	0.0040	0.0007	**6**
7				1.0000	0.9999	0.9997	0.9992	0.9980	0.9959	0.9922	0.9302	0.7608	0.5143	0.2814	0.1238	0.0435	0.0121	0.0026	**7**
8					1.0000	1.0000	0.9999	0.9996	0.9990	0.9980	0.9722	0.8713	0.6736	0.4315	0.2247	0.0940	0.0312	0.0081	**8**
9							1.0000	0.9999	0.9998	0.9995	0.9903	0.9389	0.8034	0.5888	0.3575	0.1763	0.0694	0.0214	**9**
10								1.0000	1.0000	0.9999	0.9971	0.9744	0.8943	0.7304	0.5078	0.2915	0.1350	0.0494	**10**
11										1.0000	0.9992	0.9905	0.9493	0.8407	0.6548	0.4311	0.2327	0.1002	**11**
12											0.9998	0.9969	0.9784	0.9155	0.7802	0.5785	0.3592	0.1808	**12**
13											1.0000	0.9991	0.9918	0.9599	0.8737	0.7145	0.5025	0.2923	**13**
14												0.9998	0.9973	0.9831	0.9348	0.8246	0.6448	0.4278	**14**
15												0.9999	0.9992	0.9936	0.9699	0.9029	0.7691	0.5722	**15**
16												1.0000	0.9998	0.9979	0.9876	0.9519	0.8644	0.7077	**16**
17													0.9999	0.9994	0.9955	0.9788	0.9286	0.8192	**17**
18													1.0000	0.9998	0.9986	0.9917	0.9666	0.8998	**18**
19														1.0000	0.9996	0.9971	0.9862	0.9506	**19**
20															0.9999	0.9991	0.9950	0.9786	**20**
21															1.0000	0.9998	0.9984	0.9919	**21**
22																1.0000	0.9996	0.9974	**22**
23																	0.9999	0.9993	**23**
24																	1.0000	0.9998	**24**
25																		1.0000	**25**

p	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	p
x	n=40																		x
0	0.6690	0.4457	0.2957	0.1954	0.1285	0.0842	0.0549	0.0356	0.0230	0.0148	0.0015	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	**0**
1	0.9393	0.8095	0.6615	0.5210	0.3991	0.2990	0.2201	0.1594	0.1140	0.0805	0.0121	0.0015	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	**1**
2	0.9925	0.9543	0.8822	0.7855	0.6767	0.5665	0.4625	0.3694	0.2894	0.2228	0.0486	0.0079	0.0010	0.0001	0.0000	0.0000	0.0000	0.0000	**2**
3	0.9993	0.9918	0.9686	0.9252	0.8619	0.7827	0.6937	0.6007	0.5092	0.4231	0.1302	0.0285	0.0047	0.0006	0.0001	0.0000	0.0000	0.0000	**3**
4	1.0000	0.9988	0.9933	0.9790	0.9520	0.9104	0.8546	0.7868	0.7103	0.6290	0.2633	0.0759	0.0160	0.0026	0.0003	0.0000	0.0000	0.0000	**4**
5		0.9999	0.9988	0.9951	0.9861	0.9691	0.9419	0.9033	0.8535	0.7937	0.4325	0.1613	0.0433	0.0086	0.0013	0.0001	0.0000	0.0000	**5**
6		1.0000	0.9998	0.9990	0.9966	0.9909	0.9801	0.9624	0.9361	0.9005	0.6067	0.2859	0.0962	0.0238	0.0044	0.0006	0.0001	0.0000	**6**
7			1.0000	0.9998	0.9993	0.9977	0.9942	0.9873	0.9758	0.9581	0.7559	0.4371	0.1820	0.0553	0.0124	0.0021	0.0002	0.0000	**7**
8				1.0000	0.9999	0.9995	0.9985	0.9963	0.9919	0.9845	0.8646	0.5931	0.2998	0.1110	0.0303	0.0061	0.0009	0.0001	**8**
9					1.0000	0.9999	0.9997	0.9990	0.9976	0.9949	0.9328	0.7318	0.4395	0.1959	0.0644	0.0156	0.0027	0.0003	**9**
10						1.0000	0.9999	0.9998	0.9994	0.9985	0.9701	0.8392	0.5839	0.3087	0.1215	0.0352	0.0074	0.0011	**10**
11							1.0000	1.0000	0.9999	0.9996	0.9880	0.9125	0.7151	0.4406	0.2053	0.0709	0.0179	0.0032	**11**
12									1.0000	0.9999	0.9957	0.9568	0.8209	0.5772	0.3143	0.1285	0.0386	0.0083	**12**
13										1.0000	0.9986	0.9806	0.8968	0.7032	0.4408	0.2112	0.0751	0.0192	**13**
14											0.9996	0.9921	0.9456	0.8074	0.5721	0.3174	0.1326	0.0403	**14**
15											0.9999	0.9971	0.9738	0.8849	0.6946	0.4402	0.2142	0.0769	**15**
16											1.0000	0.9990	0.9884	0.9367	0.7978	0.5681	0.3185	0.1341	**16**
17												0.9997	0.9953	0.9680	0.8761	0.6885	0.4391	0.2148	**17**
18												0.9999	0.9983	0.9852	0.9301	0.7911	0.5651	0.3179	**18**
19												1.0000	0.9994	0.9937	0.9637	0.8702	0.6844	0.4373	**19**
20													0.9998	0.9976	0.9827	0.9256	0.7870	0.5627	**20**
21													1.0000	0.9991	0.9925	0.9608	0.8669	0.6821	**21**
22														0.9997	0.9970	0.9811	0.9233	0.7852	**22**
23														0.9999	0.9989	0.9917	0.9595	0.8659	**23**
24														1.0000	0.9996	0.9966	0.9804	0.9231	**24**
25															0.9999	0.9988	0.9914	0.9597	**25**
26															1.0000	0.9996	0.9966	0.9808	**26**
27																0.9999	0.9988	0.9917	**27**
28																1.0000	0.9996	0.9968	**28**
29																	0.9999	0.9989	**29**
30																	1.0000	0.9997	**30**
31																		0.9999	**31**
32																		1.0000	**32**

