Tricky Core 2 Questions

104 marks (approx 2 hours at the rate of 75 marks per 90 minutes)

A: AQA C2 Jan 2008

1 The diagrams show a rectangle of length 6 cm and width 3 cm, and a sector of a circle of radius 6 cm and angle θ radians.

The area of the rectangle is twice the area of the sector.

(a) Show that $\theta = 0.5$. *(3 marks)*

(b) Find the perimeter of the sector. *(3 marks)*

B: AQA C2 Jan 2008

7 (a) Given that

$$\log_a x = \log_a 16 - \log_a 2$$

write down the value of x. *(1 mark)*

(b) Given that

$$\log_a y = 2\log_a 3 + \log_a 4 + 1$$

express y in terms of a, giving your answer in a form **not** involving logarithms. *(3 marks)*

C: AQA C2 Jun 2008

7 (a) The expression $\left(1 + \frac{4}{x^2}\right)^3$ can be written in the form

$$1 + \frac{p}{x^2} + \frac{q}{x^4} + \frac{64}{x^6}$$

By using the binomial expansion, or otherwise, find the values of the integers p and q. *(3 marks)*

(b) (i) Hence find $\int \left(1 + \frac{4}{x^2}\right)^3 \mathrm{d}x$. *(4 marks)*

(ii) Hence find the value of $\int_1^2 \left(1 + \frac{4}{x^2}\right)^3 \mathrm{d}x$. *(2 marks)*

D: AQA C2 Jan 2008

8 (a) Sketch the graph of $y = 3^x$, stating the coordinates of the point where the graph crosses the y-axis. *(2 marks)*

(b) Describe a single geometrical transformation that maps the graph of $y = 3^x$:

(i) onto the graph of $y = 3^{2x}$; *(2 marks)*

(ii) onto the graph of $y = 3^{x+1}$. *(2 marks)*

(c) (i) Using the substitution $Y = 3^x$, show that the equation

$$9^x - 3^{x+1} + 2 = 0$$

can be written as

$$(Y-1)(Y-2) = 0$$ *(2 marks)*

(ii) Hence show that the equation $9^x - 3^{x+1} + 2 = 0$ has a solution $x = 0$ and, by using logarithms, find the other solution, giving your answer to four decimal places. *(4 marks)*

E: AQA C2 Jan 2008

9 (a) Given that

$$\frac{3 + \sin^2\theta}{\cos\theta - 2} = 3\cos\theta$$

show that

$$\cos\theta = -\frac{1}{2}$$ *(4 marks)*

(b) Hence solve the equation

$$\frac{3 + \sin^2 3x}{\cos 3x - 2} = 3\cos 3x$$

giving all solutions in degrees in the interval $0° < x < 180°$. *(4 marks)*

F: AQA C2 Jun 2008

3 A geometric series begins

$$20 + 16 + 12.8 + 10.24 + \ldots$$

(a) Find the common ratio of the series. *(1 mark)*

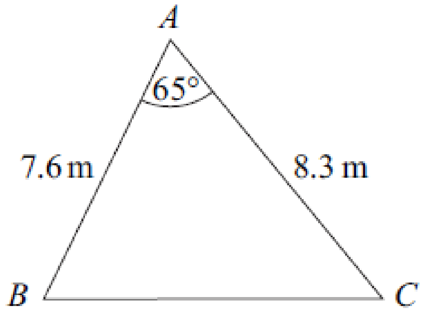
(b) Find the sum to infinity of the series. *(2 marks)*

(c) Find the sum of the first 20 terms of the series, giving your answer to three decimal places. *(2 marks)*

(d) Prove that the nth term of the series is 25×0.8^n. *(2 marks)*

G: AQA C2 Jun 2008

4 The diagram shows a triangle ABC.



The size of angle BAC is 65°, and the lengths of AB and AC are 7.6 m and 8.3 m respectively.

(a) Show that the length of BC is 8.56 m, correct to three significant figures. *(3 marks)*

(b) Calculate the area of triangle ABC, giving your answer in m^2 to three significant figures. *(2 marks)*

(c) The perpendicular from A to BC meets BC at the point D.

Calculate the length of AD, giving your answer to the nearest 0.1 m. *(3 marks)*

H: AQA C2 Jun 2008

6 The nth term of a sequence is u_n.

The sequence is defined by

$$u_{n+1} = pu_n + q$$

where p and q are constants.

The first three terms of the sequence are given by

$$u_1 = -8 \qquad u_2 = 8 \qquad u_3 = 4$$

(a) Show that $q = 6$ and find the value of p. *(5 marks)*

(b) Find the value of u_4. *(1 mark)*

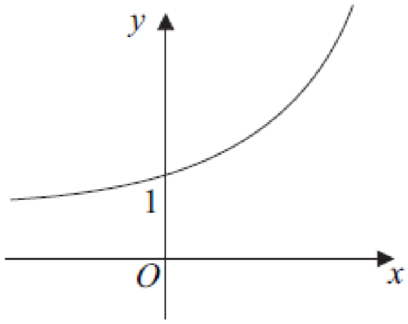
(c) The limit of u_n as n tends to infinity is L.

