Key Results Quiz – Further Maths

For each question, you need to write out the relevant mathematical result or formula from memory. You will gain a point for each correct answer, and lose a point for each incorrect answer!

Do I have to do this without my textbook?
Yes! – this exercise is designed to prepare you for your exams, so don't use your notes, don't look at anyone else's answers, and don't rely on your textbook!

Is it ok if the formula is right but written differently?
Yes! – although there are often standard conventions for common results, as long as your result is mathematically valid and useable, you'll still get the mark.

Can I use different variable names?
Yes! – provided the meaning is clear. If you use non-standard variable names, you may need to state the meaning of each variable to get the mark.

If I can't remember it, can I just work it out?
Yes! – you're welcome to derive results if you know how, although not all results are easily provable, and there won't be time to do this for every single one!

Can I use my calculator to help?
Yes! – since you'll have your calculator in the exam, feel free to use it to check your results where possible, although you won't have time to test every single one!

If it's provided in the formula book, can I skip it?
Yes! – if you know that you can look it up in the exam, just write 'formula book' or 'FB' and you still get a point.

Further Maths:

	Core Pure 1 and Core Pure 2	Further Pure 1
Correct (+1):		
Incorrect (−1):		
Blank (0):		
Total:	(max 123)	(max 46)

Use this topic list to make a note of any specific areas for development that you identify:

Core Pure 1
- ☐ CP1ch1 Complex numbers
- ☐ CP1ch2 Argand diagrams
- ☐ CP1ch3 Series
- ☐ CP1ch4 Roots of polynomials
- ☐ CP1ch5 Volumes of revolution
- ☐ CP1ch6 Matrices
- ☐ CP1ch7 Linear transformations
- ☐ CP1ch8 Proof by induction
- ☐ CP1ch9 Vectors

Core Pure 2
- ☐ CP2ch1 Complex numbers
- ☐ CP2ch2 Series
- ☐ CP2ch3 Methods in calculus
- ☐ CP2ch4 Volumes of revolution
- ☐ CP2ch5 Polar coordinates
- ☐ CP2ch6 Hyperbolic functions
- ☐ CP2ch7 Methods in differential equations
- ☐ CP2ch8 Modelling with differential equations

Further Pure 1
- ☐ FP1ch1 Vectors
- ☐ FP1ch2 Conic sections 1
- ☐ FP1ch3 Conic sections 2
- ☐ FP1ch4 Inequalities
- ☐ FP1ch5 The t-formulae
- ☐ FP1ch6 Taylor series
- ☐ FP1ch7 Methods in calculus
- ☐ FP1ch8 Numerical methods
- ☐ FP1ch9 Reducible differential equations

Core Pure 1 and Core Pure 2

1	What is the definition of the imaginary unit i?	
2	Write down the general rectangular (Cartesian) form for a complex number z.	
3	How would you compute the complex conjugate, z^*, of a given complex number z?	
4	If the discriminant of a quadratic is negative, what does that tell us about its roots?	
5	What must always be true regarding the number of roots a polynomial of order n has?	
6	What can be said about the complex roots (if any) of a polynomial with real coefficients?	
7	What is the name of the diagram used to represent complex numbers, and what do its axes represent?	
8	What is the modulus of the complex number $z = a + bi$?	
9	What is the argument of the complex number $z = a + bi$?	
10	What is the polar (modulus-argument) form of the complex number z with $\|z\| = r$, $\arg z = \theta$?	
11	What results can be used to find the modulus and argument of two complex numbers which are multiplied together?	
12	What does $\|z - w\|$ represent, geometrically, with respect to complex numbers z and w?	
13	Describe the locus of points z in the complex plane which satisfy the condition: $\|z - (a + bi)\| = k$	
14	Describe the locus of points z in the complex plane which satisfy the condition: $\|z - (a + bi)\| = \|z - (c + di)\|$	
15	Describe the locus of points z in the complex plane which satisfy the condition: $\arg\big(z - (a + bi)\big) = \alpha$	

16	What is the sum of the first n natural numbers, $1 + 2 + 3 + \cdots + n$? Give your answer in sigma notation.	
17	What is the sum of the squares of the first n natural numbers, $1^2 + 2^2 + 3^2 + \cdots + n^2$? Give your answer in sigma notation.	
18	What is the sum of the cubes of the first n natural numbers, $1^3 + 2^3 + 3^3 + \cdots + n^3$? Give your answer in sigma notation.	
19	Write down Vieta's results for the roots α, β, etc of a polynomial $ax^n + bx^{n-1} + \cdots = 0$.	
20	Write $\Sigma\frac{1}{\alpha}$ in terms of Vieta's results for a cubic equation.	
21	Write $\Sigma\frac{1}{\alpha}$ in terms of Vieta's results for a quartic equation.	
22	Write $\Sigma\alpha^2$ in terms of Vieta's results for a quadratic, cubic or quartic equation.	
23	Write $\Sigma\alpha^3$ in terms of Vieta's results for a quadratic equation.	
24	Write $\Sigma\alpha^3$ in terms of Vieta's results for a cubic equation.	
25	What is the integral for computing the volume of a solid of revolution formed by rotating a curve $y = f(x)$ one full revolution about the x-axis from $x = a$ to $x = b$?	
26	What is the integral for computing the volume of a solid of revolution formed by rotating a curve $y = f(x)$ one full revolution about the y-axis from $y = a$ to $y = b$?	
27	How is the integral for a volume of revolution modified for parametric equations $x = f(t), y = g(t)$, when rotating about the x-axis?	
28	How is the integral for a volume of revolution modified for parametric equations $x = f(t), y = g(t)$, when rotating about the x-axis?	

29	Write down the 2 by 2 zero matrix.	
30	Write down the 2 by 2 identity matrix.	
31	Write down the matrix I_3.	
32	How do you add two matrices?	
33	How do you multiply two matrices?	
34	Under what conditions can an n by m matrix be multiplied by a p by q matrix?	
35	What is the determinant of a 2 by 2 matrix?	
36	What is the test to determine if a matrix is 'singular'?	
37	What are the 'minors' of a 3 by 3 matrix?	
38	What are the 'cofactors' of a 3 by 3 matrix?	
39	What is the determinant of a 3 by 3 matrix?	
40	What is the inverse of a 3 by 3 matrix? You don't need to redefine minors, cofactors or determinant.	
41	What is the key defining feature of an inverse matrix (of any size)?	
42	How can an inverse matrix be used to solve a linear system of equations in x, y and z?	
43	What are the possible arrangements of three planes characterised by an *inconsistent* system of simultaneous equations?	

