What is i^i?

The complex number i has modulus 1 and argument $\frac{\pi}{2}$, so, written in exponential form, it becomes:

$$i = e^{i\frac{\pi}{2}}$$

Writing the base i in exponential form gives us:

$$i^i = \left(e^{i\frac{\pi}{2}}\right)^i = e^{i^2\frac{\pi}{2}} = \boldsymbol{e^{-\frac{\pi}{2}}}$$

This is, of course, a real number (approximately 0.208)

Using this technique (writing the base complex number in exponential form), write the following numbers in their simplest form:

1. 5^{5i}

Remember you can write real numbers in exponential form too, using $x = e^{\ln x}$.

2. $(3i)^{2i}$

Write the base in exponential form (first modulus argument form $re^{i\theta}$, then use the technique above to write the whole thing as a power of e).

3. $(1+i)^{1-i}$

Do the same as above – you will need to calculate the modulus and argument first.

What is i^i? SOLUTIONS

The complex number i has modulus 1 and argument $\frac{\pi}{2}$, so, written in exponential form, it becomes:

$$i = e^{i\frac{\pi}{2}}$$

Writing the base i in exponential form gives us:

$$i^i = \left(e^{i\frac{\pi}{2}}\right)^i = e^{i^2\frac{\pi}{2}} = \boldsymbol{e^{-\frac{\pi}{2}}}$$

This is, of course, a real number (approximately 0.208)

Using this technique (writing the base complex number in exponential form), write the following numbers in their simplest form:

1. 5^{5i}

Remember you can write real numbers in exponential form too, using $x = e^{\ln x}$.

$$\left(e^{\ln 5}\right)^{5i} = e^{5\ln 5 i}$$

2. $(3i)^{2i}$

Write the base in exponential form (first modulus argument form $re^{i\theta}$, then use the technique above to write the whole thing as a power of e).

$$\left(e^{\ln 3}e^{i\frac{\pi}{2}}\right)^{2i} = \left(e^{\ln 3 + i\frac{\pi}{2}}\right)^{2i} = e^{2i\ln 3 - \pi} = e^{-\pi}e^{2\ln 3 i}$$

3. $(1+i)^{1-i}$

Do the same as above – you will need to calculate the modulus and argument first.

$$\left(\sqrt{2}e^{\frac{\pi}{4}i}\right)^{1-i} = \left(e^{\ln\sqrt{2} + \frac{\pi}{4}i}\right)^{1-i} = e^{\ln\sqrt{2} - \frac{\pi}{4} + \left(\frac{\pi}{4} - \ln\sqrt{2}\right)i} = e^{\frac{\ln 2}{2} - \frac{\pi}{4}}e^{\left(\frac{\pi}{4} - \ln\sqrt{2}\right)i}$$