All About Energy

Conservation of Energy, Work Done, Kinetic, Gravitational Potential, Variable Forces, Elastic Potential

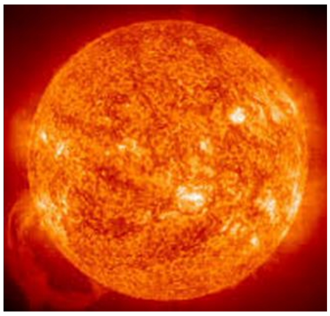
Definition: *Energy* is the capacity to do work.

Not the same as force: *A force may simply balance out another force (like when you lean on a door and it leans back on you), but work is done (energy is transferred) whenever a force makes something happen (like when you lean on a door and it swings open).*

Not the same as power: *Power is a measure of how quickly energy is transferred over time. So a car that can convert chemical energy to kinetic energy very quickly has a lot of power. Any kettle can boil water (about 300 kJ will boil a litre of water), but those with more power can do it faster (a 2 kW kettle will do the job in a minute and a half).*

Conservation of Energy

Energy can be thought of as the *currency of the universe*.
There's a fixed amount available, and it never gets lost or used up, merely transferred from one account to another.
We get most of our energy from the sun, transferred through radiation often via chemical energy stored by plants and animals.

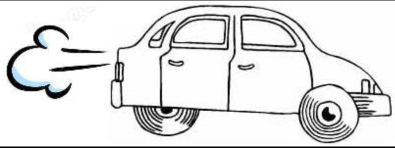


Our entire civilization depends on the controlled transfer of energy from one form to another.

We can use our personal stores of energy to 'do work'.
We exert a force, *and* make something happen as a result:

- The further you push a car, the more energy you have to spend to do it. The energy is not lost – it is converted into movement energy (kinetic) and heat and sound in the environment.
- The higher up stairs you climb, the more energy it takes. But the energy is not lost. Some is still dissipated into the environment, but much is stored as gravitational potential, and can easily be reclaimed by reducing your height. (Don't believe me? Go jump off a roof.)

Example: *What happens to energy during a drive to the shops?*

Type of Energy	Start	Middle	End
 **Chemical** *Total change:* *30 MJ less*	**1500 MJ** *10 gallons of petrol*	**1485 MJ** *5 miles at 45 mpg*	**1470 MJ** *10 miles at 45 mpg*
 **Kinetic** *Total change:* *none*	**0 MJ** *not moving*	**0.135 MJ** *1.5 tons at 30 mph*	**0 MJ** *not moving*
 **Gravitational Potential** *Total change:* *0.5 MJ less*	**1.5 MJ** *100 metres elevation*	**1.6 MJ** *110 metres elevation*	**1 MJ** *70 metres elevation*
 **Thermal, Sound, Etc.** *Total change:* *30.5 MJ more*	**0 MJ** *nothing yet*	**14.9 MJ** *transferred to environment*	**30.5 MJ** *transferred to environment*

Types of Energy

The most commonly used energy forms are: **Kinetic** (movement), **Gravitational Potential** (work done against gravity) and **Elastic Potential** (work done against tension).

Example: *What happens when you fall off a bike?*

Energy is converted from kinetic (movement) energy to heat and sound when you fall off your bike and slide to a stop. That energy is still out there, but is not easy to reclaim.

However, if you convert your kinetic energy into gravitational potential by cycling up a hill, you can easily reclaim that (turn it into kinetic energy again) by cycling back down.

How much energy?


Chemical energy stored in 4 Weetabix.


Chemical energy stored in a drill battery.


Chemical energy stored in 1 litre of petrol.

Chemical energy stored in an AA battery.


Electrical energy converted to heat and light by a single lightning strike.

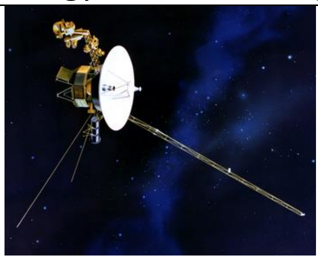
Gravitational potential energy of a penny at the top of the Eiffel Tower.


Total energy usage (gas and electricity) of an average UK household per day.


Kinetic energy of an AK-47 bullet (mass: $8g$, speed: $1600mph$).

Chemical energy stored in a bag of crisps.


Energy generated by Grand Coulee daily.


Kinetic energy of Voyager 2 space probe (mass: $700kg$, speed: $60,000\ km/h$).

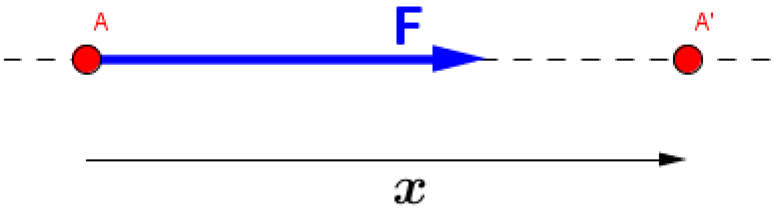
Chemical energy stored in a single pint of Guinness.

