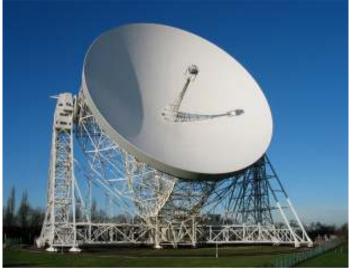
Jodrell Bank

The year is 2415. The Earth has long since been abandoned by all but a hardy civilization of cosmophobes (the space-travel averse). While the majority of the planet's population is living a life of ease on the glistening shores of Europa's shining seas, these few surviving Earthlings have congregated (out of loneliness, and the fact that the Europans took Facebook with them) at Jodrell Bank observatory:



Having originally chosen the site for ease of calling Europa, the hour-long timelag in phone conversations caused them to rethink their use of the 76m diameter satellite dish. Nowadays, it faces directly upwards and collects rainwater.

Like all satellite dishes, this dish is designed as a parabola (this bounces radio waves towards a central focus, effectively amplifying signals). This particular parabolic dish is based on the shape produced by a 360° rotation about the y-axis for the curve $y = \frac{1}{44}x^2$ between the x-axis and the line y = 33. All measurements are in metres.

How much water, in cubes (cubic metres), would the satellite dish hold when full?

How much water will the dish collect a year, given the average rainfall of the Jodrell Bank region of 800mm a year (assuming no evaporation)?

How many people would this sustainably support? (note: $1m^3 = 1000$ litres, and the average person uses 150 litres per day)

How long would it take before the dish overflows if the population of the colony is actually 40 people?

Jodrell Bank - Solutions

The year is 2415. The Earth has long since been abandoned by all but a hardy civilization of cosmophobes (the space-travel averse). While the majority of the planet's population is living a life of ease on the glistening shores of Europa's shining seas, these few surviving Earthlings have congregated (out of loneliness, and the fact that the Europans took Facebook with them) at Jodrell Bank observatory:



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How much water, in cubes (cubic metres), would the satellite dish hold when full?

$$\pi \int_0^{33} 44y \, dy = 44\pi \left[\frac{y^2}{2}\right]_0^{33} = 23958\pi = 75266m^3 \text{ to the nearest } m^3$$

How much water will the dish collect a year, given the average rainfall of the Jodrell Bank region of 800mm a year (assuming no evaporation)?

Area of opening =
$$\pi r^2 = 38^2 \pi = 1444 \pi = 4536 m^3$$
 to the nearest m^3

800mm = 0.8m so $total = 4536 \times 0.8 = 3629m^3$ to the nearest m^3

How many people would this sustainably support? (note: $1m^3 = 1000$ litres, and the average person uses 150 litres per day)

150 litres per day=0.15m3 per day=54.75m3 per year

How long would it take before the dish overflows if the population of the colony is actually 40 people?

40 people use
$$54.75 \times 40 = 2190m^3$$
 per year

This leaves $3629 - 2190 = 1439m^3$ extra per year

The dish holds $75266m^3$ so $\frac{75266}{1439} = 52$ years to the nearest year