p	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**	**0.10**	**0.15**	**0.20**	**0.25**	**0.30**	**0.35**	**0.40**	**0.45**	**0.50**	p
x	n=50																		x
0	0.6050	0.3642	0.2181	0.1299	0.0769	0.0453	0.0266	0.0155	0.0090	0.0052	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	**0**
1	0.9106	0.7358	0.5553	0.4005	0.2794	0.1900	0.1265	0.0827	0.0532	0.0338	0.0029	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	**1**
2	0.9862	0.9216	0.8108	0.6767	0.5405	0.4162	0.3108	0.2260	0.1605	0.1117	0.0142	0.0013	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	**2**
3	0.9984	0.9822	0.9372	0.8609	0.7604	0.6473	0.5327	0.4253	0.3303	0.2503	0.0460	0.0057	0.0005	0.0000	0.0000	0.0000	0.0000	0.0000	**3**
4	0.9999	0.9968	0.9832	0.9510	0.8964	0.8206	0.7290	0.6290	0.5277	0.4312	0.1121	0.0185	0.0021	0.0002	0.0000	0.0000	0.0000	0.0000	**4**
5	1.0000	0.9995	0.9963	0.9856	0.9622	0.9224	0.8650	0.7919	0.7072	0.6161	0.2194	0.0480	0.0070	0.0007	0.0001	0.0000	0.0000	0.0000	**5**
6		0.9999	0.9993	0.9964	0.9882	0.9711	0.9417	0.8981	0.8404	0.7702	0.3613	0.1034	0.0194	0.0025	0.0002	0.0000	0.0000	0.0000	**6**
7		1.0000	0.9999	0.9992	0.9968	0.9906	0.9780	0.9562	0.9232	0.8779	0.5188	0.1904	0.0453	0.0073	0.0008	0.0001	0.0000	0.0000	**7**
8			1.0000	0.9999	0.9992	0.9973	0.9927	0.9833	0.9672	0.9421	0.6681	0.3073	0.0916	0.0183	0.0025	0.0002	0.0000	0.0000	**8**
9				1.0000	0.9998	0.9993	0.9978	0.9944	0.9875	0.9755	0.7911	0.4437	0.1637	0.0402	0.0067	0.0008	0.0001	0.0000	**9**
10					1.0000	0.9998	0.9994	0.9983	0.9957	0.9906	0.8801	0.5836	0.2622	0.0789	0.0160	0.0022	0.0002	0.0000	**10**
11						1.0000	0.9999	0.9995	0.9987	0.9968	0.9372	0.7107	0.3816	0.1390	0.0342	0.0057	0.0006	0.0000	**11**
12							1.0000	0.9999	0.9996	0.9990	0.9699	0.8139	0.5110	0.2229	0.0661	0.0133	0.0018	0.0002	**12**
13								1.0000	0.9999	0.9997	0.9868	0.8894	0.6370	0.3279	0.1163	0.0280	0.0045	0.0005	**13**
14									1.0000	0.9999	0.9947	0.9393	0.7481	0.4468	0.1878	0.0540	0.0104	0.0013	**14**
15										1.0000	0.9981	0.9692	0.8369	0.5692	0.2801	0.0955	0.0220	0.0033	**15**
16											0.9993	0.9856	0.9017	0.6839	0.3889	0.1561	0.0427	0.0077	**16**
17											0.9998	0.9937	0.9449	0.7822	0.5060	0.2369	0.0765	0.0164	**17**
18											0.9999	0.9975	0.9713	0.8594	0.6216	0.3356	0.1273	0.0325	**18**
19											1.0000	0.9991	0.9861	0.9152	0.7264	0.4465	0.1974	0.0595	**19**
20												0.9997	0.9937	0.9522	0.8139	0.5610	0.2862	0.1013	**20**
21												0.9999	0.9974	0.9749	0.8813	0.6701	0.3900	0.1611	**21**
22												1.0000	0.9990	0.9877	0.9290	0.7660	0.5019	0.2399	**22**
23													0.9996	0.9944	0.9604	0.8438	0.6134	0.3359	**23**
24													0.9999	0.9976	0.9793	0.9022	0.7160	0.4439	**24**
25													1.0000	0.9991	0.9900	0.9427	0.8034	0.5561	**25**
26														0.9997	0.9955	0.9686	0.8721	0.6641	**26**
27														0.9999	0.9981	0.9840	0.9220	0.7601	**27**
28														1.0000	0.9993	0.9924	0.9556	0.8389	**28**
29															0.9997	0.9966	0.9765	0.8987	**29**
30															0.9999	0.9986	0.9884	0.9405	**30**
31															1.0000	0.9995	0.9947	0.9675	**31**
32																0.9998	0.9978	0.9836	**32**
33																0.9999	0.9991	0.9923	**33**
34																1.0000	0.9997	0.9967	**34**
35																	0.9999	0.9987	**35**
36																	1.0000	0.9995	**36**
37																		0.9998	**37**
38																		1.0000	**38**

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TABLE 2 CUMULATIVE POISSON DISTRIBUTION FUNCTION

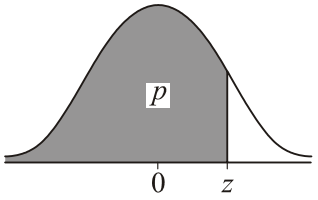
The tabulated value is P($X \leqslant x$), where X has a Poisson distribution with mean λ.

λ	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.0	1.2	1.4	1.6	1.8	λ
x															x
0	0.9048	0.8187	0.7408	0.6703	0.6065	0.5488	0.4966	0.4493	0.4066	0.3679	0.3012	0.2466	0.2019	0.1653	**0**
1	0.9953	0.9825	0.9631	0.9384	0.9098	0.8781	0.8442	0.8088	0.7725	0.7358	0.6626	0.5918	0.5249	0.4628	**1**
2	0.9998	0.9989	0.9964	0.9921	0.9856	0.9769	0.9659	0.9526	0.9371	0.9197	0.8795	0.8335	0.7834	0.7306	**2**
3	1.0000	0.9999	0.9997	0.9992	0.9982	0.9966	0.9942	0.9909	0.9865	0.9810	0.9662	0.9463	0.9212	0.8913	**3**
4		1.0000	1.0000	0.9999	0.9998	0.9996	0.9992	0.9986	0.9977	0.9963	0.9923	0.9857	0.9763	0.9636	**4**
5				1.0000	1.0000	1.0000	0.9999	0.9998	0.9997	0.9994	0.9985	0.9968	0.9940	0.9896	**5**
6							1.0000	1.0000	1.0000	0.9999	0.9997	0.9994	0.9987	0.9974	**6**
7										1.0000	1.0000	0.9999	0.9997	0.9994	**7**
8												1.0000	1.0000	0.9999	**8**
9														1.0000	**9**
λ	**2.0**	**2.2**	**2.4**	**2.6**	**2.8**	**3.0**	**3.2**	**3.4**	**3.6**	**3.8**	**4.0**	**4.5**	**5.0**	**5.5**	λ
x															x
0	0.1353	0.1108	0.0907	0.0743	0.0608	0.0498	0.0408	0.0334	0.0273	0.0224	0.0183	0.0111	0.0067	0.0041	**0**
1	0.4060	0.3546	0.3084	0.2674	0.2311	0.1991	0.1712	0.1468	0.1257	0.1074	0.0916	0.0611	0.0404	0.0266	**1**
2	0.6767	0.6227	0.5697	0.5184	0.4695	0.4232	0.3799	0.3397	0.3027	0.2689	0.2381	0.1736	0.1247	0.0884	**2**
3	0.8571	0.8194	0.7787	0.7360	0.6919	0.6472	0.6025	0.5584	0.5152	0.4735	0.4335	0.3423	0.2650	0.2017	**3**
4	0.9473	0.9275	0.9041	0.8774	0.8477	0.8153	0.7806	0.7442	0.7064	0.6678	0.6288	0.5321	0.4405	0.3575	**4**
5	0.9834	0.9751	0.9643	0.9510	0.9349	0.9161	0.8946	0.8705	0.8441	0.8156	0.7851	0.7029	0.6160	0.5289	**5**
6	0.9955	0.9925	0.9884	0.9828	0.9756	0.9665	0.9554	0.9421	0.9267	0.9091	0.8893	0.8311	0.7622	0.6860	**6**
7	0.9989	0.9980	0.9967	0.9947	0.9919	0.9881	0.9832	0.9769	0.9692	0.9599	0.9489	0.9134	0.8666	0.8095	**7**
8	0.9998	0.9995	0.9991	0.9985	0.9976	0.9962	0.9943	0.9917	0.9883	0.9840	0.9786	0.9597	0.9319	0.8944	**8**
9	1.0000	0.9999	0.9998	0.9996	0.9993	0.9989	0.9982	0.9973	0.9960	0.9942	0.9919	0.9829	0.9682	0.9462	**9**
10		1.0000	1.0000	0.9999	0.9998	0.9997	0.9995	0.9992	0.9987	0.9981	0.9972	0.9933	0.9863	0.9747	**10**
11				1.0000	1.0000	0.9999	0.9999	0.9998	0.9996	0.9994	0.9991	0.9976	0.9945	0.9890	**11**
12						1.0000	1.0000	0.9999	0.9999	0.9998	0.9997	0.9992	0.9980	0.9955	**12**
13								1.0000	1.0000	1.0000	0.9999	0.9997	0.9993	0.9983	**13**
14											1.0000	0.9999	0.9998	0.9994	**14**
15												1.0000	0.9999	0.9998	**15**
16													1.0000	0.9999	**16**
17														1.0000	**17**