44	What are the possible arrangements of three planes characterised by a *consistent* system of simultaneous equations?	
45	What 2 by 2 matrix represents a reflection in the y-axis?	
46	What 2 by 2 matrix represents a reflection in the x-axis?	
47	What 2 by 2 matrix represents a reflection in the line $y = x$?	
48	What 2 by 2 matrix represents a reflection in the line $y = -x$?	
49	What 2 by 2 matrix represents a reflection in the line $y = mx$?	
50	What 2 by 2 matrix represents a rotation by $90°$ anticlockwise about the origin?	
51	What 2 by 2 matrix represents a rotation by $90°$ clockwise about the origin?	
52	What 2 by 2 matrix represents a rotation by $180°$ about the origin?	
53	What 2 by 2 matrix represents a rotation by $\theta°$ anticlockwise about the origin?	
54	What 2 by 2 matrix represents a stretch in the x direction by scale factor a and a stretch in the y direction by scale factor b?	
55	What is the geometric interpretation of the determinant of a 2 by 2 matrix, when it represents a transformation?	
56	If matrices P and Q represent linear transformations P and Q, what matrix represents the transformation produced by applying P followed by Q?	
57	Write down the 3 by 3 matrix that represents a reflection in the y-z plane ($x = 0$).	
58	Write down the 3 by 3 matrix that represents a reflection in the x-z plane ($y = 0$).	

59	Write down the 3 by 3 matrix that represents a reflection in the x-y plane ($z = 0$).	
60	Write down the 3 by 3 matrix that represents a rotation by θ anticlockwise about the x-axis ($y = z = 0$).	
61	Write down the 3 by 3 matrix that represents a rotation by θ anticlockwise about the y-axis ($x = z = 0$).	
62	Write down the 3 by 3 matrix that represents a rotation by θ anticlockwise about the z-axis ($x = y = 0$).	
63	What are the key steps of an inductive proof?	
64	What is the parametric ('vector') equation of a line passing through the point with position vector $\boldsymbol{a}$ which is parallel to vector $\boldsymbol{b}$?	
65	What is the Cartesian equation of a line in 3D space with position vector $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and direction vector $\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$?	
66	What is the parametric ('vector') equation of a plane passing through the point with position vector $\boldsymbol{a}$ which is parallel to vectors $\boldsymbol{b}$ and $\boldsymbol{c}$?	
67	What is the Cartesian equation of a plane in 3D space with normal vector $\begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}$ passing through the point with position vector $\boldsymbol{a}$?	
68	What is the definition of the dot ('scalar') product between two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$?	
69	What is the scalar product ('dot product') form of the vector equation for a plane with normal vector $\boldsymbol{n}$ passing through point $\boldsymbol{a}$?	
70	How would you find the angle between two lines?	
71	How would you find the angle between two planes?	

72	How would you find the angle between a line and a plane?	
73	How would you show that two lines are skew?	
74	What is the perpendicular (shortest) distance from O to the plane $\boldsymbol{r} \cdot \hat{\boldsymbol{n}} = k$?	
75	What is the perpendicular (shortest) distance from the plane $\begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = d$ to the point (α, β, γ)?	
76	What is Euler's relation, connecting the trigonometric form of a complex number to the exponential form?	
77	What does De Moivre's theorem tell us about complex numbers in mod-arg form?	
78	What result links $\cos\theta$ to z, the complex number with modulus 1 and argument θ?	
79	What result links $\sin\theta$ to z, the complex number with modulus 1 and argument θ?	
80	What result links $\cos n\theta$ to z, the complex number with modulus 1 and argument θ?	
81	What result links $\sin n\theta$ to z, the complex number with modulus 1 and argument θ?	
82	What are the n^{th} roots of unity, in exponential form?	
83	What is the general Maclaurin series formula?	
84	What is the Maclaurin series expansion for e^x?	
85	What is the Maclaurin series expansion for $\ln(1+x)$?	
86	What is the Maclaurin series expansion for $\sin x$?	
87	What is the Maclaurin series expansion for $\cos x$?	
88	What is the Maclaurin series expansion for $\arctan x$?	

89	What are the conditions for an integral to be improper?	
90	What is the mean value of the function $f(x)$ between $x = a$ and $x = b$?	
91	What is the derivative of $\arcsin x$?	
92	What is the derivative of $\arccos x$?	
93	What is the derivative of $\arctan x$?	
94	What is the integral of $\frac{1}{a^2+x^2}$?	
95	What is the integral of $\frac{1}{\sqrt{a^2-x^2}}$?	
96	What results connect the Cartesian coordinates (x, y) to the Polar coordinates (r, θ)?	
97	What is the integral required to calculate the area of a sector from a polar curve?	
98	How would you find the points on a polar curve where a tangent is parallel to the initial line?	
99	How would you find the points on a polar curve where the tangent is perpendicular to the initial line?	
100	What is the exponential definition of the hyperbolic cosine $\cosh x$?	
101	What is the exponential definition of the hyperbolic sine $\sinh x$?	
102	What is the exponential definition of the hyperbolic tangent $\tanh x$?	
103	What are the domain and range of $\cosh x$?	
104	What are the domain and range of $\sinh x$?	
105	What is the equivalent of the Pythagorean identity $\cos^2\theta + \sin^2\theta \equiv 1$ for hyperbolic trig functions?	
106	What is the equivalent of the $\sin 2\theta$ double angle formula for $\sinh 2x$?	
107	What is the equivalent of the $\cos 2\theta$ double angle formula for $\cosh 2x$?	

108	What is the exact logarithmic form of the inverse function $\operatorname{arcosh} x$?	
109	What is the exact logarithmic form of the inverse function $\operatorname{arsinh} x$?	
110	What is the exact logarithmic form of the inverse function $\operatorname{artanh} x$?	
111	What is the derivative of $\sinh x$?	
112	What is the derivative of $\cosh x$?	
113	What is the derivative of $\tanh x$?	
114	What is the derivative of $\operatorname{arsinh} x$?	
115	What is the derivative of $\operatorname{arcosh} x$?	
116	What is the derivative of $\operatorname{artanh} x$?	
115	What is the general format for a first order differential equation that can be solved using the separation of variables method? And how would we solve the equation?	
116	What is the general format for a first order differential equation that can be solved using the Integrating Factor method? And how would we solve the equation?	
117	What is the general format for a homogeneous second order (linear, constant coefficient) differential equation? And what is the first step to solving it?	

118	What are the three possible cases for the general solution of a homogeneous 2nd order differential equation?	
119	When a homogeneous second order differential equation (for displacement x in terms of time t) is used to model simple harmonic motion, what type of simple harmonic motion is being exhibited for each of the three possible cases?	
120	What additional steps are required when the second order differential equation is non-homogeneous?	
121	When a non-homogeneous second order differential equation is used to model simple harmonic motion, what is the nature of the simple harmonic motion?	
122	What is the standard format of the second order differential equation for damped harmonic motion, in terms of k, the constant of resistance, and ω, the angular velocity?	
123	What is the method for solving coupled first order differential equations?	