(i) Write down an equation for L. *(1 mark)*

(ii) Hence find the value of L. *(2 marks)*

I: AQA C2 Jun 2008

8 The diagram shows a sketch of the curve with equation $y = 6^x$.



(a) (i) Use the trapezium rule with five ordinates (four strips) to find an approximate value for $\int_0^2 6^x \, dx$, giving your answer to three significant figures. *(4 marks)*

(ii) Explain, with the aid of a diagram, whether your approximate value will be an overestimate or an underestimate of the true value of $\int_0^2 6^x \, dx$. *(2 marks)*

(b) (i) Describe a single geometrical transformation that maps the graph of $y = 6^x$ onto the graph of $y = 6^{3x}$. *(2 marks)*

(ii) The line $y = 84$ intersects the curve $y = 6^{3x}$ at the point A. By using logarithms, find the x-coordinate of A, giving your answer to three decimal places. *(4 marks)*

(c) The graph of $y = 6^x$ is translated by $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ to give the graph of the curve with equation $y = f(x)$. Write down an expression for $f(x)$. *(2 marks)*

J: AQA C2 Jun 2007

8 (a) It is given that n satisfies the equation

$$\log_a n = \log_a 3 + \log_a(2n - 1)$$

Find the value of n. *(3 marks)*

(b) Given that $\log_a x = 3$ and $\log_a y - 3\log_a 2 = 4$:

(i) express x in terms of a; *(1 mark)*

(ii) express xy in terms of a. *(4 marks)*

K: AQA C2 Jun 2006

4 (a) The expression $(1-2x)^4$ can be written in the form

$$1 + px + qx^2 - 32x^3 + 16x^4$$

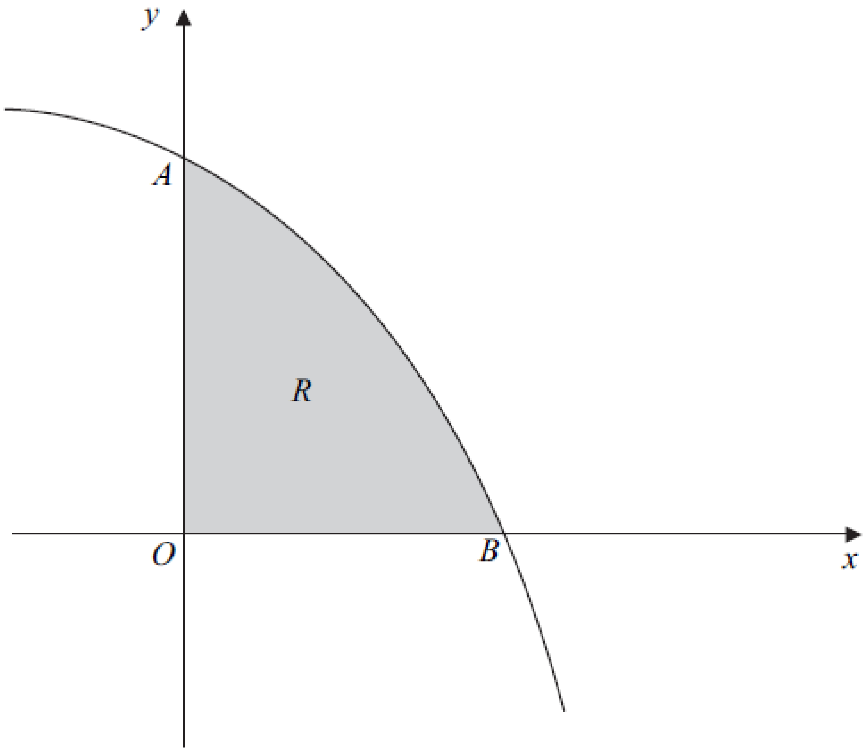
By using the binomial expansion, or otherwise, find the values of the integers p and q. *(3 marks)*

(b) Find the coefficient of x in the expansion of $(2+x)^9$. *(2 marks)*

(c) Find the coefficient of x in the expansion of $(1-2x)^4(2+x)^9$. *(3 marks)*

L: AQA C2 Jun 2006

6 The diagram shows a sketch of the curve with equation $y = 27 - 3^x$.



The curve $y = 27 - 3^x$ intersects the y-axis at the point A and the x-axis at the point B.

(a) (i) Find the y-coordinate of point A. *(2 marks)*

(ii) Verify that the x-coordinate of point B is 3. *(1 mark)*

(b) The region, R, bounded by the curve $y = 27 - 3^x$ and the coordinate axes is shaded. Use the trapezium rule with four ordinates (three strips) to find an approximate value for the area of R. *(4 marks)*

(c) (i) Use logarithms to solve the equation $3^x = 13$, giving your answer to four decimal places. *(3 marks)*

(ii) The line $y = k$ intersects the curve $y = 27 - 3^x$ at the point where $3^x = 13$. Find the value of k. *(1 mark)*

(d) (i) Describe the single geometrical transformation by which the curve with equation $y = -3^x$ can be obtained **from** the curve $y = 27 - 3^x$. *(2 marks)*

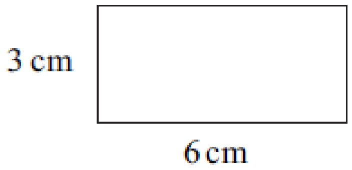
(ii) Sketch the curve $y = -3^x$. *(2 marks)*

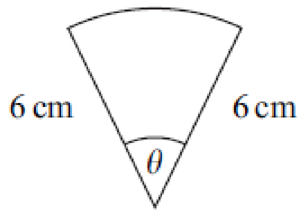
Tricky Core 2 Questions: HINTS

104 marks (approx 2 hours at the rate of 75 marks per 90 minutes)

A: AQA C2 Jan 2008

1 The diagrams show a rectangle of length 6 cm and width 3 cm, and a sector of a circle of radius 6 cm and angle θ radians.





The area of the rectangle is twice the area of the sector.