λ	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0	11.0	12.0	13.0	14.0	15.0	λ
x															x
0	0.0025	0.0015	0.0009	0.0006	0.0003	0.0002	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	**0**
1	0.0174	0.0113	0.0073	0.0047	0.0030	0.0019	0.0012	0.0008	0.0005	0.0002	0.0001	0.0000	0.0000	0.0000	**1**
2	0.0620	0.0430	0.0296	0.0203	0.0138	0.0093	0.0062	0.0042	0.0028	0.0012	0.0005	0.0002	0.0001	0.0000	**2**
3	0.1512	0.1118	0.0818	0.0591	0.0424	0.0301	0.0212	0.0149	0.0103	0.0049	0.0023	0.0011	0.0005	0.0002	**3**
4	0.2851	0.2237	0.1730	0.1321	0.0996	0.0744	0.0550	0.0403	0.0293	0.0151	0.0076	0.0037	0.0018	0.0009	**4**
5	0.4457	0.3690	0.3007	0.2414	0.1912	0.1496	0.1157	0.0885	0.0671	0.0375	0.0203	0.0107	0.0055	0.0028	**5**
6	0.6063	0.5265	0.4497	0.3782	0.3134	0.2562	0.2068	0.1649	0.1301	0.0786	0.0458	0.0259	0.0142	0.0076	**6**
7	0.7440	0.6728	0.5987	0.5246	0.4530	0.3856	0.3239	0.2687	0.2202	0.1432	0.0895	0.0540	0.0316	0.0180	**7**
8	0.8472	0.7916	0.7291	0.6620	0.5925	0.5231	0.4557	0.3918	0.3328	0.2320	0.1550	0.0998	0.0621	0.0374	**8**
9	0.9161	0.8774	0.8305	0.7764	0.7166	0.6530	0.5874	0.5218	0.4579	0.3405	0.2424	0.1658	0.1094	0.0699	**9**
10	0.9574	0.9332	0.9015	0.8622	0.8159	0.7634	0.7060	0.6453	0.5830	0.4599	0.3472	0.2517	0.1757	0.1185	**10**
11	0.9799	0.9661	0.9467	0.9208	0.8881	0.8487	0.8030	0.7520	0.6968	0.5793	0.4616	0.3532	0.2600	0.1848	**11**
12	0.9912	0.9840	0.9730	0.9573	0.9362	0.9091	0.8758	0.8364	0.7916	0.6887	0.5760	0.4631	0.3585	0.2676	**12**
13	0.9964	0.9929	0.9872	0.9784	0.9658	0.9486	0.9261	0.8981	0.8645	0.7813	0.6815	0.5730	0.4644	0.3632	**13**
14	0.9986	0.9970	0.9943	0.9897	0.9827	0.9726	0.9585	0.9400	0.9165	0.8540	0.7720	0.6751	0.5704	0.4657	**14**
15	0.9995	0.9988	0.9976	0.9954	0.9918	0.9862	0.9780	0.9665	0.9513	0.9074	0.8444	0.7636	0.6694	0.5681	**15**
16	0.9998	0.9996	0.9990	0.9980	0.9963	0.9934	0.9889	0.9823	0.9730	0.9441	0.8987	0.8355	0.7559	0.6641	**16**
17	0.9999	0.9998	0.9996	0.9992	0.9984	0.9970	0.9947	0.9911	0.9857	0.9678	0.9370	0.8905	0.8272	0.7489	**17**
18	1.0000	0.9999	0.9999	0.9997	0.9993	0.9987	0.9976	0.9957	0.9928	0.9823	0.9626	0.9302	0.8826	0.8195	**18**
19		1.0000	1.0000	0.9999	0.9997	0.9995	0.9989	0.9980	0.9965	0.9907	0.9787	0.9573	0.9235	0.8752	**19**
20				1.0000	0.9999	0.9998	0.9996	0.9991	0.9984	0.9953	0.9884	0.9750	0.9521	0.9170	**20**
21					1.0000	0.9999	0.9998	0.9996	0.9993	0.9977	0.9939	0.9859	0.9712	0.9469	**21**
22						1.0000	0.9999	0.9999	0.9997	0.9990	0.9970	0.9924	0.9833	0.9673	**22**
23							1.0000	0.9999	0.9999	0.9995	0.9985	0.9960	0.9907	0.9805	**23**
24								1.0000	1.0000	0.9998	0.9993	0.9980	0.9950	0.9888	**24**
25										0.9999	0.9997	0.9990	0.9974	0.9938	**25**
26										1.0000	0.9999	0.9995	0.9987	0.9967	**26**
27											0.9999	0.9998	0.9994	0.9983	**27**
28											1.0000	0.9999	0.9997	0.9991	**28**
29												1.0000	0.9999	0.9996	**29**
30													0.9999	0.9998	**30**
31													1.0000	0.9999	**31**
32														1.0000	**32**

TABLE 3 NORMAL DISTRIBUTION FUNCTION

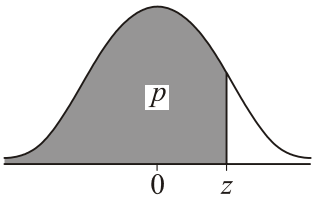
The table gives the probability, p, that a normally distributed random variable Z, with mean = 0 and variance = 1, is less than or equal to z.



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	z
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586	**0.0**
0.1	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535	**0.1**
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409	**0.2**
0.3	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173	**0.3**
0.4	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793	**0.4**
0.5	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240	**0.5**
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490	**0.6**
0.7	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524	**0.7**
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327	**0.8**
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891	**0.9**
1.0	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214	**1.0**
1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298	**1.1**
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147	**1.2**
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91309	0.91466	0.91621	0.91774	**1.3**
1.4	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189	**1.4**
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408	**1.5**
1.6	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449	**1.6**
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327	**1.7**
1.8	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062	**1.8**
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670	**1.9**
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169	**2.0**
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574	**2.1**
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899	**2.2**
2.3	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158	**2.3**
2.4	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361	**2.4**
2.5	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520	**2.5**
2.6	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643	**2.6**
2.7	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736	**2.7**
2.8	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807	**2.8**
2.9	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861	**2.9**
3.0	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.99900	**3.0**
3.1	0.99903	0.99906	0.99910	0.99913	0.99916	0.99918	0.99921	0.99924	0.99926	0.99929	**3.1**
3.2	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950	**3.2**
3.3	0.99952	0.99953	0.99955	0.99957	0.99958	0.99960	0.99961	0.99962	0.99964	0.99965	**3.3**
3.4	0.99966	0.99968	0.99969	0.99970	0.99971	0.99972	0.99973	0.99974	0.99975	0.99976	**3.4**
3.5	0.99977	0.99978	0.99978	0.99979	0.99980	0.99981	0.99981	0.99982	0.99983	0.99983	**3.5**
3.6	0.99984	0.99985	0.99985	0.99986	0.99986	0.99987	0.99987	0.99988	0.99988	0.99989	**3.6**
3.7	0.99989	0.99990	0.99990	0.99990	0.99991	0.99991	0.99992	0.99992	0.99992	0.99992	**3.7**
3.8	0.99993	0.99993	0.99993	0.99994	0.99994	0.99994	0.99994	0.99995	0.99995	0.99995	**3.8**
3.9	0.99995	0.99995	0.99996	0.99996	0.99996	0.99996	0.99996	0.99996	0.99997	0.99997	**3.9**

TABLE 4 PERCENTAGE POINTS OF THE NORMAL DISTRIBUTION

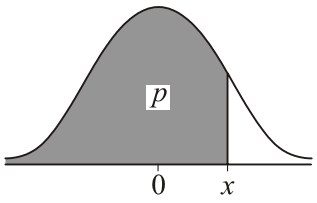
The table gives the values of z satisfying $P(Z \leqslant z) = p$, where Z is the normally distributed random variable with mean = 0 and variance = 1.