Further Pure 1

1	What is the fundamental definition of the vector ('cross') product $\boldsymbol{u} \times \boldsymbol{v}$?	
2	What is the relationship between $\boldsymbol{u} \times \boldsymbol{v}$ and $\boldsymbol{v} \times \boldsymbol{u}$?	
3	What is the cross product of two parallel vectors?	
4	How would you calculate the cross product of two 3D vectors $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$?	
5	What is the geometric interpretation of the size of the cross product $\lvert \boldsymbol{u} \times \boldsymbol{v} \rvert$?	
6	What is the geometric interpretation of the direction of the cross product $\boldsymbol{u} \times \boldsymbol{v}$?	
7	How is the scalar triple product of vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ calculated?	
8	What is the geometric interpretation of the scalar triple product?	
9	How would you calculate the volume of a tetrahedron whose corners have position vectors $\boldsymbol{0}$, $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$?	
10	What is the cross product form of a vector line equation with position vector $\boldsymbol{a}$ and direction vector $\boldsymbol{b}$?	
11	What are the direction cosines of the vector $\boldsymbol{v} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$?	
12	What is the geometric interpretation of the direction cosines of a vector?	
13	What is one additional property of the direction cosines?	

14	How would you calculate the shortest distance between two skew lines, using the cross product?	
15	What are the standard and parametric forms of the equation of a parabola?	
16	What are the standard and parametric forms of the equation of an ellipse?	
17	What are the standard and parametric forms of the equation of a hyperbola? Give both the trigonometric and hyperbolic forms of the parametrization.	
18	What are the standard and parametric forms of the equation of a rectangular hyperbola?	
19	What is the relationship between the distance of a point P from the focus S and from the directrix D, for any conic?	
20	What methods can be used to determine the gradient of a point on a conic?	
21	What is the parametric equation of a tangent to the standard parabola?	
22	What is the parametric equation of a normal to the standard parabola?	
23	What is the parametric equation of a tangent to the standard rectangular hyperbola?	
24	What is the parametric equation of a normal to the standard rectangular hyperbola?	
25	What is the parametric equation of a tangent to the standard ellipse?	
26	What is the parametric equation of a normal to the standard ellipse?	
27	What is the parametric equation of a tangent to the standard hyperbola?	

28	What is the parametric equation of a normal to the standard hyperbola?	
29	What is the range of possible values for the eccentricity of: a circle, ellipse, parabola, hyperbola and rectangular hyperbola?	
30	How is eccentricity calculated for an ellipse provided in the standard format?	
31	How is eccentricity calculated for a hyperbola provided in the standard format?	
32	Where are the foci and directrices for an ellipse provided in the standard format?	
33	Where are the foci and directrices for a hyperbola provided in the standard format?	
34	What key step is usually required to deal with fractional inequalities, and why is it necessary?	
35	How would you go about solving an inequality involving the modulus function?	
36	What are the t-formulae for $\sin\theta$, $\cos\theta$ and $\tan\theta$ when $t = \tan\left(\frac{\theta}{2}\right)$?	
37	What is the Taylor series expansion formula for $f(x)$ in ascending powers of $(x-a)$, centred at $x = a$?	
38	What is the Taylor series expansion formula for $f(x+a)$ in ascending powers of x, centred at $x = 0$?	

39	State Leibnitz' theorem.	
40	State L'Hospital's rule.	
41	In addition to the general result $\lim_{x\to a}\big(Af(x)+Bg(x)\big)=A\lim_{x\to a}f(x)+B\lim_{x\to a}g(x)$ what other results govern the behaviour of limits?	
42	What is Euler's method?	
43	What is the midpoint method?	
44	What is Euler's method for second order differential equations?	
45	What is Simpson's rule? You may use abbreviations provided your intention is clear.	
46	When using a substitution to reduce a differential equation to a solvable form, what is the first thing you need to look for?	

SOLUTIONS Key Results Quiz SOLUTIONS

Core Pure 1 and Core Pure 2

1	What is the definition of the imaginary unit i?	$i = \sqrt{-1}$
2	Write down the general rectangular (Cartesian) form for a complex number Type equation here..	$z = a + bi \quad a, b \in \mathbb{R}$
3	How would you compute the complex conjugate, z^*, of a given complex number z?	$z = a + bi \implies z^* = a - bi$
4	If the discriminant of a quadratic is negative, what does that tell us about its roots?	They are complex conjugates.
5	What must always be true regarding the number of roots a polynomial of order n has?	A polynomial of order n has n roots (not necessarily distinct, or real).
6	What can be said about the complex roots (if any) of a polynomial with real coefficients?	Complex roots always occur in conjugate pairs: if z is a root, so is z^*.
7	What is the name of the diagram used to represent complex numbers, and what do its axes represent?	An 'Argand' diagram, with a horizontal real axis (Re) and vertical imaginary axis (Im).
8	What is the modulus of the complex number $z = a + bi$?	The size of z: $\lvert z\rvert = \sqrt{a^2 + b^2}$
9	What is the argument of the complex number $z = a + bi$?	The angle from the positive real axis to the vector representing the complex number, measured anti-clockwise. For $a, b > 0$, $\arg(a + bi) = \operatorname{atan}\left(\frac{b}{a}\right)$. For points in other quadrants, a diagram can be used to adjust.
10	What is the polar (modulus-argument) form of the complex number z with $\lvert z\rvert = r$, $\arg z = \theta$?	$z = r(\cos\theta + i\sin\theta)$
11	What results can be used to find the modulus and argument of two complex numbers which are multiplied together?	Multiply the moduli, add the arguments: $\lvert zw\rvert = \lvert z\rvert\lvert w\rvert$ $\arg(zw) = \arg(z) + \arg(w)$
12	What does $\lvert z - w\rvert$ represent, geometrically, with respect to complex numbers z and w?	The distance between the numbers on an Argand diagram.
13	Describe the locus of points z in the complex plane which satisfy the condition: $\lvert z - (a + bi)\rvert = k$	A circle, centred at $a + bi$, radius k.
14	Describe the locus of points z in the complex plane which satisfy the condition: $\lvert z - (a + bi)\rvert = \lvert z - (c + di)\rvert$	The perpendicular bisector of the line joining the points $a + bi$ and $c + di$. Equivalently, the line of equidistance between $a + bi$ and $c + di$.