(a) Show that $\theta = 0.5$. *(3 marks)*

(b) Find the perimeter of the sector. *(3 marks)*

Use the standard formulae for sector area and arc length: $A = \frac{1}{2}r^2\theta$ and $l = r\theta$ (these ought to be memorised, but you can also derive them easily from the area and circumference of a circle formule by considering a suitable fraction of the full circle).

B: AQA C2 Jan 2008

7 (a) Given that

$$\log_a x = \log_a 16 - \log_a 2$$

write down the value of x. *(1 mark)*

(b) Given that

$$\log_a y = 2\log_a 3 + \log_a 4 + 1$$

express y in terms of a, giving your answer in a form **not** involving logarithms. *(3 marks)*

Use the log rule $\log A - \log B = \log\frac{A}{B}$ to turn the right-hand side into a single logarithm for part a), and use the rules $n\log A = \log A^n$ and $\log A + \log B = \log AB$ to simplify the right-hand side for part b). You will also need to deal with the 1, either by doing a to the power of each side and using index laws or by rewriting as $1 = \log_a a$ in order to combine with the rest to write as a single logarithm, and then raising a to the power of each side.

7 (a) The expression $\left(1+\frac{4}{x^2}\right)^3$ can be written in the form

$$1+\frac{p}{x^2}+\frac{q}{x^4}+\frac{64}{x^6}$$

By using the binomial expansion, or otherwise, find the values of the integers p and q.

(3 marks)

(b) (i) Hence find $\int\left(1+\frac{4}{x^2}\right)^3 \mathrm{d}x$. *(4 marks)*

(ii) Hence find the value of $\int_1^2\left(1+\frac{4}{x^2}\right)^3 \mathrm{d}x$. *(2 marks)*

Use the binomial expansion formula given in the formula book, taking care to treat $\frac{4}{x^2}$ as you normally would the x term (ie, ascending powers of $\frac{4}{x^2}$ rather than simply ascending powers of x as you would get from $(1+x)^3$). To integrate, write your expression from part a) in index form (ie $\frac{64}{x^6}=64x^{-6}$, etc) then use $\int x^n\,dx=\frac{x^{n+1}}{n+1}+C$. Finally, for the last part, substitute the limits into your expression.

D: AQA C2 Jan 2008

8 (a) Sketch the graph of $y = 3^x$, stating the coordinates of the point where the graph crosses the y-axis. *(2 marks)*

(b) Describe a single geometrical transformation that maps the graph of $y = 3^x$:

(i) onto the graph of $y = 3^{2x}$; *(2 marks)*

(ii) onto the graph of $y = 3^{x+1}$. *(2 marks)*

(c) (i) Using the substitution $Y = 3^x$, show that the equation

$$9^x - 3^{x+1} + 2 = 0$$

can be written as

$$(Y-1)(Y-2) = 0$$ *(2 marks)*

(ii) Hence show that the equation $9^x - 3^{x+1} + 2 = 0$ has a solution $x = 0$ and, by using logarithms, find the other solution, giving your answer to four decimal places. *(4 marks)*

Recall the general shape of an exponential curve, and if in doubt substitute some values for x into the function. In particular, consider $x = 0$ to determine where the curve crosses the y-axis, and consider what happens as x approaches $\pm\infty$. To determine the correct graph transformations for part b), consider what x has been replaced by. Use the fact that $y = f\left(\frac{x}{a}\right)$ represents a stretch by scale factor a in the x direction, and $y = f(x-a)$ represents a translation by $\begin{bmatrix} a \\ 0 \end{bmatrix}$. You may also, for part ii, rewrite in the form $y = k\,f(x)$ using index rules, and consider the transformation a stretch by scale factor k in the y direction. For part c), take care to write 9^x and 3^{x+1} in terms of 3^x by using index laws, in order to make the substitution. The resulting quadratic should factorise as shown. For the final part, you must use what you have previously proven (that's what 'hence' tells you). You have already found a way to write this equation in the form of a quadratic (which you have also factorised), so find solutions for Y, then use the connection $Y = 3^x$ to convert these into solutions for x.

E: AQA C2 Jan 2008

9 (a) Given that

$$\frac{3+\sin^2\theta}{\cos\theta - 2} = 3\cos\theta$$

show that

$$\cos\theta = -\frac{1}{2}$$ *(4 marks)*

(b) Hence solve the equation

$$\frac{3+\sin^2 3x}{\cos 3x - 2} = 3\cos 3x$$

giving all solutions in degrees in the interval $0° < x < 180°$. *(4 marks)*

Multiply both sides of the equation by $\cos\theta - 2$ and use the identity $\sin^2\theta + \cos^2\theta = 1$ to rewrite as a quadratic in $\cos\theta$. By factorising you should find two potential solutions for $\cos\theta$ and you should make sure you justify rejecting one of them. For part b), use what you have shown to simplify this closely related equation, and – taking care to modify your solution range to pick up all values of x between $0°$ and $180°$ – you should solve for $3x$ then convert to find solutions for x.