p	**0.00**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**	p
0.5	0.0000	0.0251	0.0502	0.0753	0.1004	0.1257	0.1510	0.1764	0.2019	0.2275	**0.5**
0.6	0.2533	0.2793	0.3055	0.3319	0.3585	0.3853	0.4125	0.4399	0.4677	0.4958	**0.6**
0.7	0.5244	0.5534	0.5828	0.6128	0.6433	0.6745	0.7063	0.7388	0.7722	0.8064	**0.7**
0.8	0.8416	0.8779	0.9154	0.9542	0.9945	1.0364	1.0803	1.1264	1.1750	1.2265	**0.8**
0.9	1.2816	1.3408	1.4051	1.4758	1.5548	1.6449	1.7507	1.8808	2.0537	2.3263	**0.9**
p	**0.000**	**0.001**	**0.002**	**0.003**	**0.004**	**0.005**	**0.006**	**0.007**	**0.008**	**0.009**	p
0.95	1.6449	1.6546	1.6646	1.6747	1.6849	1.6954	1.7060	1.7169	1.7279	1.7392	**0.95**
0.96	1.7507	1.7624	1.7744	1.7866	1.7991	1.8119	1.8250	1.8384	1.8522	1.8663	**0.96**
0.97	1.8808	1.8957	1.9110	1.9268	1.9431	1.9600	1.9774	1.9954	2.0141	2.0335	**0.97**
0.98	2.0537	2.0749	2.0969	2.1201	2.1444	2.1701	2.1973	2.2262	2.2571	2.2904	**0.98**
0.99	2.3263	2.3656	2.4089	2.4573	2.5121	2.5758	2.6521	2.7478	2.8782	3.0902	**0.99**

TABLE 5 PERCENTAGE POINTS OF THE STUDENT'S *t*-DISTRIBUTION

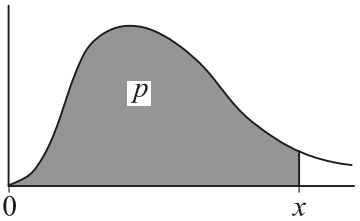
The table gives the values of x satisfying $P(X \leqslant x) = p$, where X is a random variable having the Student's t-distribution with ν degrees of freedom.



p	0.9	0.95	0.975	0.99	0.995
ν					
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.121	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
31	1.309	1.696	2.040	2.453	2.744
32	1.309	1.694	2.037	2.449	2.738
33	1.308	1.692	2.035	2.445	2.733
34	1.307	1.691	2.032	2.441	2.728
35	1.306	1.690	2.030	2.438	2.724
36	1.306	1.688	2.028	2.434	2.719
37	1.305	1.687	2.026	2.431	2.715
38	1.304	1.686	2.024	2.429	2.712
39	1.304	1.685	2.023	2.426	2.708
40	1.303	1.684	2.021	2.423	2.704
45	1.301	1.679	2.014	2.412	2.690
50	1.299	1.676	2.009	2.403	2.678
55	1.297	1.673	2.004	2.396	2.668
60	1.296	1.671	2.000	2.390	2.660
65	1.295	1.669	1.997	2.385	2.654
70	1.294	1.667	1.994	2.381	2.648
75	1.293	1.665	1.992	2.377	2.643
80	1.292	1.664	1.990	2.374	2.639
85	1.292	1.663	1.998	2.371	2.635
90	1.291	1.662	1.987	2.368	2.632
95	1.291	1.661	1.985	2.366	2.629
100	1.290	1.660	1.984	2.364	2.626
125	1.288	1.657	1.979	2.357	2.616
150	1.287	1.655	1.976	2.351	2.609
200	1.286	1.653	1.972	2.345	2.601
∞	1.282	1.645	1.960	2.326	2.576

TABLE 6 PERCENTAGE POINTS OF THE χ^2 DISTRIBUTION

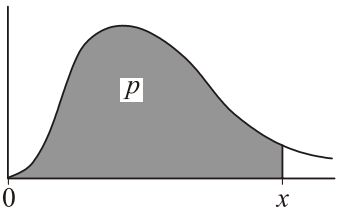
The table gives the values of x satisfying $P(X \leqslant x) = p$, where X is a random variable having the χ^2 distribution with ν degrees of freedom.



p	0.005	0.01	0.025	0.05	0.1	0.9	0.95	0.975	0.99	0.995	p
ν											ν
1	0.00004	0.0002	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879	**1**
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597	**2**
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838	**3**
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860	**4**
5	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.833	15.086	16.750	**5**
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548	**6**
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278	**7**
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955	**8**
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589	**9**
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188	**10**
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757	**11**
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300	**12**
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819	**13**
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319	**14**
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801	**15**
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267	**16**
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718	**17**
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156	**18**
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582	**19**
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997	**20**
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401	**21**
22	8.643	9.542	10.982	12.338	14.041	30.813	33.924	36.781	40.289	42.796	**22**
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181	**23**
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559	**24**
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928	**25**
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290	**26**
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.195	46.963	49.645	**27**
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993	**28**
29	13.121	14.256	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336	**29**
30	13.787	14.953	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672	**30**
31	14.458	15.655	17.539	19.281	21.434	41.422	44.985	48.232	52.191	55.003	**31**
32	15.134	16.362	18.291	20.072	22.271	42.585	46.194	49.480	53.486	56.328	**32**
33	15.815	17.074	19.047	20.867	23.110	43.745	47.400	50.725	54.776	57.648	**33**
34	16.501	17.789	19.806	21.664	23.952	44.903	48.602	51.996	56.061	58.964	**34**
35	17.192	18.509	20.569	22.465	24.797	46.059	49.802	53.203	57.342	60.275	**35**
36	17.887	19.223	21.336	23.269	25.643	47.212	50.998	54.437	58.619	61.581	**36**
37	18.586	19.960	22.106	24.075	26.492	48.363	52.192	55.668	59.892	62.883	**37**
38	19.289	20.691	22.878	24.884	27.343	49.513	53.384	56.896	61.162	64.181	**38**
39	19.996	21.426	23.654	25.695	28.196	50.660	54.572	58.120	62.428	65.476	**39**
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766	**40**
45	24.311	25.901	28.366	30.612	33.350	57.505	61.656	65.410	69.957	73.166	**45**
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490	**50**
55	31.735	33.570	36.398	38.958	42.060	68.796	73.311	77.380	82.292	85.749	**55**
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	91.952	**60**
65	39.383	41.444	44.603	47.450	50.883	79.973	84.821	89.177	94.422	98.105	**65**
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215	**70**
75	47.206	49.475	52.942	56.054	59.795	91.061	96.217	100.839	106.393	110.286	**75**
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.321	**80**
85	55.170	57.634	61.389	64.749	68.777	102.079	107.522	112.393	118.236	122.325	**85**
90	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116	128.299	**90**
95	63.250	65.898	69.925	73.520	77.818	113.038	118.752	123.858	129.973	134.247	**95**
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.169	**100**

TABLE 7 PERCENTAGE POINTS OF THE *F*-DISTRIBUTION

The tables give the values of x satisfying $P(X \leqslant x) = p$, where X is a random variable having the F-distribution with ν_1 degrees of freedom in the numerator and ν_2 degrees of freedom in the denominator.



F-Distribution (p=0.995)

Use for one-tail tests at significance level 0.5% or two-tail tests at significance level 1%.