Definition: *Work done* is energy transferred by a force.

The work is done by a force in a given direction, and the greater the distance moved in the direction of the force, the more work is said to be done by the force.

$$Work\ Done = Force \times Distance$$

One Newton of force exerted over a distance of **one metre** transfers **one joule** of energy.



Example: *How does work done affect stored energy?*

Pulling a sledge $4m$ across the ice by a horizontal rope with a tension of $70N$ will transfer a total of $70 \times 4 = 280\ Joules$ of energy. If we assume the ice is completely frictionless, that means all the energy is converted into kinetic energy.

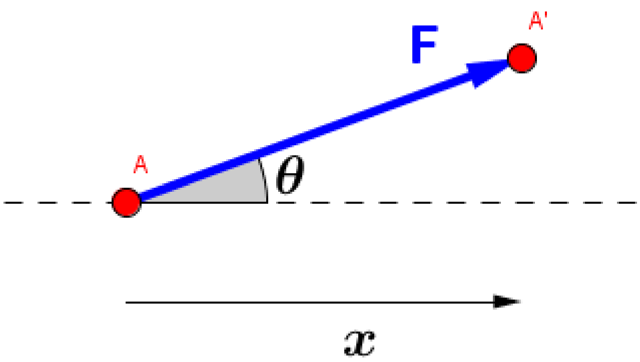
Since all this energy is transferred into kinetic energy for the sledge, it must have $280\ J$ of kinetic energy by the end.

OK, but what if you don't move *exactly* in the direction of the force?

Or, equivalently, what if the force doesn't move you *exactly* in the direction it's pulling?

There are two ways to think of this. Either, like in our original definition, as the force (F) multiplied by the distance moved *in that direction* ($x \cos\theta$) or as the *component of force in our direction* ($F \cos\theta$) multiplied by the distance (x). (We can ignore the perpendicular component since it doesn't do any work). Either way, we get:

$$Work\ Done = Fx\cos\theta$$



Note: a consequence of this is that a force acting in the *opposite* direction to motion does a *negative* amount of work.

We say work is done *against* friction, for instance. This reduces the overall energy of the system (transferring it out into the environment in the form of heat, etc.)

Gravitational Potential Energy

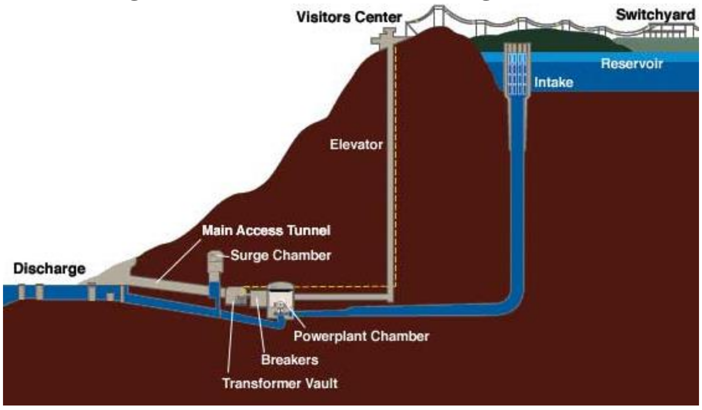
If you move in the direction of your weight (vertically) then the force of gravity is doing work. Since the total distance moved in the direction of that force is your height, we have:

$$Gravitational\ Potential\ Energy\ (GPE) = mgh$$

$$m = mass\ (kg) \quad g = acceleration\ due\ to\ gravity\ (ms^{-2}) \quad h = height\ (m)$$

We treat the force of gravity slightly differently to other forces because it is so easy for work to be done both *by* the force of weight and *against* the force of weight. This means you can effectively store energy by lifting things up, and then release the stored energy by letting them come back down.

Pumped-storage hydroelectric facilities use GPE to store energy, pumping water to a high reservoir during off-peak times when energy is cheap, and releasing it to sell back to the grid when demand is high:



Kinetic Energy

We have already considered kinetic energy as a consequence of a force doing work. By combining $F = ma$ and $v^2 = u^2 + 2as$ we can analyse the situation to determine the formula directly based on speed:

$$F = ma \quad \Longrightarrow \quad a = \frac{F}{m} \quad \Longrightarrow \quad v^2 = u^2 + 2\left(\frac{F}{m}\right)x \quad \Longrightarrow \quad Fx = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

