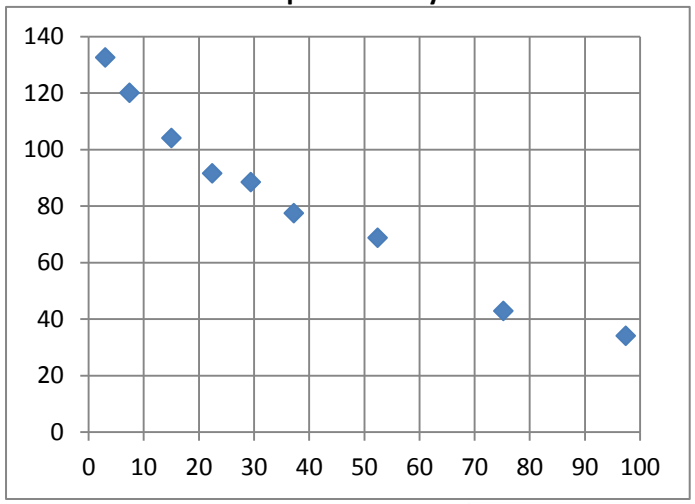
Determining an exponential relationship using linear laws.

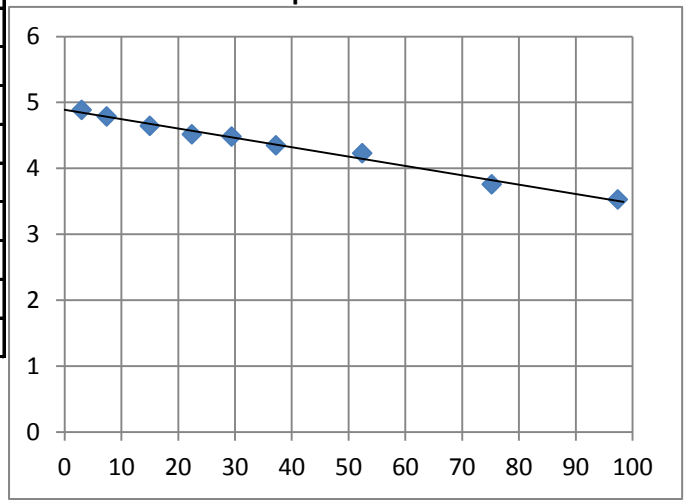
x	y
3	132.6
7.4	120.1
15	104.1
22.4	91.6
29.4	88.5
37.2	77.5
52.4	68.8
75.2	42.9
97.4	34.1

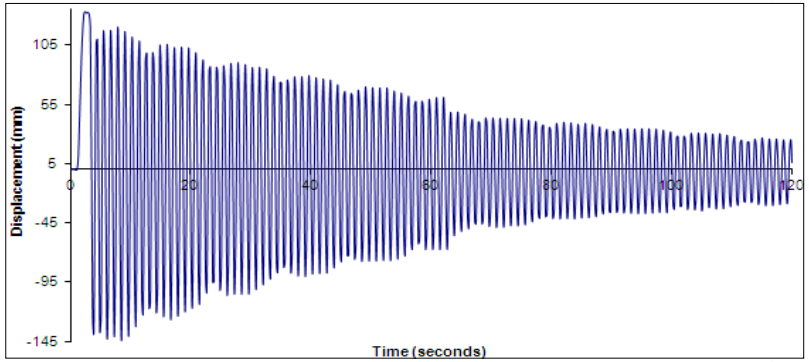
Graph 1: x and y



X	Y
3	4.887
7.4	4.788
15	4.645
22.4	4.517
29.4	4.483
37.2	4.35
52.4	4.231
75.2	3.759
97.4	3.529

Graph 2: X and Y





These data are taken from an A-level Physics experiment on damped oscillations. The variable x represents time in seconds, y displacement in mm.
Each point marked is an observed maximum, selected from the graph above.

$$y = ke^{cx}$$

$$\ln y = \ln ke^{cx}$$

$$\ln y = \ln k + \ln e^{cx}$$

$$\mathbf{\ln y = \ln k + cx}$$

$$Y = \ln y \quad X = x \quad a = \ln k \quad b = c \quad \Longrightarrow \quad Y = a + bX$$

The variables have been altered as shown above. Using your knowledge of straight line graphs, determine the equation of the line of best fit, and hence the values of a and b. Use this to write an equation for the original relationship between x and y.

Solutions

Note: in answer to the question 'is there an exponential relationship?', the straight line graph has an R-squared value of 0.9893, indicating strong positive correlation.

The equation of the straight line (using linear regression techniques) is given by
y = 4.8891 - 0.0142x.
Students should use data from two specific points to determine their best fit line equation, so it may vary.
Using a = ln(k) and b = c gives values for k and c of 132.83 and -0.0142 respectively.

This will give a relationship between x and y as: $y = 132.83e^{-0.0142x}$