F: AQA C2 Jun 2008

3 A geometric series begins

$$20 + 16 + 12.8 + 10.24 + \ldots$$

(a) Find the common ratio of the series. *(1 mark)*

(b) Find the sum to infinity of the series. *(2 marks)*

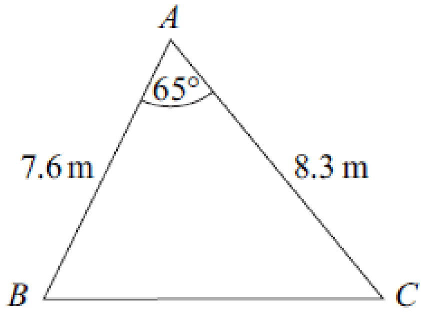
(c) Find the sum of the first 20 terms of the series, giving your answer to three decimal places. *(2 marks)*

(d) Prove that the nth term of the series is 25×0.8^n. *(2 marks)*

The common ratio can be found by dividing any term by the previous one. The sum to infinity can be found using the formula (provided in the formula book) $S_\infty = \frac{a}{1-r}$. The sum of the first n terms (also given in the formula book) can be found using $S_n = \frac{a(1-r^n)}{1-r}$. Finally, using the formula for the n^{th} term of a geometric series and rearranging using index laws, you should be able to prove the result from part d).

G: AQA C2 Jun 2008

4 The diagram shows a triangle ABC.



The size of angle BAC is 65°, and the lengths of AB and AC are 7.6 m and 8.3 m respectively.

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(b) Calculate the area of triangle ABC, giving your answer in m^2 to three significant figures. *(2 marks)*

(c) The perpendicular from A to BC meets BC at the point D.

Calculate the length of AD, giving your answer to the nearest 0.1 m. *(3 marks)*

Use the cosine rule (provided in the formula book) $a^2 = b^2 + c^2 - 2bc\cos A$ to find the length of the missing side. Use the rule $Area = \frac{1}{2}ab\sin C$ to find the area. Finally, use the first two pieces of information to deduce the length of the perpendiular by using the fact that the area of a triangle is also given by $\frac{1}{2}(base \times height)$.

H: AQA C2 Jun 2008

6 The nth term of a sequence is u_n.

The sequence is defined by

$$u_{n+1} = pu_n + q$$

where p and q are constants.

The first three terms of the sequence are given by

$$u_1 = -8 \qquad u_2 = 8 \qquad u_3 = 4$$

(a) Show that $q = 6$ and find the value of p. *(5 marks)*

(b) Find the value of u_4. *(1 mark)*

(c) The limit of u_n as n tends to infinity is L.

(i) Write down an equation for L. *(1 mark)*

(ii) Hence find the value of L. *(2 marks)*

Use the sequence definition and substitute example values of terms to form two equations in p and q. Solve simultaneously (elimination works well in this case) to determine the values of p and q. Use these values to write the sequence definition in full, then use this to calculate the fourth term, using the third one. Finally, recall that if the n^{th} term of a sequence tends towards some limit L, so does the $(n+1)^{th}$ term. Another way to put this is that as the terms of the sequence approach a limit, the terms of the sequence approach one another (become indistinguishably close). The limit can be found by replacing U_n and U_{n+1} both by L.

I: AQA C2 Jun 2008

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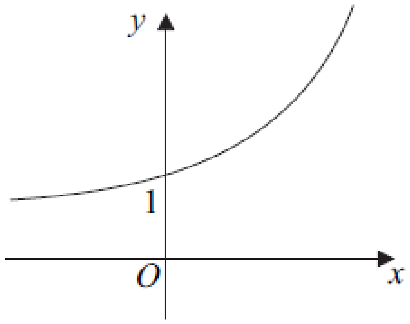

(a) (i) Use the trapezium rule with five ordinates (four strips) to find an approximate value for $\int_0^2 6^x \, dx$, giving your answer to three significant figures. *(4 marks)*

(ii) Explain, with the aid of a diagram, whether your approximate value will be an overestimate or an underestimate of the true value of $\int_0^2 6^x \, dx$. *(2 marks)*

(b) (i) Describe a single geometrical transformation that maps the graph of $y = 6^x$ onto the graph of $y = 6^{3x}$. *(2 marks)*

(ii) The line $y = 84$ intersects the curve $y = 6^{3x}$ at the point A. By using logarithms, find the x-coordinate of A, giving your answer to three decimal places. *(4 marks)*

(c) The graph of $y = 6^x$ is translated by $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ to give the graph of the curve with equation $y = f(x)$. Write down an expression for $f(x)$. *(2 marks)*

Use the formula for Trapezium Rule given in the formula book, taking care to use the correct number of ordinates and calculate the correct 'height' of the strips. The five x ordinates will be $x_0 = 0$, $x_1 = 0.5$, …, $x_4 = 2$. To determine whether your estimate is larger or smaller than the real area, draw a diagram which exaggerates the curve sufficiently to show whether straight lines to form the trapezia gives a larger or a smaller area than that enclosed by the curve. To find the transformation for part b), consider how the input value x has been altered. Recall that $y = f\left(\frac{x}{a}\right)$ represents a stretch of the original function $y = f(x)$ by scale factor a in the x direction. For part c), use the fact that a translation by vector $\begin{bmatrix} a \\ b \end{bmatrix}$ transforms $y = f(x)$ into $y = f(x - a) + b$.

J: AQA C2 Jun 2007

8 (a) It is given that n satisfies the equation

$$\log_a n = \log_a 3 + \log_a(2n - 1)$$

Find the value of n. *(3 marks)*

(b) Given that $\log_a x = 3$ and $\log_a y - 3\log_a 2 = 4$:

(i) express x in terms of a; *(1 mark)*

(ii) express xy in terms of a. *(4 marks)*

Use $\log A + \log B = \log AB$ to combine the logarithms on the right-hand side, then raise a to the power of both sides to get rid of the logs. For part b), raise a to the power of both sides to find x, and use $n \log A = \log A^n$ along with $\log A - \log B = \log \frac{A}{B}$ to combine the logarithms on the left-hand side, then raise a to the power of each side and rearrange to find y. Use your values (both in terms of a) to find xy.