15	Describe the locus of points z in the complex plane which satisfy the condition: $\arg(z-(a+bi))=\alpha$	The half-line (ray), starting at the point $a+bi$ at an angle of α to the positive real axis.
16	What is the sum of the first n natural numbers, $1+2+3+\cdots+n$? Give your answer in sigma notation.	$\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$ *Note: this could be derived from the S_n formula for an arithmetic series, which is in the formula book:* $S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n[2a+(n-1)d]$
17	What is the sum of the squares of the first n natural numbers, $1^2+2^2+3^2+\cdots+n^2$? Give your answer in sigma notation.	In the formula book: $\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$
18	What is the sum of the cubes of the first n natural numbers, $1^3+2^3+3^3+\cdots+n^3$? Give your answer in sigma notation.	In the formula book: $\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$
19	Write down Vieta's results for the roots α, β, etc of a polynomial $ax^n+bx^{n-1}+\cdots=0$.	$\Sigma\alpha = -\frac{b}{a}$ $\Sigma\alpha\beta = \frac{c}{a}$ $\Sigma\alpha\beta\gamma = -\frac{d}{a}$ $\Sigma\alpha\beta\gamma\delta = \frac{e}{a}$
20	Write $\Sigma\frac{1}{\alpha}$ in terms of Vieta's results for a cubic equation.	$\Sigma\frac{1}{\alpha} = \frac{\Sigma\alpha\beta}{\alpha\beta\gamma}$
21	Write $\Sigma\frac{1}{\alpha}$ in terms of Vieta's results for a quartic equation.	$\Sigma\frac{1}{\alpha} = \frac{\Sigma\alpha\beta\gamma}{\alpha\beta\gamma\delta}$
22	Write $\Sigma\alpha^2$ in terms of Vieta's results for a quadratic, cubic or quartic equation.	$\Sigma\alpha^2 = (\Sigma\alpha)^2 - 2\Sigma\alpha\beta$
23	Write $\Sigma\alpha^3$ in terms of Vieta's results for a quadratic equation.	$\Sigma\alpha^3 = (\Sigma\alpha)^3 - 3(\Sigma\alpha\beta)(\Sigma\alpha)$
24	Write $\Sigma\alpha^3$ in terms of Vieta's results for a cubic equation.	$\Sigma\alpha^3 = (\Sigma\alpha)^3 - 3(\Sigma\alpha\beta)(\Sigma\alpha) + 3\alpha\beta\gamma$
25	What is the integral for computing the volume of a solid of revolution formed by rotating a curve $y=f(x)$ one full revolution about the x-axis from $x=a$ to $x=b$?	$V = \pi\int_{x=a}^{x=b} y^2\,dx$
26	What is the integral for computing the volume of a solid of revolution formed by rotating a curve $y=f(x)$ one full revolution about the y-axis from $y=a$ to $y=b$?	$V = \pi\int_{y=a}^{y=b} x^2\,dy$

27	How is the integral for a volume of revolution modified for parametric equations $x = f(t), y = g(t)$, when rotating about the x-axis?	$V = \pi \int_{t=f^{-1}(a)}^{t=f^{-1}(b)} y^2 \frac{dx}{dt}\, dt$
28	How is the integral for a volume of revolution modified for parametric equations $x = f(t), y = g(t)$, when rotating about the x-axis?	$V = \pi \int_{t=g^{-1}(a)}^{t=g^{-1}(b)} x^2 \frac{dy}{dt}\, dt$
29	Write down the 2 by 2 zero matrix.	$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$
30	Write down the 2 by 2 identity matrix.	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
31	Write down the matrix I_3.	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$
32	How do you add two matrices?	Add corresponding elements. Commutative, so: $A + B = B + A$.
33	How do you multiply two matrices?	Each element is the sum-product of the corresponding *rows* of the first matrix and corresponding *columns* of the second matrix. Eg: $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} w & x \\ y & z \end{pmatrix} = \begin{pmatrix} aw + by & ax + bz \\ cw + dy & cx + dz \end{pmatrix}$ Not commutative, so in general $AB \neq BA$
34	Under what conditions can an n by m matrix be multiplied by a p by q matrix?	When $m = p$. That is, when the number of columns of the first matrix is equal to the number of rows of the second matrix.
35	What is the determinant of a 2 by 2 matrix?	$\left\lvert \begin{pmatrix} a & b \\ c & d \end{pmatrix} \right\rvert = ad - bc$
36	What is the test to determine if a matrix is 'singular'?	If singular, the determinant is zero. And if $det = 0$, the matrix is singular.
37	What are the 'minors' of a 3 by 3 matrix?	The determinants of the 2 by 2 matrix which remains when an element's row and column are deleted.
38	What are the 'cofactors' of a 3 by 3 matrix?	The matrix of minors, where alternate elements (in a checkerboard pattern) are multiplied by -1.
39	What is the determinant of a 3 by 3 matrix?	The sum-product of the cofactors of any row or column with the elements of that row or column.
40	What is the inverse of a 3 by 3 matrix? You don't need to redefine minors, cofactors or determinant.	The transpose of the matrix of cofactors, divided by the determinant.

41	What is the key defining feature of an inverse matrix (of any size)?	M^{-1} is such that $MM^{-1} = M^{-1}M = I$
42	How can an inverse matrix be used to solve a linear system of equations in x, y and z?	If $M\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}a\\b\\c\end{pmatrix}$ then $\begin{pmatrix}x\\y\\z\end{pmatrix} = M^{-1}\begin{pmatrix}a\\b\\c\end{pmatrix}$
43	What are the possible arrangements of three planes characterised by an *inconsistent* system of simultaneous equations?	Two or more parallel planes. No parallel planes (a 'prism').
44	What are the possible arrangements of three planes characterised by a *consistent* system of simultaneous equations?	Meet at a single point ($\det M \neq 0$). Meet at a common line (a 'sheaf'). All planes coincident.
45	What 2 by 2 matrix represents a reflection in the y-axis?	$\begin{pmatrix}-1 & 0\\0 & 1\end{pmatrix}$ *Note: you can use the formula book for reflection in $y = \tan\theta\, x$, with $\theta = \frac{\pi}{2}$.*
46	What 2 by 2 matrix represents a reflection in the x-axis?	$\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$ *Note: you can use the formula book for reflection in $y = \tan\theta\, x$, with $\theta = 0$.*
47	What 2 by 2 matrix represents a reflection in the line $y = x$?	$\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$ *Note: you can use the formula book for reflection in $y = \tan\theta\, x$, with $\theta = \frac{\pi}{4}$.*
48	What 2 by 2 matrix represents a reflection in the line $y = -x$?	$\begin{pmatrix}0 & -1\\-1 & 0\end{pmatrix}$ *Note: you can use the formula book for reflection in $y = \tan\theta\, x$, with $\theta = -\frac{\pi}{4}$.*
49	What 2 by 2 matrix represents a reflection in the line $y = mx$?	In the formula book: Reflection in the line $y = (\tan\theta)x$: $\begin{pmatrix}\cos 2\theta & \sin 2\theta\\\sin 2\theta & -\cos 2\theta\end{pmatrix}$
50	What 2 by 2 matrix represents a rotation by 90° anticlockwise about the origin?	$\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}$ *Note: you can use the formula book for rotation with $\theta = 90°$.*
51	What 2 by 2 matrix represents a rotation by 90° clockwise about the origin?	$\begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix}$ *Note: you can use the formula book for rotation by θ with $\theta = -90°$ or $270°$.*
52	What 2 by 2 matrix represents a rotation by 180° about the origin?	$\begin{pmatrix}-1 & 0\\0 & -1\end{pmatrix}$ *Note: you can use the formula book for rotation by θ with $\theta = 180°$.*
53	What 2 by 2 matrix represents a rotation by θ° anticlockwise about the origin?	From the formula book: Anticlockwise rotation through θ about O: $\begin{pmatrix}\cos\theta & -\sin\theta\\\sin\theta & \cos\theta\end{pmatrix}$
54	What 2 by 2 matrix represents a stretch in the x direction by scale factor a and a stretch in the y direction by scale factor b?	$\begin{pmatrix}a & 0\\0 & b\end{pmatrix}$