ν_1	1	2	3	4	5	6	7	8	9	10	11	12	15	20	25	30	40	50	100	∞	ν_1
ν_2																					ν_2
1	16211	20000	21615	22500	23056	23437	23715	23925	24091	24224	24334	24426	24630	24836	24960	25044	25148	25211	25337	25464	1
2	198.5	199.0	199.2	199.2	199.3	199.3	199.4	199.4	199.4	199.4	199.4	199.4	199.4	199.4	199.5	199.5	199.5	199.5	199.5	199.5	2
3	55.55	49.80	47.47	46.19	45.39	44.84	44.43	44.13	43.88	43.69	43.52	43.39	43.08	42.78	42.59	42.47	42.31	42.21	42.02	41.83	3
4	31.33	26.28	24.26	23.15	22.46	21.97	21.62	21.35	21.14	20.97	20.82	20.70	20.44	20.17	20.00	19.89	19.75	19.67	19.50	19.32	4
5	22.78	18.31	16.53	15.56	14.94	14.51	14.20	13.96	13.77	13.62	13.49	13.38	13.15	12.90	12.76	12.66	12.53	12.45	12.30	12.14	5
6	18.635	14.544	12.917	12.028	11.464	11.073	10.786	10.566	10.391	10.250	10.133	10.034	9.814	9.589	9.451	9.358	9.241	9.170	9.026	8.879	6
7	16.236	12.404	10.882	10.050	9.522	9.155	8.885	8.678	8.514	8.380	8.270	8.176	7.968	7.754	7.623	7.534	7.422	7.354	7.217	7.076	7
8	14.688	11.042	9.596	8.805	8.302	7.952	7.694	7.496	7.339	7.211	7.104	7.015	6.814	6.608	6.482	6.396	6.288	6.222	6.088	5.951	8
9	13.614	10.107	8.717	7.956	7.471	7.134	6.885	6.693	6.541	6.417	6.314	6.227	6.032	5.832	5.708	5.625	5.519	5.454	5.322	5.188	9
10	12.826	9.427	8.081	7.343	6.872	6.545	6.302	6.116	5.968	5.847	5.746	5.661	5.471	5.274	5.153	5.071	4.966	4.902	4.772	4.639	10
11	12.226	8.912	7.600	6.881	6.422	6.102	5.865	5.682	5.537	5.418	5.320	5.236	5.049	4.855	4.736	4.654	4.551	4.488	4.359	4.226	11
12	11.754	8.510	7.226	6.521	6.071	5.757	5.525	5.345	5.202	5.085	4.988	4.906	4.721	4.530	4.412	4.331	4.228	4.165	4.037	3.904	12
13	11.374	8.186	6.926	6.233	5.791	5.482	5.253	5.076	4.935	4.820	4.724	4.643	4.460	4.270	4.153	4.073	3.970	3.908	3.780	3.647	13
14	11.060	7.922	6.680	5.998	5.562	5.257	5.031	4.857	4.717	4.603	4.508	4.428	4.247	4.059	3.942	3.862	3.760	3.697	3.569	3.436	14
15	10.798	7.701	6.476	5.803	5.372	5.071	4.847	4.674	4.536	4.424	4.329	4.250	4.070	3.883	3.766	3.687	3.585	3.523	3.394	3.260	15
20	9.944	6.986	5.818	5.174	4.762	4.472	4.257	4.090	3.956	3.847	3.756	3.678	3.502	3.318	3.203	3.123	3.022	2.959	2.828	2.690	20
25	9.475	6.598	5.462	4.835	4.433	4.150	3.939	3.776	3.645	3.537	3.447	3.370	3.196	3.013	2.898	2.819	2.716	2.652	2.519	2.377	25
30	9.180	6.355	5.239	4.623	4.228	3.949	3.742	3.580	3.450	3.344	3.255	3.179	3.006	2.823	2.708	2.628	2.524	2.459	2.323	2.176	30
40	8.828	6.066	4.976	4.374	3.986	3.713	3.509	3.350	3.222	3.117	3.028	2.953	2.781	2.598	2.482	2.401	2.296	2.230	2.088	1.932	40
50	8.626	5.902	4.826	4.232	3.849	3.579	3.376	3.219	3.092	2.988	2.900	2.825	2.653	2.470	2.353	2.272	2.164	2.097	1.951	1.786	50
100	8.241	5.589	4.542	3.963	3.589	3.325	3.127	2.972	2.847	2.744	2.657	2.583	2.411	2.227	2.108	2.024	1.912	1.840	1.681	1.485	100
∞	7.879	5.298	4.279	3.715	3.350	3.091	2.897	2.744	2.621	2.519	2.432	2.358	2.187	2.000	1.877	1.789	1.669	1.590	1.402	1.001	∞

F-Distribution (p=0.99)

Use for one-tail tests at significance level 1% or two-tail tests at significance level 2%.

ν_1	1	2	3	4	5	6	7	8	9	10	11	12	15	20	25	30	40	50	100	∞	ν_1
ν_2																					ν_2
1	4052	5000	5403	5625	5764	5859	5928	5981	6022	6056	6083	6106	6157	6209	6240	6261	6287	6303	6334	6366	1
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39	99.40	99.41	99.42	99.43	99.45	99.46	99.47	99.47	99.48	99.49	99.50	2
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23	27.13	27.05	26.87	26.69	26.58	26.50	26.41	26.35	26.24	26.13	3
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55	14.45	14.37	14.20	14.02	13.91	13.84	13.75	13.69	13.58	13.46	4
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05	9.96	9.89	9.72	9.55	9.45	9.38	9.29	9.24	9.13	9.02	5
6	13.745	10.925	9.780	9.148	8.746	8.466	8.260	8.102	7.976	7.874	7.790	7.718	7.559	7.396	7.296	7.229	7.143	7.091	6.987	6.880	6
7	12.246	9.547	8.451	7.847	7.460	7.191	6.993	6.840	6.719	6.620	6.538	6.469	6.314	6.155	6.058	5.992	5.908	5.858	5.755	5.650	7
8	11.259	8.649	7.591	7.006	6.632	6.371	6.178	6.029	5.911	5.814	5.734	5.667	5.515	5.359	5.263	5.198	5.116	5.065	4.963	4.859	8
9	10.561	8.022	6.992	6.422	6.057	5.802	5.613	5.467	5.351	5.257	5.178	5.111	4.962	4.808	4.713	4.649	4.567	4.517	4.415	4.311	9
10	10.044	7.559	6.552	5.994	5.636	5.386	5.200	5.057	4.942	4.849	4.772	4.706	4.558	4.405	4.311	4.247	4.165	4.115	4.014	3.909	10
11	9.646	7.206	6.217	5.668	5.316	5.069	4.886	4.744	4.632	4.539	4.462	4.397	4.251	4.099	4.005	3.941	3.860	3.810	3.708	3.602	11
12	9.330	6.927	5.953	5.412	5.064	4.821	4.640	4.499	4.388	4.296	4.220	4.155	4.010	3.858	3.765	3.701	3.619	3.569	3.467	3.361	12
13	9.074	6.701	5.739	5.205	4.862	4.620	4.441	4.302	4.191	4.100	4.025	3.960	3.815	3.665	3.571	3.507	3.425	3.375	3.272	3.165	13
14	8.862	6.515	5.564	5.035	4.695	4.456	4.278	4.140	4.030	3.939	3.864	3.800	3.656	3.505	3.412	3.348	3.266	3.215	3.112	3.004	14
15	8.683	6.359	5.417	4.893	4.556	4.318	4.142	4.004	3.895	3.805	3.730	3.666	3.522	3.372	3.278	3.214	3.132	3.081	2.977	2.868	15
20	8.096	5.849	4.938	4.431	4.103	3.871	3.699	3.564	3.457	3.368	3.294	3.231	3.088	2.938	2.843	2.778	2.695	2.643	2.535	2.421	20
25	7.770	5.568	4.675	4.177	3.855	3.627	3.457	3.324	3.217	3.129	3.056	2.993	2.850	2.699	2.604	2.538	2.453	2.400	2.289	2.169	25
30	7.562	5.390	4.510	4.018	3.699	3.473	3.304	3.173	3.067	2.979	2.906	2.843	2.700	2.549	2.453	2.386	2.299	2.245	2.131	2.006	30
40	7.314	5.179	4.313	3.828	3.514	3.291	3.124	2.993	2.888	2.801	2.727	2.665	2.522	2.369	2.271	2.203	2.114	2.058	1.938	1.805	40
50	7.171	5.057	4.199	3.720	3.408	3.186	3.020	2.890	2.785	2.698	2.625	2.562	2.419	2.265	2.167	2.098	2.007	1.949	1.825	1.683	50
100	6.895	4.824	3.984	3.513	3.206	2.988	2.823	2.694	2.590	2.503	2.430	2.368	2.223	2.067	1.965	1.893	1.797	1.735	1.598	1.427	100
∞	6.635	4.605	3.782	3.319	3.017	2.802	2.639	2.511	2.407	2.321	2.248	2.185	2.039	1.878	1.773	1.696	1.592	1.523	1.358	1.000	∞

F-Distribution (p=0.975)

Use for one-tail tests at significance level 2.5% or two-tail tests at significance level 5%.

