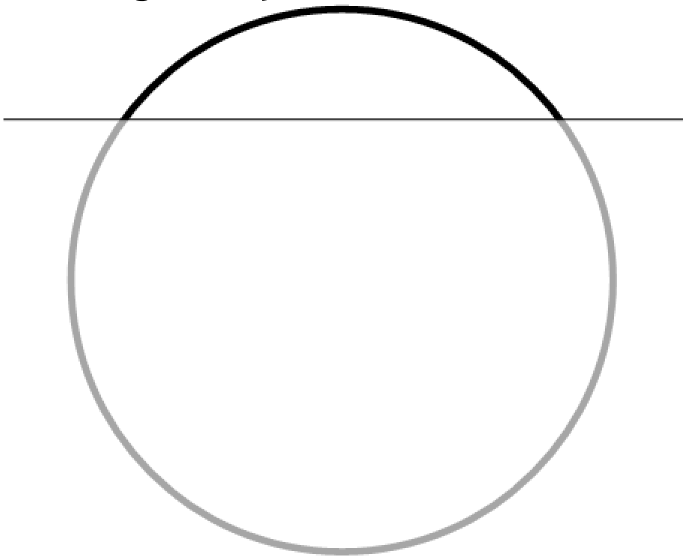
Europa

Jupiter's moons have long been of interest to both scientists and science fiction writers. Europa, with its Earth-like characteristics, is considered one of the most likely places in the solar system to support life (ranking slightly above the Outer Hebrides in a recent study).

Now, suppose a research base were to be constructed on the surface of Europa. It is built in the form of a light, but durable, flexible bubble. Not quite as large as a full hemisphere, it is the shape of a slice cut from the top of a sphere. The grey line indicates the complete sphere (although the only part being built is the bit above the line of the ground):



In order to determine oxygen requirements, we need to be able to calculate the volume of the space inside the dome.
Clearly, the volume of a full sphere would be given by: $V = \frac{4}{3}\pi r^3$ and the volume of a hemisphere would be: $V = \frac{2}{3}\pi r^3$

But what about a dome with height h, cut from a sphere of radius r?

And can you come up with a valid method for calculating the height required, for a radius of 100m, for a dome to take up ¼ of the total sphere in volume?

Europa - Solutions

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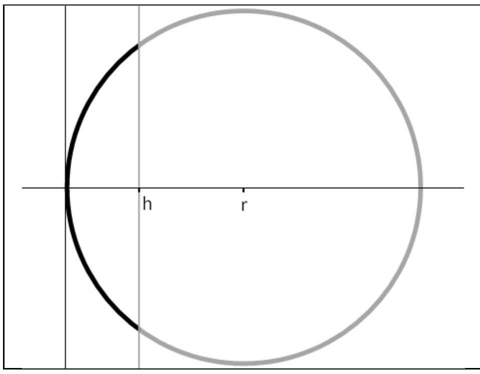

$$(x-r)^2 + y^2 = r^2 \quad so \quad y^2 = r^2 - (x-r)^2$$

$$V = \int_0^h \pi y^2\, dx = \pi \int_0^h r^2 - (x-r)^2\, dx = \pi \int_0^h r^2 - x^2 + 2rx - r^2\, dx$$

$$= \pi \int_0^h 2rx - x^2\, dx = \pi \left[rx^2 - \frac{x^3}{3} \right]_0^h = \boldsymbol{\frac{\pi h^2}{3}(3r - h)}$$

And can you come up with a valid method for calculating the height required, for a radius of 100m, for a dome to take up ¼ of the total sphere in volume?

$$\frac{\pi h^2}{3}(3(100) - h) = \frac{1}{4}\left(\frac{4}{3}\pi(100)^3\right) \quad \Rightarrow \quad h^2(300 - h) = 1000000$$

$$\textit{Since this is a cubic and may be difficult to solve analytically, an iterative formula is created:}$$

$$h = \sqrt{\frac{1000000}{300 - h}} = 1000\sqrt{\frac{1}{300 - h}} \quad \textit{so we apply:} \quad h_{n+1} = 1000\sqrt{\frac{1}{300 - h_n}}$$

$$\textit{Letting } h_1 = 50, \quad h_2 = 63.24 \textit{ to 2 d.p., etc and to 2 d.p.}, h_6 = h_7 = \mathbf{65.27}$$