55	What is the geometric interpretation of the determinant of a 2 by 2 matrix, when it represents a transformation?	The determinant is the signed area scale factor of the corresponding transformation (eg if negative, it implies a reflection).
56	If matrices P and Q represent linear transformations P and Q, what matrix represents the transformation produced by applying P followed by Q?	QP
57	Write down the 3 by 3 matrix that represents a reflection in the y-z plane ($x = 0$).	$\begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$
58	Write down the 3 by 3 matrix that represents a reflection in the x-z plane ($y = 0$).	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$
59	Write down the 3 by 3 matrix that represents a reflection in the x-y plane ($z = 0$).	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$
60	Write down the 3 by 3 matrix that represents a rotation by θ anticlockwise about the x-axis ($y = z = 0$).	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$ *Note: the 2 by 2 rotation matrix from the formula book can be used to help construct this.*
61	Write down the 3 by 3 matrix that represents a rotation by θ anticlockwise about the y-axis ($x = z = 0$).	$\begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$ *Note: the 2 by 2 rotation matrix from the formula book can be used to help construct this.*
62	Write down the 3 by 3 matrix that represents a rotation by θ anticlockwise about the z-axis ($x = y = 0$).	$\begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$ *Note: the 2 by 2 rotation matrix from the formula book can be used to help construct this.*
63	What are the key steps of an inductive proof?	Base case: Let $n = 1$... Hypothesis: Assume true for $n = k$... Inductive step: Consider $n = k + 1$... Conclusion: Therefore, since true for $n = 1$, and if true for $n = k$ also true for $n = k + 1$, ...
64	What is the parametric ('vector') equation of a line passing through the point with position vector $\boldsymbol{a}$ which is parallel to vector $\boldsymbol{b}$?	$\boldsymbol{r} = \boldsymbol{a} + \lambda\boldsymbol{b}$
65	What is the Cartesian equation of a line in 3D space with position vector $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and direction vector $\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$?	In the formula book: If A is the point with position vector $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and the direction vector $\mathbf{b}$ is given by $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$, then the straight line through A with direction vector $\mathbf{b}$ has cartesian equation $\frac{x - a_1}{b_1} = \frac{y - a_2}{b_2} = \frac{z - a_3}{b_3} (= \lambda)$

66	What is the parametric ('vector') equation of a plane passing through the point with position vector $\boldsymbol{a}$ which is parallel to vectors $\boldsymbol{b}$ and $\boldsymbol{c}$?	In the formula book: The plane through the point with position vector $\mathbf{a}$ and parallel to $\mathbf{b}$ and $\mathbf{c}$ has equation $\mathbf{r} = \mathbf{a} + s\mathbf{b} + t\mathbf{c}$
67	What is the Cartesian equation of a plane in 3D space with normal vector $\begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}$ passing through the point with position vector $\boldsymbol{a}$?	In the formula book: The plane through A with normal vector $\mathbf{n} = n_1\mathbf{i} + n_2\mathbf{j} + n_3\mathbf{k}$ has cartesian equation $n_1x + n_2y + n_3z + d = 0$ where $d = -\mathbf{a}.\mathbf{n}$
68	What is the definition of the dot ('scalar') product between two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$?	$\boldsymbol{a} \cdot \boldsymbol{b} = \lvert\boldsymbol{a}\rvert\lvert\boldsymbol{b}\rvert \cos\theta$
69	What is the scalar product ('dot product') form of the vector equation for a plane with normal vector $\boldsymbol{n}$ passing through point $\boldsymbol{a}$?	$\boldsymbol{r} \cdot \boldsymbol{n} = k \ \ where\ k = \boldsymbol{a} \cdot \boldsymbol{n}$ $or\ \ \boldsymbol{r} \cdot \hat{\boldsymbol{n}} = d \ \ where\ d = \boldsymbol{a} \cdot \hat{\boldsymbol{n}}$
70	How would you find the angle between two lines?	Use the dot product of their direction vectors: $\cos\theta = \frac{\boldsymbol{d_1} \cdot \boldsymbol{d_2}}{\lvert\boldsymbol{d_1}\rvert\lvert\boldsymbol{d_2}\rvert}$
71	How would you find the angle between two planes?	Use the dot product of their normal vectors: $\cos\theta = \frac{\boldsymbol{n_1} \cdot \boldsymbol{n_2}}{\lvert\boldsymbol{n_1}\rvert\lvert\boldsymbol{n_2}\rvert}$
72	How would you find the angle between a line and a plane?	Use the dot product of the direction vector of the line and the normal vector of the plane, *then subtract from* $90°$... $\cos(\theta - 90) = \frac{\boldsymbol{n_1} \cdot \boldsymbol{d_1}}{\lvert\boldsymbol{n_1}\rvert\lvert\boldsymbol{d_1}\rvert}$
73	How would you show that two lines are skew?	Demonstrate that they are not parallel (ie that their direction vectors are not scalar multiples of one another), and that they do not intersect (ie that there are no consistent solutions to $\boldsymbol{r_1} = \boldsymbol{r_2}$).
74	What is the perpendicular (shortest) distance from O to the plane $\boldsymbol{r} \cdot \hat{\boldsymbol{n}} = k$?	k
75	What is the perpendicular (shortest) distance from the plane $\begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = d$ to the point (α, β, γ)?	In the formula book: The perpendicular distance of (α, β, γ) from $n_1x + n_2y + n_3z + d = 0$ is $\frac{\lvert n_1\alpha + n_2\beta + n_3\gamma + d\rvert}{\sqrt{n_1^2 + n_2^2 + n_3^2}}$
76	What is Euler's relation, connecting the trigonometric form of a complex number to the exponential form?	$re^{i\theta} = r(\cos\theta + i\sin\theta)$
77	What does De Moivre's theorem tell us about complex numbers in mod-arg form?	In the formula book: $\{r(\cos\theta + \mathrm{i}\sin\theta)\}^n = r^n(\cos n\theta + \mathrm{i}\sin n\theta)$
78	What result links $\cos\theta$ to z, the complex number with modulus 1 and argument θ?	$2\cos\theta = z + \frac{1}{z}$
79	What result links $\sin\theta$ to z, the complex number with modulus 1 and argument θ?	$2i\sin\theta = z - \frac{1}{z}$
80	What result links $\cos n\theta$ to z, the complex number with modulus 1 and argument θ?	$2\cos n\theta = z^n + \frac{1}{z^n}$