ν_1	1	2	3	4	5	6	7	8	9	10	11	12	15	20	25	30	40	50	100	∞	ν_1
ν_2																					ν_2
1	647.8	799.5	864.2	899.6	921.8	937.1	948.2	956.7	963.3	968.6	973.0	976.7	984.9	993.1	998.1	1001.4	1005.6	1008.1	1013.2	1018.3	**1**
2	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39	39.40	39.41	39.41	39.43	39.45	39.46	39.46	39.47	39.48	39.49	39.50	**2**
3	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47	14.42	14.37	14.34	14.25	14.17	14.12	14.08	14.04	14.01	13.96	13.90	**3**
4	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84	8.79	8.75	8.66	8.56	8.50	8.46	8.41	8.38	8.32	8.26	**4**
5	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62	6.57	6.52	6.43	6.33	6.27	6.23	6.18	6.14	6.08	6.02	**5**
6	8.813	7.260	6.599	6.227	5.988	5.820	5.695	5.600	5.523	5.461	5.410	5.366	5.269	5.168	5.107	5.065	5.012	4.980	4.915	4.849	**6**
7	8.073	6.542	5.890	5.523	5.285	5.119	4.995	4.899	4.823	4.761	4.709	4.666	4.568	4.467	4.405	4.362	4.309	4.276	4.210	4.142	**7**
8	7.571	6.059	5.416	5.053	4.817	4.652	4.529	4.433	4.357	4.295	4.243	4.200	4.101	3.999	3.937	3.894	3.840	3.807	3.739	3.670	**8**
9	7.209	5.715	5.078	4.718	4.484	4.320	4.197	4.102	4.026	3.964	3.912	3.868	3.769	3.667	3.604	3.560	3.505	3.472	3.403	3.333	**9**
10	6.937	5.456	4.826	4.468	4.236	4.072	3.950	3.855	3.779	3.717	3.665	3.621	3.522	3.419	3.355	3.311	3.255	3.221	3.152	3.080	**10**
11	6.724	5.256	4.630	4.275	4.044	3.881	3.759	3.664	3.588	3.526	3.474	3.430	3.330	3.226	3.162	3.118	3.061	3.027	2.956	2.883	**11**
12	6.554	5.096	4.474	4.121	3.891	3.728	3.607	3.512	3.436	3.374	3.321	3.277	3.177	3.073	3.008	2.963	2.906	2.871	2.800	2.725	**12**
13	6.414	4.965	4.347	3.996	3.767	3.604	3.483	3.388	3.312	3.250	3.197	3.153	3.053	2.948	2.882	2.837	2.780	2.744	2.671	2.595	**13**
14	6.298	4.857	4.242	3.892	3.663	3.501	3.380	3.285	3.209	3.147	3.095	3.050	2.949	2.844	2.778	2.732	2.674	2.638	2.565	2.487	**14**
15	6.200	4.765	4.153	3.804	3.576	3.415	3.293	3.199	3.123	3.060	3.008	2.963	2.862	2.756	2.689	2.644	2.585	2.549	2.474	2.395	**15**
20	5.871	4.461	3.859	3.515	3.289	3.128	3.007	2.913	2.837	2.774	2.721	2.676	2.573	2.464	2.396	2.349	2.287	2.249	2.170	2.085	**20**
25	5.686	4.291	3.694	3.353	3.129	2.969	2.848	2.753	2.677	2.613	2.560	2.515	2.411	2.300	2.230	2.182	2.118	2.079	1.996	1.906	**25**
30	5.568	4.182	3.589	3.250	3.026	2.867	2.746	2.651	2.575	2.511	2.458	2.412	2.307	2.195	2.124	2.074	2.009	1.968	1.882	1.787	**30**
40	5.424	4.051	3.463	3.126	2.904	2.744	2.624	2.529	2.452	2.388	2.334	2.288	2.182	2.068	1.994	1.943	1.875	1.832	1.741	1.637	**40**
50	5.340	3.975	3.390	3.054	2.833	2.674	2.553	2.458	2.381	2.317	2.263	2.216	2.109	1.993	1.919	1.866	1.796	1.752	1.656	1.545	**50**
100	5.179	3.828	3.250	2.917	2.696	2.537	2.417	2.321	2.244	2.179	2.125	2.077	1.968	1.849	1.770	1.715	1.640	1.592	1.483	1.347	**100**
∞	5.024	3.689	3.116	2.786	2.567	2.408	2.288	2.192	2.114	2.048	1.993	1.945	1.833	1.708	1.626	1.566	1.484	1.428	1.296	1.000	∞

F-Distribution (p=0.95)

Use for one-tail tests at significance level 5% or two-tail tests at significance level 10%.

ν_1	1	2	3	4	5	6	7	8	9	10	11	12	15	20	25	30	40	50	100	∞	ν_1
ν_2																					ν_2
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.0	243.9	245.9	248.0	249.3	250.1	251.1	251.8	253.0	254.3	**1**
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.40	19.41	19.43	19.45	19.46	19.46	19.47	19.48	19.49	19.50	**2**
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.76	8.74	8.70	8.66	8.63	8.62	8.59	8.58	8.55	8.53	**3**
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.94	5.91	5.86	5.80	5.77	5.75	5.72	5.70	5.66	5.63	**4**
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.70	4.68	4.62	4.56	4.52	4.50	4.46	4.44	4.41	4.36	**5**
6	5.987	5.143	4.757	4.534	4.387	4.284	4.207	4.147	4.099	4.060	4.027	4.000	3.938	3.874	3.835	3.808	3.774	3.754	3.712	3.669	**6**
7	5.591	4.737	4.347	4.120	3.972	3.866	3.787	3.726	3.677	3.637	3.603	3.575	3.511	3.445	3.404	3.376	3.340	3.319	3.275	3.230	**7**
8	5.318	4.459	4.066	3.838	3.688	3.581	3.500	3.438	3.388	3.347	3.313	3.284	3.218	3.150	3.108	3.079	3.043	3.020	2.975	2.928	**8**
9	5.117	4.256	3.863	3.633	3.482	3.374	3.293	3.230	3.179	3.137	3.102	3.073	3.006	2.936	2.893	2.864	2.826	2.803	2.756	2.707	**9**
10	4.965	4.103	3.708	3.478	3.326	3.217	3.135	3.072	3.020	2.978	2.943	2.913	2.845	2.774	2.730	2.700	2.661	2.637	2.588	2.538	**10**
11	4.844	3.982	3.587	3.357	3.204	3.095	3.012	2.948	2.896	2.854	2.818	2.788	2.719	2.646	2.601	2.570	2.531	2.507	2.457	2.404	**11**
12	4.747	3.885	3.490	3.259	3.106	2.996	2.913	2.849	2.796	2.753	2.717	2.687	2.617	2.544	2.498	2.466	2.426	2.401	2.350	2.296	**12**
13	4.667	3.806	3.411	3.179	3.025	2.915	2.832	2.767	2.714	2.671	2.635	2.604	2.533	2.459	2.412	2.380	2.339	2.314	2.261	2.206	**13**
14	4.600	3.739	3.344	3.112	2.958	2.848	2.764	2.699	2.646	2.602	2.565	2.534	2.463	2.388	2.341	2.308	2.266	2.241	2.187	2.131	**14**
15	4.543	3.682	3.287	3.056	2.901	2.790	2.707	2.641	2.588	2.544	2.507	2.475	2.403	2.328	2.280	2.247	2.204	2.178	2.123	2.066	**15**
20	4.351	3.493	3.098	2.866	2.711	2.599	2.514	2.447	2.393	2.348	2.310	2.278	2.203	2.124	2.074	2.039	1.994	1.966	1.907	1.843	**20**
25	4.242	3.385	2.991	2.759	2.603	2.490	2.405	2.337	2.282	2.236	2.198	2.165	2.089	2.007	1.955	1.919	1.872	1.842	1.779	1.711	**25**
30	4.171	3.316	2.922	2.690	2.534	2.421	2.334	2.266	2.211	2.165	2.126	2.092	2.015	1.932	1.878	1.841	1.792	1.761	1.695	1.622	**30**
40	4.085	3.232	2.839	2.606	2.449	2.336	2.249	2.180	2.124	2.077	2.038	2.003	1.924	1.839	1.783	1.744	1.693	1.660	1.589	1.509	**40**
50	4.034	3.183	2.790	2.557	2.400	2.286	2.199	2.130	2.073	2.026	1.986	1.952	1.871	1.784	1.727	1.687	1.634	1.599	1.525	1.438	**50**
100	3.936	3.087	2.696	2.463	2.305	2.191	2.103	2.032	1.975	1.927	1.886	1.850	1.768	1.676	1.616	1.573	1.515	1.477	1.392	1.283	**100**
∞	3.841	2.996	2.605	2.372	2.214	2.099	2.010	1.938	1.880	1.831	1.789	1.752	1.666	1.571	1.506	1.459	1.394	1.350	1.243	1.000	∞

TABLE 8 CRITICAL VALUES OF THE PRODUCT MOMENT CORRELATION COEFFICIENT

The table gives the critical values, for different significance levels, of the product moment correlation coefficient, r, for varying sample sizes, n.