K: AQA C2 Jun 2006

4 (a) The expression $(1-2x)^4$ can be written in the form

$$1 + px + qx^2 - 32x^3 + 16x^4$$

By using the binomial expansion, or otherwise, find the values of the integers p and q. *(3 marks)*

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L: AQA C2 Jun 2006

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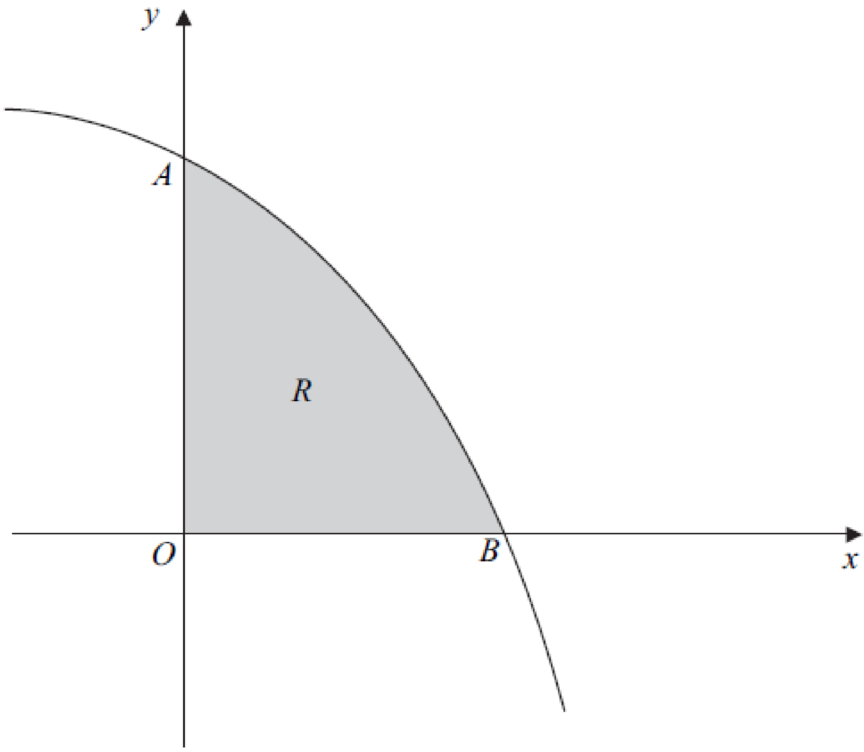


The curve $y = 27 - 3^x$ intersects the y-axis at the point A and the x-axis at the point B.

(a) (i) Find the y-coordinate of point A. *(2 marks)*

(ii) Verify that the x-coordinate of point B is 3. *(1 mark)*

(b) The region, R, bounded by the curve $y = 27 - 3^x$ and the coordinate axes is shaded. Use the trapezium rule with four ordinates (three strips) to find an approximate value for the area of R. *(4 marks)*

(c) (i) Use logarithms to solve the equation $3^x = 13$, giving your answer to four decimal places. *(3 marks)*

(ii) The line $y = k$ intersects the curve $y = 27 - 3^x$ at the point where $3^x = 13$. Find the value of k. *(1 mark)*

(d) (i) Describe the single geometrical transformation by which the curve with equation $y = -3^x$ can be obtained **from** the curve $y = 27 - 3^x$. *(2 marks)*

(ii) Sketch the curve $y = -3^x$. *(2 marks)*

Tricky Core 2 Questions MARK SCHEME

104 marks (approx 2 hours at the rate of 75 marks per 90 minutes)

A: AQA C2 Jan 2008

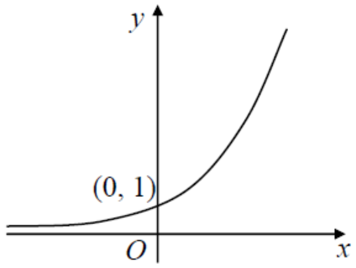
Q	Solution	Marks	Total	Comments
1(a)	Area of sector = $\frac{1}{2}r^2\theta = \frac{1}{2}\times 6^2 \times \theta$	M1		$\frac{1}{2}r^2\theta$ seen or used
	$6 \times 3 = 2 \times \frac{1}{2}\times 6^2 \times \theta$	m1		OE Forming equation
	$36\theta = 18 \Rightarrow \theta = 0.5$	A1	3	AG
(b)	Arc = 6θ;	M1		$r\theta$ seen or used
	….. = 3 cm	A1		PI by a correct perimeter
	$\Rightarrow$ Perimeter = 12 + arc = 15 cm	A1F	3	Ft wrong evaluation of 6θ. Condone missing/wrong units throughout the question.
	Total		**6**	

B: AQA C2 Jan 2008

Q	Solution	Marks	Total	Comments
7(a)	$x = 8$	B1	1	No clear log law errors seen. Condone answer left as $\frac{16}{2}$
(b)	$\log_a y = \log_a 3^2 + \log_a 4 + 1$	M1		One law of logs used correctly
	$\log_a y = \log_a \left(3^2 \times 4\right) + 1$	M1		Either a second law of logs used correctly or the 1 written as $\log_a a$
	$\log_a y = \log_a \left(3^2 \times 4\right) + \log_a a = \log_a 36a$			
	$\Rightarrow y = 36a$	A1	3	CSO
	Total		**4**	

