Geostationary Orbit

A number of communication and weather satellites are currently in orbit around the earth at such a height and in such a position as to always be over a single point on the surface.

The trick is to locate over the equator, and choose the correct distance from Earth to cause circular motion to proceed at the same angular speed as the rotation of the Earth itself.

In modelling the circular motion of a satellite around our planet, the centripetal force is provided solely by the gravitational attraction between the satellite and the Earth. The centripetal force acting on a satellite of mass m orbiting the Earth at a distance from the centre of the earth r:

$$F = \frac{4 \times 10^{14} m}{r^2}$$

The period of rotation of the Earth is about 23 hours 56 minutes (a standard 24-hour day is relative to the sun, which we are slowly rotating around – a sidereal day is our rotation relative to 'fixed' stars). Calculate the angular speed of the Earth (in terms of π).

Use the centripetal force equation above to calculate the height required for a satellite to have the same angular speed. Note: the radius of the earth is $6380km$.

Calculate the speed that a geostationary satellite must be travelling at to remain in a stable orbit.

Geostationary Orbit Solutions

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$$\omega = \frac{2\pi}{T} = \frac{2\pi}{86160} = \boldsymbol{\frac{\pi}{43080}}$$

Use the centripetal force equation above to calculate the height required for a satellite to have the same angular speed. Note: the radius of the earth is $6380km$.

$$F = mr\omega^2 \quad \Rightarrow \quad \frac{4 \times 10^{14} m}{r^2} = mr\left(\frac{\pi}{43080}\right)^2 \quad \Rightarrow \quad r^3 = (4 \times 10^{14})\left(\frac{43080}{\pi}\right)^2$$

$$\Rightarrow \quad r = 42212125m \quad \Rightarrow \quad height = 42212.125 - 6380 = \boldsymbol{35800km\ to\ 3\ s.f.}$$

Calculate the speed that a geostationary satellite must be travelling at to remain in a stable orbit.

$$v = r\omega = 42212125 \times \frac{\pi}{43080} = \boldsymbol{3080ms^{-1}\ to\ 3\ s.f.}$$