Loop The Loop

In 2009 stuntman Steve Truglia attempted to drive a Toyota Aygo through a $12m$ diameter loop-the-loop for the show Fifth Gear.

By modelling the car as a particle in contact with the inside surface of a cylinder, answer the following questions:

Calculate the minimum speed required at the bottom for the vehicle to remain in contact with the loop all the way around. *Note: ignore the effect of continued forwards acceleration from the car – we will initially assume that the only forces acting on the car during the motion are its weight and the normal reaction from the track.*

What is the maximum normal reaction force Steve will experience?

Since in reality energy will be lost as a result of work done against friction and air resistance, calculate the greatest speed he could enter the loop at such that his reaction force at that point is $8mg$ (more or less the greatest he could cope with outside a G-suit), and calculate his speed at a height of h (giving your answer in terms of h).

Calculate the normal reaction he experiences in terms of h and hence calculate the minimum normal reaction.

Loop The Loop SOLUTIONS

In 2009 stuntman Steve Truglia attempted to drive a Toyota Aygo through a $12m$ diameter loop-the-loop for the show Fifth Gear.

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Resolving forces radially at the top:

$$mg = \frac{mv^2}{r} \quad \Rightarrow \quad mg = \frac{mv^2}{6} \quad \Rightarrow \quad v = \sqrt{6g} \approx 7.67ms^{-1} \text{ at the top}$$

Using conservation of energy between the bottom and the top:

$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mgh \quad \Rightarrow \quad \frac{1}{2}mu^2 = \frac{1}{2}m(6g) + mg(12)$$

$$\Rightarrow \quad u = \sqrt{\mathbf{30g}} \approx \mathbf{17.1ms^{-1}} \approx \mathbf{38.4mph \text{ at the bottom}}$$

What is the maximum normal reaction force Steve will experience?

Maximum reaction force occurs at the bottom of the loop (both on the way up and on the way down) since the normal reaction must counteract weight as well as provide a centripetal force large enough for the large speed. Resolving radially at the bottom:

$$R - mg = \frac{mu^2}{r} \quad \Rightarrow \quad R = mg + \frac{m(30g)}{6} = \mathbf{6mg}$$

That is, a force whose associated acceleration is $6g$. Note that a bungee jumper will rarely experience more than $2g$ or $3g$ of acceleration, and a fighter pilot could experience $10g$, but will be equipped with a special suit (and chair) to deal with this.

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Resolving forces at the bottom:

$$R - mg = \frac{mu^2}{r} \quad \Rightarrow \quad 7mg = \frac{mu^2}{6} \quad \Rightarrow \quad u = \sqrt{42g}$$

Using conservation of energy:

$$\frac{1}{2}mu^2 = mgh + \frac{1}{2}mv^2 \quad \Rightarrow \quad 21g = gh + \frac{1}{2}v^2 \quad \Rightarrow \quad \boldsymbol{v = \sqrt{2g(21-h)}}$$

Calculate the normal reaction he experiences in terms of h and hence calculate the minimum normal reaction.

Calculating the cosine of the angle from the downward vertical at height h:

$$h = r - r\cos\theta \quad \Rightarrow \quad h = 6(1-\cos\theta) \quad \Rightarrow \quad \cos\theta = 1 - \frac{h}{6}$$

Resolving forces at height h:

$$R - mg\cos\theta = \frac{mv^2}{r} \quad \Rightarrow \quad R - mg\left(1 - \frac{h}{6}\right) = \frac{2mg(21-h)}{6}$$

$$\Rightarrow \quad R = 7mg - \frac{mgh}{3} + mg - \frac{mgh}{6} = 8mg - \frac{mgh}{2} = \boldsymbol{mg\left(8 - \frac{h}{2}\right)}$$

Minimum reaction occurs at the top:

$$h = 12 \quad \Rightarrow \quad R = mg\left(8 - \frac{12}{2}\right) = \mathbf{2mg}$$