Q	Solution	Marks	Total	Comments
7(a)	$\left(1+\frac{4}{x^2}\right)^3=$			
	$[1^3]+3(1^2)\left(\frac{4}{x^2}\right)+3(1)\left(\frac{4}{x^2}\right)^2+\left[\left(\frac{4}{x^2}\right)^3\right]$	M1		Any valid method as far as term(s) in $1/x^2$ and term(s) in $1/x^4$
	$=[1]+\frac{12}{x^2}+\frac{48}{x^4}+\left[\frac{64}{x^6}\right]$	A1		$p=12$ Accept $\frac{12}{x^2}$ even within a series
		A1	3	$q=48$ Accept $\frac{48}{x^4}$ even within a series
(b)(i)	$\int\left(1+\frac{4}{x^2}\right)^3\,\mathrm{d}x$			
	$=\int\ (1+\frac{p}{x^2}+\frac{q}{x^4}+\frac{64}{x^6})\,\mathrm{d}x$	M1		Integral of an 'expansion', at least 3 terms PI by the next line
	$=x-px^{-1}-\frac{q}{3}x^{-3}-\frac{64}{5}x^{-5}\ (+c)$	m1 A2F,1	4	At least two powers correctly obtained Ft on c's non-zero integer values for p and q (A1F for two terms correct; can be unsimplified)
	$=x-12x^{-1}-16x^{-3}-\frac{64}{5}x^{-5}\ (+c)$			Condone missing c but check that signs have been simplified at some stage before the award of both A marks.
(ii)	$\left(2-\frac{p}{2}-\frac{q}{3(8)}-\frac{64}{5(32)}\right)-$ $\left(1-p-\frac{q}{3}-\frac{64}{5}\right)$	M1		F(2) − F(1), where F(x) is cand's answer or the correct answer to (b)(i).
	$=\ 33.4$	A1	2	CSO
	Total		**9**	

D: AQA C2 Jan 2008

Q	Solution	Marks	Total	Comments
8(a)		B1		Shape (graph must clearly go below the intersection pt.). Condone if x-axis is a tangent
		B1	2	Only intersection with y-axis at (0, 1) stated/indicated … (accept 1 on y-axis as equivalent) 0
(b)(i)	Stretch **(I)** in x-direction **(II)** scale factor 0.5 **(III)**	M1 A1	2	Need**(I)** & one of **(II)**,**(III)** M0 if >1 transformation
(ii)	Translation;	B1;		Must be 'Translation' or 'translate(d)' for 1st B mark
	$\begin{bmatrix} -1 \\ 0 \end{bmatrix}$	B1	2	Accept **full** equivalent to vector in words provided linked to 'translation/ move/shift' and **negative** x-direction (Note: B0 B1 is possible)
	ALTn: Stretch **(I)** in y-direction **(II)** scale factor 3 **(III)**			[Mark the alternative as in (b)(i).]
(c)(i)	$9^x = (3^2)^x = 3^{2x} = (3^x)^2 = Y^2$; $3^{x+1} = 3^x \times 3^1 = 3Y$	M1		Justifying either $9^x = Y^2$ or $3^{x+1} = 3Y$
	$9^x - 3^{x+1} + 2 = 0 \Rightarrow Y^2 - 3Y + 2 = 0$ $\Rightarrow (Y-1)(Y-2) = 0$	A1	2	AG
(ii)	$Y = 1 \Rightarrow \quad 3^x = 1 \Rightarrow x = 0$	B1		AG (Accept direct substitution if convinced)
	$Y = 2 \Rightarrow \quad 3^x = 2$ $\log_{10} 3^x = \log_{10} 2$	M1		Takes logs of both, PI by 'correct' value(s) later. or $x = \log_3 2$ seen
	$x \log_{10} 3 = \log_{10} 2$	m1		Use of $\log 3^x = x \log 3$ or $\log_3 2 = \dfrac{\lg 2}{\lg 3}$ OE (PI by $\log_3 2 = 0.630$ or 0.631 or better)
	$x = \dfrac{\lg 2}{\lg 3} = 0.630929....$ =0.6309 to 4dp	A1	4	Must show that logarithms have been used otherwise 0/3
	Total		**12**	

Q	Solution	Marks	Total	Comments
9(a)	$\dfrac{3+\sin^2\theta}{\cos\theta-2}=3\cos\theta$			
	$\Rightarrow \dfrac{3+(1-\cos^2\theta)}{\cos\theta-2}=3\cos\theta$	M1		$\cos^2\theta + \sin^2\theta = 1$ stated or used [If cand starts with cos $\theta = -½$ and gets $\sin^2\theta = ¾$ without explicitly finding value for θ and verifies 1st equation is true, award M1moA0]
	$\Rightarrow \dfrac{4-\cos^2\theta}{\cos\theta-2}=3\cos\theta$			
	$\Rightarrow \dfrac{(2-\cos\theta)(2+\cos\theta)}{\cos\theta-2}=3\cos\theta$	m1		Difference of two squares or division (PI by next line)
	$\Rightarrow -1(2+\cos\theta)=3\cos\theta$	A1		
	$\Rightarrow -2=4\cos\theta \Rightarrow \cos\theta=-\dfrac{1}{2}$	A1	4	CSO AG
	Alternative for (a)			
	$3+1-\cos^2\theta=3\cos^2\theta-6\cos\theta$	(M1)		$\cos^2\theta + \sin^2\theta = 1$
	$(4\cos\theta+2)(\cos\theta-2)=0$	(m1)		Factorising or formula
	$\cos\theta - 2 \neq 0$	(A1)		Indicates rejection of $\cos\theta=2$
	$\Rightarrow 4\cos\theta=-2 \Rightarrow \cos\theta=-\dfrac{1}{2}$	(A1)		AG Be convinced
(b)	$\theta = 3x \Rightarrow \cos 3x = -\dfrac{1}{2}$	M1		Uses part (a) to reach either $\cos 3x = -0.5$ or $\cos 3x = 0.5$
	$\cos^{-1}\left(-\dfrac{1}{2}\right) = 120°$	m1		Or $\cos^{-1}(0.5) = 60°$ Condone radians here
	$3x = 120°, 240°, 480°, \ldots$			
	$x = 40°, 80°, 160°$	A2,1,0	4	A1 for at least two correct. If >3 solutions in the interval $0° < x < 180°$, deduct 1 mark from any A marks for each extra solution. Deduct 1 mark from any A marks if answers in radians. Ignore extra values outside the given interval.
	Total		**8**	