One tail Two tail	10% 20%	5% 10%	2.5% 5%	1% 2%	0.5% 1%	One tail Two tail
n						n
4	0.8000	0.9000	0.9500	0.9800	0.9900	4
5	0.6870	0.8054	0.8783	0.9343	0.9587	5
6	0.6084	0.7293	0.8114	0.8822	0.9172	6
7	0.5509	0.6694	0.7545	0.8329	0.8745	7
8	0.5067	0.6215	0.7067	0.7887	0.8343	8
9	0.4716	0.5822	0.6664	0.7498	0.7977	9
10	0.4428	0.5494	0.6319	0.7155	0.7646	10
11	0.4187	0.5214	0.6021	0.6851	0.7348	11
12	0.3981	0.4973	0.5760	0.6581	0.7079	12
13	0.3802	0.4762	0.5529	0.6339	0.6835	13
14	0.3646	0.4575	0.5324	0.6120	0.6614	14
15	0.3507	0.4409	0.5140	0.5923	0.6411	15
16	0.3383	0.4259	0.4973	0.5742	0.6226	16
17	0.3271	0.4124	0.4821	0.5577	0.6055	17
18	0.3170	0.4000	0.4683	0.5425	0.5897	18
19	0.3077	0.3887	0.4555	0.5285	0.5751	19
20	0.2992	0.3783	0.4438	0.5155	0.5614	20
21	0.2914	0.3687	0.4329	0.5034	0.5487	21
22	0.2841	0.3598	0.4227	0.4921	0.5368	22
23	0.2774	0.3515	0.4132	0.4815	0.5256	23
24	0.2711	0.3438	0.4044	0.4716	0.5151	24
25	0.2653	0.3365	0.3961	0.4622	0.5052	25
26	0.2598	0.3297	0.3882	0.4534	0.4958	26
27	0.2546	0.3233	0.3809	0.4451	0.4869	27
28	0.2497	0.3172	0.3739	0.4372	0.4785	28
29	0.2451	0.3115	0.3673	0.4297	0.4705	29
30	0.2407	0.3061	0.3610	0.4226	0.4629	30
31	0.2366	0.3009	0.3550	0.4158	0.4556	31
32	0.2327	0.2960	0.3494	0.4093	0.4487	32
33	0.2289	0.2913	0.3440	0.4032	0.4421	33
34	0.2254	0.2869	0.3388	0.3972	0.4357	34
35	0.2220	0.2826	0.3338	0.3916	0.4296	35
36	0.2187	0.2785	0.3291	0.3862	0.4238	36
37	0.2156	0.2746	0.3246	0.3810	0.4182	37
38	0.2126	0.2709	0.3202	0.3760	0.4128	38
39	0.2097	0.2673	0.3160	0.3712	0.4076	39
40	0.2070	0.2638	0.3120	0.3665	0.4026	40
41	0.2043	0.2605	0.3081	0.3621	0.3978	41
42	0.2018	0.2573	0.3044	0.3578	0.3932	42
43	0.1993	0.2542	0.3008	0.3536	0.3887	43
44	0.1970	0.2512	0.2973	0.3496	0.3843	44
45	0.1947	0.2483	0.2940	0.3457	0.3801	45
46	0.1925	0.2455	0.2907	0.3420	0.3761	46
47	0.1903	0.2429	0.2876	0.3384	0.3721	47
48	0.1883	0.2403	0.2845	0.3348	0.3683	48
49	0.1863	0.2377	0.2816	0.3314	0.3646	49
50	0.1843	0.2353	0.2787	0.3281	0.3610	50
60	0.1678	0.2144	0.2542	0.2997	0.3301	60
70	0.1550	0.1982	0.2352	0.2776	0.3060	70
80	0.1448	0.1852	0.2199	0.2597	0.2864	80
90	0.1364	0.1745	0.2072	0.2449	0.2702	90
100	0.1292	0.1654	0.1966	0.2324	0.2565	100

TABLE 9 CRITICAL VALUES OF SPEARMAN'S RANK CORRELATION COEFFICIENT

The table gives the critical values, for different significance levels, of Spearman's rank correlation coefficient, r_s, for varying sample sizes, n.

Since r_s is discrete, exact significance levels cannot be obtained in most cases. The critical values given are those with significance levels closest to the stated value.

One tail Two tail	10% 20%	5% 10%	2.5% 5%	1% 2%	0.5% 1%	One tail Two tail
n						n
4	1.0000	1.0000	1.0000	1.0000	1.0000	4
5	0.7000	0.9000	0.9000	1.0000	1.0000	5
6	0.6571	0.7714	0.8286	0.9429	0.9429	6
7	0.5714	0.6786	0.7857	0.8571	0.8929	7
8	0.5476	0.6429	0.7381	0.8095	0.8571	8
9	0.4833	0.6000	0.6833	0.7667	0.8167	9
10	0.4424	0.5636	0.6485	0.7333	0.7818	10
11	0.4182	0.5273	0.6091	0.7000	0.7545	11
12	0.3986	0.5035	0.5874	0.6713	0.7273	12
13	0.3791	0.4780	0.5604	0.6484	0.6978	13
14	0.3670	0.4593	0.5385	0.6220	0.6747	14
15	0.3500	0.4429	0.5179	0.6000	0.6536	15
16	0.3382	0.4265	0.5029	0.5824	0.6324	16
17	0.3271	0.4124	0.4821	0.5577	0.6055	17
18	0.3170	0.4000	0.4683	0.5425	0.5897	18
19	0.3077	0.3887	0.4555	0.5285	0.5751	19
20	0.2992	0.3783	0.4438	0.5155	0.5614	20
21	0.2914	0.3687	0.4329	0.5034	0.5487	21
22	0.2841	0.3598	0.4227	0.4921	0.5368	22
23	0.2774	0.3515	0.4132	0.4815	0.5256	23
24	0.2711	0.3438	0.4044	0.4716	0.5151	24
25	0.2653	0.3365	0.3961	0.4622	0.5052	25
26	0.2598	0.3297	0.3882	0.4534	0.4958	26
27	0.2546	0.3233	0.3809	0.4451	0.4869	27
28	0.2497	0.3172	0.3739	0.4372	0.4785	28
29	0.2451	0.3115	0.3673	0.4297	0.4705	29
30	0.2407	0.3061	0.3610	0.4226	0.4629	30
31	0.2366	0.3009	0.3550	0.4158	0.4556	31
32	0.2327	0.2960	0.3494	0.4093	0.4487	32
33	0.2289	0.2913	0.3440	0.4032	0.4421	33
34	0.2254	0.2869	0.3388	0.3972	0.4357	34
35	0.2220	0.2826	0.3338	0.3916	0.4296	35
36	0.2187	0.2785	0.3291	0.3862	0.4238	36
37	0.2156	0.2746	0.3246	0.3810	0.4182	37
38	0.2126	0.2709	0.3202	0.3760	0.4128	38
39	0.2097	0.2673	0.3160	0.3712	0.4076	39
40	0.2070	0.2638	0.3120	0.3665	0.4026	40
41	0.2043	0.2605	0.3081	0.3621	0.3978	41
42	0.2018	0.2573	0.3044	0.3578	0.3932	42
43	0.1993	0.2542	0.3008	0.3536	0.3887	43
44	0.1970	0.2512	0.2973	0.3496	0.3843	44
45	0.1947	0.2483	0.2940	0.3457	0.3801	45
46	0.1925	0.2455	0.2907	0.3420	0.3761	46
47	0.1903	0.2429	0.2876	0.3384	0.3721	47
48	0.1883	0.2403	0.2845	0.3348	0.3683	48
49	0.1863	0.2377	0.2816	0.3314	0.3646	49
50	0.1843	0.2353	0.2787	0.3281	0.3610	50
60	0.1678	0.2144	0.2542	0.2997	0.3301	60
70	0.1550	0.1982	0.2352	0.2776	0.3060	70
80	0.1448	0.1852	0.2199	0.2597	0.2864	80
90	0.1364	0.1745	0.2072	0.2449	0.2702	90
100	0.1292	0.1654	0.1966	0.2324	0.2565	100

TABLE 10 CRITICAL VALUES OF THE WILCOXON SIGNED RANK STATISTIC

The table gives the lower tail critical values of the statistic T.