81	What result links $\sin n\theta$ to z, the complex number with modulus 1 and argument θ?	$2i \sin n\theta = z^n - \frac{1}{z^n}$
82	What are the n^{th} roots of unity, in exponential form?	In the formula book: The roots of $z^n = 1$ are given by $z = e^{\frac{2\pi ki}{n}}$, for $k = 0, 1, 2, \ldots, n-1$
83	What is the general Maclaurin series formula?	In the formula book: **Maclaurin's and Taylor's Series** $f(x) = f(0) + x\,f'(0) + \frac{x^2}{2!}f''(0) + \ldots + \frac{x^r}{r!}f^{(r)}(0) + \ldots$
84	What is the Maclaurin series expansion for e^x?	In the formula book: $e^x = \exp(x) = 1 + x + \frac{x^2}{2!} + \ldots + \frac{x^r}{r!} + \ldots$ for all x
85	What is the Maclaurin series expansion for $\ln(1+x)$?	In the formula book: $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots + (-1)^{r+1}\frac{x^r}{r} + \ldots \quad (-1 < x \leqslant 1)$
86	What is the Maclaurin series expansion for $\sin x$?	In the formula book: $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + (-1)^r\frac{x^{2r+1}}{(2r+1)!} + \ldots$ for all x
87	What is the Maclaurin series expansion for $\cos x$?	In the formula book: $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots + (-1)^r\frac{x^{2r}}{(2r)!} + \ldots$ for all x
88	What is the Maclaurin series expansion for $\arctan x$?	In the formula book: $\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \ldots + (-1)^r\frac{x^{2r+1}}{2r+1} + \ldots \quad (-1 \leqslant x \leqslant 1)$
89	What are the conditions for an integral to be improper?	One or both limits are infinite, or the function is undefined at one of the limits, or at some point between them.
90	What is the mean value of the function $f(x)$ between $x = a$ and $x = b$?	$\frac{1}{b-a}\int_a^b f(x)\,dx$
91	What is the derivative of $\arcsin x$?	In the formula book: $\arcsin x \qquad \frac{1}{\sqrt{1-x^2}}$
92	What is the derivative of $\arccos x$?	In the formula book: $\arccos x \qquad -\frac{1}{\sqrt{1-x^2}}$
93	What is the derivative of $\arctan x$?	In the formula book: $\arctan x \qquad \frac{1}{1+x^2}$
94	What is the integral of $\frac{1}{a^2+x^2}$?	In the formula book: $\frac{1}{a^2+x^2} \qquad \frac{1}{a}\arctan\left(\frac{x}{a}\right)$
95	What is the integral of $\frac{1}{\sqrt{a^2-x^2}}$?	In the formula book: $\frac{1}{\sqrt{a^2-x^2}} \qquad \arcsin\left(\frac{x}{a}\right) \quad (\lvert x\rvert < a)$

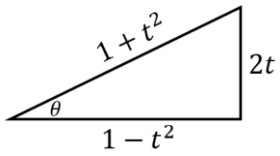
96	What results connect the Cartesian coordinates (x, y) to the Polar coordinates (r, θ)?	$x = r\cos\theta$ $y = r\sin\theta$ $x^2 + y^2 = r^2$
97	What is the integral required to calculate the area of a sector from a polar curve?	In the formula book: **Area of a sector** $A = \frac{1}{2}\int r^2\,\mathrm{d}\theta$ (polar coordinates)
98	How would you find the points on a polar curve where a tangent is parallel to the initial line?	Set $\frac{dy}{d\theta} = 0$, using $y = r\sin\theta$, and solve for θ.
99	How would you find the points on a polar curve where the tangent is perpendicular to the initial line?	Set $\frac{dx}{d\theta} = 0$, using $x = r\cos\theta$, and solve for θ.
100	What is the exponential definition of the hyperbolic cosine $\cosh x$?	$\cosh x = \frac{e^x + e^{-x}}{2}$
101	What is the exponential definition of the hyperbolic sine $\sinh x$?	$\sinh x = \frac{e^x - e^{-x}}{2}$
102	What is the exponential definition of the hyperbolic tangent $\tanh x$?	$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$
103	What are the domain and range of $\cosh x$?	$x \in \mathbb{R}$ $\cosh x \geq 1$
104	What are the domain and range of $\sinh x$?	$x \in \mathbb{R}$ $\sinh x \in \mathbb{R}$
105	What is the equivalent of the Pythagorean identity $\cos^2\theta + \sin^2\theta \equiv 1$ for hyperbolic trig functions?	In the formula book: $\cosh^2 x - \sinh^2 x = 1$
106	What is the equivalent of the $\sin 2\theta$ double angle formula for $\sinh 2x$?	In the formula book: $\sinh 2x = 2\sinh x \cosh x$
107	What is the equivalent of the $\cos 2\theta$ double angle formula for $\cosh 2x$?	In the formula book: $\cosh 2x = \cosh^2 x + \sinh^2 x$
108	What is the exact logarithmic form of the inverse function $\operatorname{arcosh} x$?	In the formula book: $\operatorname{arcosh} x = \ln\{x + \sqrt{x^2 - 1}\}$
109	What is the exact logarithmic form of the inverse function $\operatorname{arsinh} x$?	In the formula book: $\operatorname{arsinh} x = \ln\{x + \sqrt{x^2 + 1}\}$
110	What is the exact logarithmic form of the inverse function $\operatorname{artanh} x$?	In the formula book: $\operatorname{artanh} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$
111	What is the derivative of $\sinh x$?	In the formula book: **f(x)** $\sinh x$ — **f′(x)** $\cosh x$