F: AQA C2 Jun 2008

3(a)	$r = 16 \div 20 = 0.8$	B1	1	OE
(b)	$\dfrac{a}{1-r} = \dfrac{20}{1-0.8}$	M1		OE Using a correct formula with $a = 20$ or $r =$ c's 0.8
	$= 100$	A1F	2	ft on c's value of r provided $\lvert r \rvert < 1$
(c)	$\{S_{20} =\} \dfrac{a(1-r^{20})}{1-r}$	M1		OE Using a correct formula with $n = 20$
	$= 100(1-0.8^{20}) = 98.847\{07..\}$	A1	2	Condone > 3dp
(d)	$n\text{th term} = 20\, r^{n-1} = 20(0.8)^{n-1}$	M1		Ft on c's r . Award even if 16^{n-1} seen
	$= 20 \times 0.8^{-1} \times 0.8^n$			
	$= 25 \times 0.8^n$	A1	2	CSO; AG
	Total		**7**	

G: AQA C2 Jun 2008

Q	Solution	Marks	Total	Comments
4(a)	$\{BC^2 =\}\ 7.6^2 + 8.3^2 - 2 \times 7.6 \times 8.3 \cos 65$	M1		RHS of cosine rule used
	….. $= 57.76 + 68.89 - 53.3175...$	m1		Correct order of evaluation
	$BC = \sqrt{73.33..} = 8.563..\quad (= 8.56 \text{ m})$	A1	3	AG; must see $\sqrt{73.33....}$ or > 3sf value
(b)	Area triangle $= \dfrac{1}{2} \times 7.6 \times 8.3 \times \sin 65$	M1		Use of $\dfrac{1}{2} bc \sin A$ OE
	$= 28.58\ldots = 28.6\ (\text{m}^2)$	A1	2	Condone > 3sf
(c)	Area of triangle $= 0.5 \times BC \times AD$	M1		Or valid method to find sin$B$ or sin$C$
	AD = [Ans (b)] ÷ [0.5×Ans (a)]	m1		Or AD = 7.6sin$B$; Or AD = 8.3sin$C$
	$AD = 6.67..\ = 6.7$ (m)	A1	3	If not 6.7 accept 6.65 to 6.69 inclusive.
	Total		**8**	

H: AQA C2 Jun 2008

6(a)	$8 = -8p + q$	M1		Either equation. PI eg by combined eqn.
	$4 = \ 8p + q$	A1		Both (condone embedded values for the M1A1)
		m1		Valid method to solve two simultaneous equations in p **and** q to find either p or q
	$q = 6$	A1		AG (condone if left as a fraction)
	$p = -0.25$	B1	5	OE
(b)	$u_4 = 5$	B1F	1	Ft on $(6 + 4p)$
(c)(i)	$L = pL + q\,;\quad (L = -0.25\,L + 6)$	M1	1	OE
(ii)	$L = \dfrac{q}{1-p}$	m1		Rearranging
	$L = \dfrac{6}{1.25} = 4.8$	A1F	2	Ft on $\dfrac{6}{1-p}$ Dependent on previous two marks
	Total		**9**	

I: AQA C2 Jun 2008

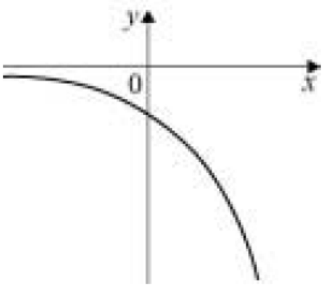
Q	Solution	Marks	Total	Comments
8(a)(i)	$h = 0.5$ Integral = $h/2$ {……}	B1		PI
	$\{..\}=f(0)+2[f\left(\frac{1}{2}\right)+f(1)+f\left(\frac{3}{2}\right)]+f(2)$	M1		OE summing of areas of the four traps.
	$\{\}= 1+2\left[\sqrt{6}+6+6\sqrt{6}\right]+36$ $= 1+2[2.449..+ 6 + 14.6969..] + 36$ $= 37 + 2 \times 23.146.. = 83.292\ldots$	A1		Condone 1 numerical slip. Accept 3sf values if not exact.
	Integral = $0.25 \times 83.292.. = 20.8$ (3sf)	A1	4	CAO; must be 20.8
(ii)	Relevant trapezia drawn on a copy of given graph	M1		Accept single trapezium with its sloping side above the curve
	{Approximation is an}overestimate	A1	2	Dep. on 4 trapezia with each of their upper vertices lying on the curve
(b)(i)	Stretch **(I)** in x-direction **(II)**	M1		Need **(I)** and one of **(II)**, **(III)** M0 if more than one transformation
	(scale factor) $\frac{1}{3}$ **(III)**	A1	2	
(ii)	$6^{3x} = 84$	M1		PI
	$\log_{10} 6^{3x} = \log_{10} 84$	M1		Take logs of both sides of $a^x = b$, PI by 'correct' value(s) later or $3x = \log_6 84$
	$3x\log_{10} 6 = \log_{10} 84$	m1		Use of $\log 6^{3x} = 3x\log 6$ OE or $3x = \log_6 84$ seen
	$x = \frac{\lg 84}{3\lg 6}$			
	$x =$ 0.82429…. $= 0.824$ (to 3dp)	A1	4	Must see that logs have been used before any of the last 3 marks are awarded in (b)(ii). Condone > 3dp
(c)	$f(x) = 6^{x-1} - 2$	B2,1	2	B1 for either $6^{x-1}+2$ or for $6^{x+1}-2$
	Total		**14**	