The upper tail critical values are given by $\frac{1}{2}n(n+1)-T$.

T is the sum of the ranks of observations with the same sign.
Since T is discrete, exact significance levels cannot usually be obtained.
The critical values tabulated are those with significance levels closest to the stated value.
The critical region includes the tabulated value.

One tail	**10%**	**5%**	**2.5%**	**1%**	**0.5%**
Two tail	**20%**	**10%**	**5%**	**2%**	**1%**
n					
3	0				
4	1	0			
5	2	1	0		
6	4	2	1	0	
7	6	4	2	0	0
8	8	6	4	2	0
9	11	8	6	3	2
10	14	11	8	5	3
11	18	14	11	7	5
12	22	17	14	10	7
13	26	21	17	13	10
14	31	26	21	16	13
15	37	30	25	20	16
16	42	36	30	24	19
17	49	41	35	28	23
18	55	47	40	33	28
19	62	54	46	38	32
20	70	60	52	43	37

TABLE 11 CRITICAL VALUES OF THE MANN–WHITNEY STATISTIC

The table gives the lower tail critical values of the statistic U.
The upper tail critical values are given by $mn - U$.

$U = T - \frac{n(n+1)}{2}$ where T is the sum of the ranks of the sample of size n.

Since U is discrete, exact significance levels cannot be obtained.
The critical values tabulated are those with significance levels closest to the stated value.
The critical region includes the tabulated value.

One tail 5% Two tail 10%

m	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
n											
2		0	0	0	0	1	1	1	2	2	2
3	0	0	1	1	2	3	3	4	5	5	6
4	0	1	2	3	4	5	6	7	8	9	10
5	0	1	3	4	5	7	8	10	11	12	14
6	0	2	4	5	7	9	11	12	14	16	18
7	1	3	5	7	9	11	13	15	18	20	22
8	1	3	6	8	11	13	16	18	21	24	26
9	1	4	7	10	12	15	18	21	24	27	30
10	2	5	8	11	14	18	21	24	28	31	34
11	2	5	9	12	16	20	24	27	31	35	39
12	2	6	10	14	18	22	26	30	34	39	43

One tail 2.5% Two tail 5%

m	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
n											
2				0	0	0	0	0	1	1	1
3			0	0	1	2	2	3	3	4	4
4		0	1	2	2	3	4	5	6	7	8
5	0	0	2	3	4	5	6	7	9	10	11
6	0	1	2	4	5	7	8	10	12	13	15
7	0	2	3	5	7	9	11	13	15	17	18
8	0	2	4	6	8	11	13	15	18	20	22
9	0	3	5	7	10	13	15	18	21	23	26
10	1	3	6	9	12	15	18	21	24	27	30
11	1	4	7	10	13	17	20	23	27	30	34
12	1	4	8	11	15	18	22	26	30	34	38

TABLE 12 CONTROL CHARTS FOR VARIABILITY

For range charts, multiply σ by the appropriate value of D.
For standard deviation charts, multiply σ by the appropriate value of E.
To obtain an estimate of σ, multiply the mean range by the appropriate value of b.
Normal distribution is assumed.

Sample size	$D_{0.999}$	$D_{0.975}$	$D_{0.025}$	$D_{0.001}$	$E_{0.999}$	$E_{0.975}$	$E_{0.025}$	$E_{0.001}$	b
2			3.170	4.654			2.24	3.29	0.8862
3	0.060	0.303	3.682	5.063	0.03	0.16	1.92	2.63	0.5908
4	0.199	0.595	3.984	5.309	0.09	0.27	1.76	2.33	0.4857
5	0.367	0.850	4.197	5.484	0.15	0.35	1.67	2.15	0.4299
6	0.535	1.066	4.361	5.619	0.20	0.41	1.60	2.03	0.3946
7	0.691	1.251	4.494	5.730	0.25	0.45	1.55	1.93	0.3698
8	0.835	1.410	4.605	5.823	0.29	0.49	1.51	1.86	0.3512
10	1.085	1.674	4.784	5.973	0.36	0.55	1.45	1.76	0.3249
12	1.293	1.884	4.925	6.096	0.41	0.59	1.41	1.69	0.3069

TABLE 13 RANDOM NUMBERS

29927	66187	80784	37542	62446	13481	72730	48511	42315
62506	22780	30720	79338	68358	62765	33401	82758	42929
83752	10664	12193	88766	76763	90977	46881	59089	39648
75703	27522	79504	06662	25468	92407	19626	61173	52793
99617	59120	33554	32904	95312	61763	68868	94179	73442
17490	64900	12690	95474	53849	64791	35843	44832	01296
02384	16680	76637	42437	27994	24718	09566	43821	89315
44031	51668	85907	22683	06119	25360	35480	91334	01522
94058	36466	99717	57651	02512	98785	86491	76812	10324
88783	77127	95783	40666	82539	84224	94354	41979	32823
33380	47444	02936	57303	31458	28669	22538	66884	38370
95198	41684	89066	17963	39042	50791	44683	15134	19909
03183	62706	65531	47767	42347	51899	33582	28098	43168
26623	00550	52329	90292	37508	97310	92049	47365	80242
57929	14290	08118	95473	91586	58953	74998	73950	54662
49103	92150	78211	27762	18135	43479	61698	77768	00223
98634	31870	56839	60478	62129	87149	60240	09079	38567
90593	76248	60379	98204	59254	51616	41091	11818	11001
68604	90298	38595	52048	95137	73363	53307	37914	27903
72711	43441	87108	82155	43650	81967	56348	19878	75813
08193	05302	11352	48369	55731	81158	21037	29534	98074
74829	51695	51682	97660	97110	69540	69776	22736	54635
25900	81265	25339	43875	38563	43530	36289	78810	18959
42417	50106	24752	94664	11611	05720	77091	96338	68507
32291	57653	42135	36440	79427	11660	15666	55682	25449
24096	57419	35611	91179	51464	94284	92449	97347	22184
50344	22824	09193	98771	30963	02876	97671	56397	91677
76672	52872	48610	31314	21545	23601	18278	93530	02142
50385	70112	26897	00077	04803	98326	88933	17710	75750
64222	95920	80534	55090	04105	01415	11376	20709	78887
11602	06891	07924	42959	73124	36830	70559	55739	73191
87962	92071	13405	05057	85947	73043	94208	52829	88272
41595	07611	36646	70863	57797	82033	19236	74608	14324
34917	58038	47230	38817	70605	62771	02851	23195	20204
50622	76133	54065	34055	13961	07604	30260	92240	40736
14422	58282	73673	04535	03557	40036	85475	16021	77173
48254	71043	44942	12252	59557	53013	26170	21980	18582
59322	65251	84379	05985	45765	38349	68661	18129	29338
04224	19593	72554	54239	44870	38726	51297	82412	65799
17264	41154	16019	70481	97716	53185	53901	89036	01253
09632	07182	78111	19253	12414	73496	24090	54974	48941
54282	74626	40866	91371	44589	31478	58842	71961	38487
80207	43497	37079	53974	20241	62576	15660	68405	57982
93899	94309	56732	59858	28457	74546	45424	92496	71035
46869	52284	00000	42554	58770	83458	58425	60956	21595