112	What is the derivative of $\cosh x$?	In the formula book: $\mathbf{f(x)}$: $\cosh x$; $\mathbf{f'(x)}$: $\sinh x$
113	What is the derivative of $\tanh x$?	In the formula book: $\mathbf{f(x)}$: $\tanh x$; $\mathbf{f'(x)}$: $\operatorname{sech}^2 x$
114	What is the derivative of $\operatorname{arsinh} x$?	In the formula book: $\mathbf{f(x)}$: $\operatorname{arsinh} x$; $\mathbf{f'(x)}$: $\frac{1}{\sqrt{1+x^2}}$
115	What is the derivative of $\operatorname{arcosh} x$?	In the formula book: $\mathbf{f(x)}$: $\operatorname{arcosh} x$; $\mathbf{f'(x)}$: $\frac{1}{\sqrt{x^2-1}}$
116	What is the derivative of $\operatorname{artanh} x$?	In the formula book: $\mathbf{f(x)}$: $\operatorname{artanh} x$; $\mathbf{f'(x)}$: $\frac{1}{1-x^2}$
115	What is the general format for a first order differential equation that can be solved using the separation of variables method? And how would we solve the equation?	$$\frac{dy}{dx} = f(x)g(y)$$ $$\Rightarrow \int \frac{1}{g(y)}\, dy = \int f(x)\, dx$$
116	What is the general format for a first order differential equation that can be solved using the Integrating Factor method? And how would we solve the equation?	$$\frac{dy}{dx} + P(x)y = Q(x)$$ The I.F. is $e^{\int P(x)dx}$. Multiplying gives: $$e^{\int P(x)}\frac{dy}{dx} + P(x)e^{\int P(x)}\, y = e^{\int P(x)}\, Q(x)$$ This is 'exact', so we can integrate the LHS using 'reverse product rule': $$e^{\int P(x)}y = \int e^{\int P(x)}Q(x)\, dx$$
117	What is the general format for a homogeneous second order (linear, constant coefficient) differential equation? And what is the first step to solving it?	$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$$ First, find the roots of the Auxiliary Equation $a\lambda^2 + b\lambda + c = 0$
118	What are the three possible cases for the general solution of a homogeneous 2nd order differential equation?	If the AE has real distinct roots α, β: $$y = Ae^{\alpha x} + Be^{\beta x}$$ If the AE has a repeated real root α: $$y = (A + Bx)e^{\alpha x}$$ If the AE has complex roots $p \pm qi$: $$y = e^{px}(A\cos qx + B\sin qx)$$

119	When a homogeneous second order differential equation (for displacement x in terms of time t) is used to model simple harmonic motion, what type of simple harmonic motion is being exhibited for each of the three possible cases?	Heavy damping: $x = Ae^{\alpha t} + Be^{\beta t}$ Critical damping: $x = (A + Bx)e^{\alpha t}$ Light damping: $x = e^{pt}(A\cos qt + B\sin qt)$
120	What additional steps are required when the second order differential equation is non-homogeneous?	First find a Particular Integral, of a corresponding format to the RHS, and add it to the Complementary Function (the GS of the homogeneous case).
121	When a non-homogeneous second order differential equation is used to model simple harmonic motion, what is the nature of the simple harmonic motion?	Forced simple harmonic motion (the function on the RHS corresponds to the driving force applied to the system).
122	What is the standard format of the second order differential equation for damped harmonic motion, in terms of k, the constant of resistance, and ω, the angular velocity?	$\frac{d^2x}{dt^2} + k\frac{dx}{dt} + \omega^2 x = 0$
123	What is the method for solving coupled first order differential equations?	Differentiate one of the equations in order to eliminate one of the dependent variables and its first derivative, producing a second order differential equation involving the other variable.

1	What is the fundamental definition of the vector ('cross') product $\boldsymbol{u} \times \boldsymbol{v}$?	$\boldsymbol{u} \times \boldsymbol{v} = \lvert\boldsymbol{u}\rvert\lvert\boldsymbol{v}\rvert \sin\theta\, \hat{\boldsymbol{n}}$
2	What is the relationship between $\boldsymbol{u} \times \boldsymbol{v}$ and $\boldsymbol{v} \times \boldsymbol{u}$?	$\boldsymbol{u} \times \boldsymbol{v} = -\boldsymbol{v} \times \boldsymbol{u}$ (this product is 'anti-commutative')
3	What is the cross product of two parallel vectors?	The zero vector, eg in 3D: $\begin{pmatrix}0\\0\\0\end{pmatrix}$
4	How would you calculate the cross product of two 3D vectors $\begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix}$ and $\begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix}$?	In the formula book: Vector product: $\mathbf{a} \times \mathbf{b} = \lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert \sin\theta\, \hat{\mathbf{n}} = \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix} = \begin{pmatrix}a_2b_3 - a_3b_2\\ a_3b_1 - a_1b_3\\ a_1b_2 - a_2b_1\end{pmatrix}$
5	What is the geometric interpretation of the size of the cross product $\lvert\boldsymbol{u} \times \boldsymbol{v}\rvert$?	$\lvert\boldsymbol{u} \times \boldsymbol{v}\rvert$ gives the area of the parallelogram formed by $\boldsymbol{u}$ and $\boldsymbol{v}$.
6	What is the geometric interpretation of the direction of the cross product $\boldsymbol{u} \times \boldsymbol{v}$?	The direction of $\boldsymbol{u} \times \boldsymbol{v}$, denoted $\hat{\boldsymbol{n}}$, is perpendicular to both $\boldsymbol{u}$ and $\boldsymbol{v}$, and is directed according to the 'right-hand rule' (if turning anticlockwise from $\boldsymbol{u}$ to $\boldsymbol{v}$, then $\hat{\boldsymbol{n}}$ points towards you).
7	How is the scalar triple product of vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ calculated?	In the formula book: $\mathbf{a}.(\mathbf{b} \times \mathbf{c}) = \begin{vmatrix}a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{vmatrix} = \mathbf{b}.(\mathbf{c} \times \mathbf{a}) = \mathbf{c}.(\mathbf{a} \times \mathbf{b})$
8	What is the geometric interpretation of the scalar triple product?	It represents the volume of a parallelopiped formed by $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$.
9	How would you calculate the volume of a tetrahedron whose corners have position vectors $\mathbf{0}$, $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$?	$\frac{1}{6}\lvert(\boldsymbol{a} \cdot (\boldsymbol{b} \times \boldsymbol{c}))\rvert$
10	What is the cross product form of a vector line equation with position vector $\boldsymbol{a}$ and direction vector $\boldsymbol{b}$?	$(\boldsymbol{r} - \boldsymbol{a}) \times \boldsymbol{b} = \mathbf{0}$ May also be written as: $\boldsymbol{r} \times \boldsymbol{b} = \boldsymbol{a} \times \boldsymbol{b}$
11	What are the direction cosines of the vector $\boldsymbol{v} = \begin{pmatrix}x\\y\\z\end{pmatrix}$?	$\cos\alpha = \frac{x}{\lvert\boldsymbol{v}\rvert}$ $\cos\beta = \frac{y}{\lvert\boldsymbol{v}\rvert}$ $\cos\gamma = \frac{z}{\lvert\boldsymbol{v}\rvert}$
12	What is the geometric interpretation of the direction cosines of a vector?	The angles α, β and γ are those made by the vector with the $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$ directions.
13	What is one additional property of the direction cosines?	$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$
14	How would you calculate the shortest distance between two skew lines, using the cross product?	Project a vector between the lines (eg from one position vector to the other) onto a unit vector perpendicular to both lines (formed by taking the cross product of the two direction vectors).