J: AQA C2 Jun 2007

Q	Solution	Marks	Total	Comments
8(a)	$\log_a n = \log_a 3(2n-1)$	M1		OE Log law used PI by next line
	$\Rightarrow n = 3(2n-1)$	m1		OE, but must **not** have any logs.
	$\Rightarrow 3 = 5n \Rightarrow n = \frac{3}{5}$	A1	3	
(b)(i)	$\log_a x = 3 \Rightarrow x = a^3$	B1	1	
(ii)	$\log_a y - \log_a 2^3 = 4$	M1		$3\log 2 = \log 2^3$ seen or used any time in **(ii)**
	$\log_a \frac{y}{2^3} = 4 \begin{cases} xy = a^7 \times a^{(3\log_a 2)} \\ \text{or} \\ y = a^4 \times a^{(3\log_a 2)} \end{cases}$	M1		Correct method leading to an equation involving y (or xy) and a log but **not** involving + or −
	$\frac{y}{2^3} = a^4 \quad \begin{cases} xy = a^7 \times 2^3 \\ \text{or} \\ y = a^4 \times 2^3 \end{cases}$	ml		Correct method to eliminate **ALL** logs e.g. using $\log_a N = k \Rightarrow N = a^k$ or using $a^{\log_a c} = c$
	$by = a^3 \times 8a^4$ or $8a^7$	A1	4	
	Total		**8**	

K: AQA C2 Jun 2006

Question	Solution	Marks	Total	Comments
4(a)	$(1-2x)^4 = (1)^4 + 4(1)^3(-2x) + 6(1^2)(-2x)^2 + [4(1)(-2x)^3 + (-2x)^4]$	M1		Any valid method as far as term(s) in x and term(s) in x^2.
	$= [1] - 8x + 24x^2 + [-32x^3 + 16x^4]$	A1		$p = -8$ Accept $-8x$ even within a series.
		A1	3	$q = 24$ Accept $24x^2$ even within a series.
(b)	x term is $\binom{9}{1} 2^8 x$	M1		OE
	Coefficient of x term is $= 9 \times 2^8 = 2304$ $(=k)$	A1	2	Condone $2304x$
(c)	$(1-2x)^4 (2+x)^9 = (1+px+..)(2^9+kx...)$	M1		Uses (**a**) and (**b**) oe (PI)
	$= \ldots$			
	$= \ldots..+ kx + px(2^9) + \ldots..$	M1		Multiply the two expansions to get x terms
	Coefficient of x is $k + 512p$			
	$= 2304 - 4096 = -1792$	A1ft	3	ft on candidate's values of k and p. Condone $-1792x$ SC If 0/3 award B1ft for $p+k$ evaluated
	Total		**8**	

Question	Solution	Marks	Total	Comments
6(a)(i)	y-coordinate of A is $27-3^0$; $=26$	M1A1	2	
(ii)	When $x=3$, $y=27-3^3=0 \Rightarrow B(3,0)$	B1	1	AG; be convinced
(b)	$h=1$	B1		PI
	Area $\approx h/2\{\ldots\}$ $\{\ldots\}=$ f(0)+f(3)+2[f(1)+f(2)] $\{\ldots\}=$ "26" + 0 + 2(24 + 18)	 M1 A1√		 OE summing of areas of the 'trapezia'.. on (a)(i) ($\sum$trap= "25"+21+9)
	(Area $\approx$) 55	A1√	4	on [42 + 0.5× "(a)(i)"]
(c)(i)	$\log_{10} 3^x = \log_{10} 13$	M1		Takes ln or $\log_{10}$ on both or $x=\log_3 13$
	$x\log_{10} 3 = \log_{10} 13$	m1		Use of $\log 3^x = x\log 3$ or $\log_3 13 = \frac{\lg 13}{\lg 3}$ OE (PI by $\log_3 13 = 2.335$ or better)
	$x=\frac{\lg 13}{\lg 3}=2.334717\ldots$ = 2.3347 to 4dp	A1	3	Must show that logarithms have been used
(ii)	$\{k=\}$ 14	B1	1	Condone $y=14$; Accept final answer 14 with only zeros after decimal point eg 14.000
(d)(i)	Translation;	B1;		'Translation'/'translate(d)' B0 if more than one transformation
	$\begin{bmatrix} 0 \\ -27 \end{bmatrix}$	B1	2	Accept full equivalent in words provided linked to 'translation/move/shift' and negative y-direction (Note: B0 B1 is possible)
(ii)		B1 B1	 2	Correct shape (translation of given curve vertically downwards) Only point of intersection with coord axes is on negative y-axis and curve is asymptotic to the negative x-axis
	Total		**15**	