# Deriving the SUVAT (kinematics) equations

Assumptions:

$$average\ speed = \frac{distance}{time} \qquad (1)$$

$$acceleration = \frac{change\ in\ speed}{time} \qquad (2)$$

Variables:

$$\begin{aligned} \boldsymbol{s} &= displacement\ (m) \\ \boldsymbol{u} &= initial\ velocity\ (ms^{-1}) \\ \boldsymbol{v} &= final\ velocity\ (ms^{-1}) \\ \boldsymbol{a} &= acceleration\ (ms^{-2}) \\ t &= time\ (s) \end{aligned} \qquad (3)$$

**Deriving $\boldsymbol{v = u + at}$:**

Writing $(2)$ using the variables from $(3)$:

$$a = \frac{v-u}{t}$$

Rearranging:

$$\boldsymbol{v = u + at} \qquad (4)$$

***Intuition behind the formula:*** *The speed increases according to the size of the acceleration and the time for which the particle is accelerating.*

**Deriving $\boldsymbol{s = \left(\frac{u+v}{2}\right)t}$:**

Writing $(1)$ using the variables from $(3)$:

$$\frac{u+v}{2} = \frac{s}{t}$$

Rearranging:

$$\boldsymbol{s = \left(\frac{u+v}{2}\right)t} \qquad (5)$$

***Intuition behind the formula:*** *The distance travelled is the average speed multiplied by the time. Since acceleration is constant, the average speed is halfway between $u$ and $v$.*

**Deriving $\boldsymbol{s = ut + \frac{1}{2}at^2}$:**

Substituting an expression for $v$ from $(4)$ into $(5)$:

$$s = \frac{u + (u + at)}{2}t$$

Rearranging:

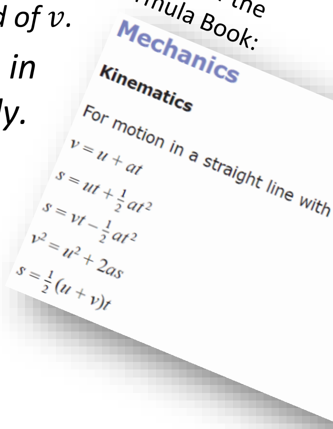
$$s = \frac{2u + at}{2}t$$

$$\boldsymbol{s = ut + \frac{1}{2}at^2}$$

*Note: $s = vt - \frac{1}{2}at^2$ can be derived by subbing in $u$ instead of $v$.*

***Intuition behind the formula:*** *A fixed speed corresponds to a linear increase in displacement, but if speed is increasing, displacement increases quadratically.*

***Note:*** *All on p.4 of the Edexcel Formula Book:*


**Mechanics**

**Kinematics**

For motion in a straight line with

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = vt - \frac{1}{2}at^2$

$v^2 = u^2 + 2as$

$s = \frac{1}{2}(u + v)t$

**Deriving $\boldsymbol{v^2 = u^2 + 2as}$:**

Substituting an expression for $t$ from $(4)$ into $(5)$:

$$s = \frac{u+v}{2}\left(\frac{v-u}{a}\right) = \frac{(v+u)(v-u)}{2a}$$

$$s = \frac{v^2 - u^2}{2a}$$

$$\boldsymbol{v^2 = u^2 + 2as}$$

***Intuition behind the formula:*** *The kinetic energy of a body is given by $\frac{1}{2}mv^2$, and energy transferred is given by $ma \times s$, so this is just "Final KE equals initial KE plus work done".*

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