15	What are the standard and parametric forms of the equation of a parabola?	$y^2 = 4ax$ $(at^2, 2at)$
16	What are the standard and parametric forms of the equation of an ellipse?	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $(a\cos\theta, b\sin\theta)$
17	What are the standard and parametric forms of the equation of a hyperbola? Give both the trigonometric and hyperbolic forms of the parametrization.	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ $(a\sec\theta, b\tan\theta)$ $(\pm a\cosh\theta, b\sinh\theta)$
18	What are the standard and parametric forms of the equation of a rectangular hyperbola?	$xy = c^2$ $\left(ct, \frac{c}{t}\right)$
19	What is the relationship between the distance of a point P from the focus S and from the directrix D, for any conic?	$PS = ePD$, where e is the eccentricity
20	What methods can be used to determine the gradient of a point on a conic?	Implicit or parametric differentiation
21	What is the parametric equation of a tangent to the standard parabola?	$ty = x + at^2$
22	What is the parametric equation of a normal to the standard parabola?	$y + tx = 2at + at^2$
23	What is the parametric equation of a tangent to the standard rectangular hyperbola?	$x + t^2y = 2ct$
24	What is the parametric equation of a normal to the standard rectangular hyperbola?	$t^3x - ty = c(t^4 - 1)$
25	What is the parametric equation of a tangent to the standard ellipse?	$(b\cos t)x + (a\sin t)y = ab$
26	What is the parametric equation of a normal to the standard ellipse?	$(a\sin t)x - (b\cos t)y = (a^2 - b^2)\cos t\sin t$
27	What is the parametric equation of a tangent to the standard hyperbola?	$(a\sinh t)y + ab = (b\cosh t)x$ Or: $(b\sec\theta)x - (a\tan\theta)y = ab$
28	What is the parametric equation of a normal to the standard hyperbola?	$(a\sinh t)x + (b\cosh t)y = (a^2 + b^2)\sinh t\cosh t$ Or: $by + (a\sin\theta)x = (a^2 + b^2)\tan\theta$
29	What is the range of possible values for the eccentricity of: a circle, ellipse, parabola, hyperbola and rectangular hyperbola?	Circle: $e = 0$ Ellipse: $0 < e < 1$ Parabola: $e = 1$ Hyperbola: $e > 1$ Rectangular hyperbola: $e = \sqrt{2}$

30	How is eccentricity calculated for an ellipse provided in the standard format?	In the formula book: $b^2 = a^2(1 - e^2)$
31	How is eccentricity calculated for a hyperbola provided in the standard format?	In the formula book: $b^2 = a^2(e^2 - 1)$
32	Where are the foci and directrices for an ellipse provided in the standard format?	In the formula book: $(\pm ae, 0)$ $x = \pm\frac{a}{e}$
33	Where are the foci and directrices for a hyperbola provided in the standard format?	In the formula book: $(\pm ae, 0)$ $x = \pm\frac{a}{e}$
34	What key step is usually required to deal with fractional inequalities, and why is it necessary?	Multiply by the square of the denominator so that its sign is known, and the inequality sign is preserved.
35	How would you go about solving an inequality involving the modulus function?	Find critical values by solving both equations (using $\lvert f(x)\rvert = f(x)$ for one and $\lvert f(x)\rvert = -f(x)$ for the other), then use a graph to determine solutions.
36	What are the t-formulae for $\sin\theta$, $\cos\theta$ and $\tan\theta$ when $t = \tan\left(\frac{\theta}{2}\right)$?	 $\sin\theta = \frac{2t}{1+t^2}$ $\cos\theta = \frac{1-t^2}{1+t^2}$ $\tan\theta = \frac{2t}{1-t^2}$
37	What is the Taylor series expansion formula for $f(x)$ in ascending powers of $(x-a)$, centred at $x = a$?	Not in the formula book, but *is* generally provided with the exam question: $f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)(x-a)^2}{2!} + \cdots$
38	What is the Taylor series expansion formula for $f(x+a)$ in ascending powers of x, centred at $x = 0$?	Not in the formula book, but *is* generally provided with the exam question: $f(x+a) = f(a) + f'(a)x + \frac{f''(a)x^2}{2!} + \cdots$
39	State Leibnitz' theorem.	$y^{(n)} = \sum_{k=0}^{n} \binom{n}{k} u^{(k)} v^{(n-k)}$
40	State L'Hospital's rule.	If $\frac{f(x)}{g(x)}$ evaluates to either $\frac{0}{0}$ or $\frac{\pm\infty}{\pm\infty}$ in the limit, then $\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\left(\frac{f'(x)}{g'(x)}\right)$

41	In addition to the general result $\lim_{x\to a}\big(Af(x)+Bg(x)\big)=A\lim_{x\to a}f(x)+B\lim_{x\to a}g(x)$ what other results govern the behaviour of limits?	If the limits of $f(x)$ and $g(x)$ exist, then $\lim_{x\to a}\big(f(x)g(x)\big)=\lim_{x\to a}f(x)\times\lim_{x\to a}g(x)$. If the limit of $f(x)$ exists, then $\lim_{x\to a}e^{f(x)}=e^{lim_{x\to a}f(x)}$
42	What is Euler's method?	Not in the formula book, but *is* generally provided with the exam question: $y_1=y_0+hy_1'$
43	What is the midpoint method?	Not in the formula book, but *is* generally provided with the exam question: $y_1=y_{-1}+2hy_0'$
44	What is Euler's method for second order differential equations?	Not in the formula book, but *is* generally provided with the exam question: $y_1=2y_0-y_{-1}+h^2y_0''$
45	What is Simpson's rule? You may use abbreviations provided your intention is clear.	$\int_a^b f(x)\,dx\approx\frac{h}{3}\big(ends+4(odds)+2(evens)\big)$
46	When using a substitution to reduce a differential equation to a solvable form, what is the first thing you need to look for?	Which variables are being changed. In particular, whether the independent variable is changing or not. This will help you decide what to differentiate and how.