Geometric Management

Due to a mix-up at head office, for a number of weeks there were two managers in a particular high street shop. Both had very different views on how the shop should be run, and until the situation could be sorted out it was rather confusing for everyone. Every time one of the managers sent a memo round instructing staff on a particular policy, the other one would immediately circulate his own memo saying the opposite. Fortunately this meant that the shop continued to function much as it always had, with every new initiative reversed as soon as it was suggested.

However, neither manager was very good at maths. And so when one of them sent orders to the sales department to reduce the price of all electrical goods by 20%, the other ordered Sales to increase the price of the goods by 20%. As mathematicians will be aware, a 20% increase is not the exact opposite of a 20% reduction, and so as a result the final price of electrical goods was not the same as the original.

If a toaster initially costs £40, how much will it cost after the two sets of orders have been carried out?

More generally, what is the overall percentage change for electrical goods as a result of these orders?

Now, once the first manager discovered his order had been reversed (as he thought), he immediately reissued it, and the second manager reissued his order. This happened daily for 3 weeks. How much would a fridge cost at the end of the three weeks if it cost £250 originally? If a wholesaler bought one of these fridges the day before the changes, then bought another every single day for the 3 weeks, buying 22 in total, how much did the fridges cost him each, on average?

How long will it be before all electrical goods are no more than a tenth of their original price?

Geometric Management Solutions

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$$£40 \times 0.8 \times 1.2 = £\mathbf{38.40}$$

More generally, what is the overall percentage change for electrical goods as a result of these orders?

$$0.8 \times 1.2 = 0.96 \quad so \quad \mathbf{4\%\ \textit{reduction}}$$

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$$Geometric\ Series: \quad a = 250 \quad r = 0.96 \quad n = 22$$

$$U_n = ar^{n-1} \quad so \quad U_{22} = 250 \times 0.96^{21} = £\mathbf{106.08\ \textit{to}\ 2\ \textit{d.p.}}$$

$$S_n = \frac{a(1-r^n)}{1-r} \quad so \quad S_{22} = \frac{250(1-0.96^{22})}{1-0.96} = £3704.07\ to\ 2\ d.p.$$

$$22\ fridges\ at\ a\ total\ of\ £3704.07\ gives\ an\ average\ of\ £\mathbf{168.37\ \textit{per fridge}}$$

How long will it be before all electrical goods are no more than a tenth of their original price?

$$U_n = ar^{n-1} \quad so \quad U_n = a \times 0.96^{n-1} < 0.1a$$

$$0.96^{n-1} < 0.1 \quad \Rightarrow \quad (n-1)\ln(0.96) < \ln(0.1) \quad \Rightarrow \quad n > 1 + \frac{\ln(0.1)}{\ln(0.96)} \quad \Rightarrow n > 57.4 \quad so \quad n = 58$$

Note: Since the log of a number less than the base will be negative, the inequality changes when we divide