Taking $u = 0$ as no kinetic energy, this gives:

$$Kinetic\ Energy\ (KE) = \frac{1}{2}mv^2$$

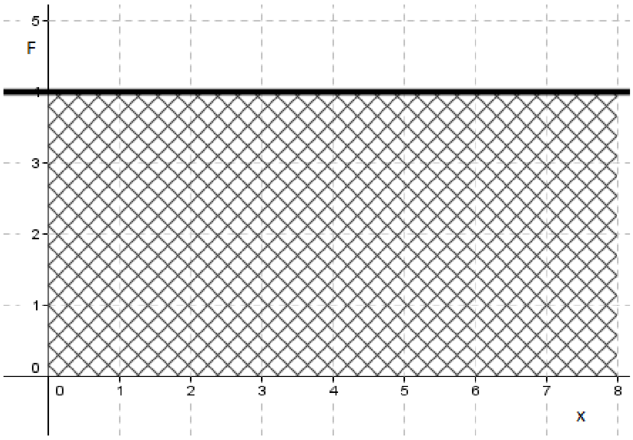
$$m = mass\ (kg) \quad v = speed\ (ms^{-1})$$

OK, but what if the force *isn't* constant?

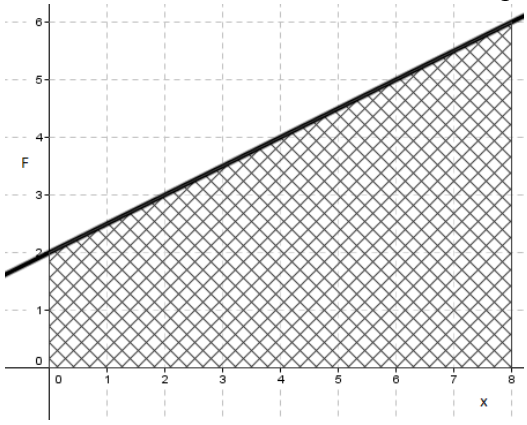
Most real situations will involve forces that change. Friction varies with the normal reaction, air resistance typically varies with speed, and the tension in an elastic string varies depending on how much you've stretched it.

So we have to extend our definition. Imagine that instead of a constant force, we had the next best thing: a constantly increasing force.

Constant force

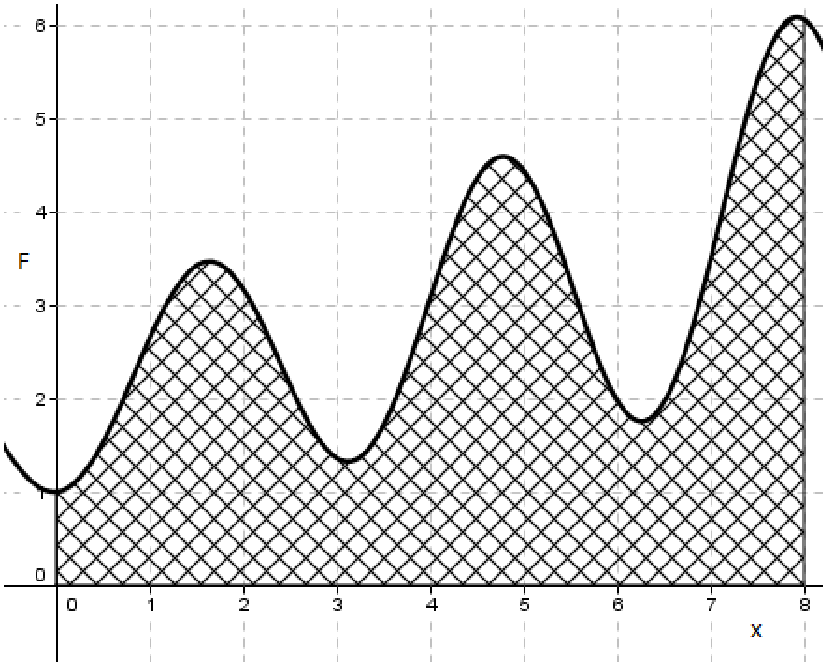


Force with constant rate of change



In the first case, our original definition, $Work\ Done = Fx$, is sufficient. In the second case, the *average force* still allows us to calculate Work Done in the same way: Not as much energy is transferred during the first half of the journey, but more is transferred in the second half to make up for it. Note that these are both just the area under the graph.

For any force whatsoever, we will need a better way to calculate the area. Considering the instantaneous rate at which energy is transferred is equivalent to using calculus. Imagine splitting the shape below into thin, almost-rectangular strips. To find the total area we need to integrate the force, F, with respect to x. This is equivalent to adding up the energy transferred in every single moment:



In general, for a force given as a function of distance $F(x)$, the work done by the force between $x = a$ and $x = b$ is given by:

$$\int_a^b F(x)\,dx$$

Elastic Potential Energy

This is an example of work done against a force which changes as the distance changes. Hooke's law tells us that the tension in an elastic string with natural length l and modulus of elasticity λ is given by:

$$T = \frac{\lambda x}{l}$$

Where x is the extension of the string (which is equivalent to distance travelled in the (opposite) direction of the force.

By integrating this force between $x = 0$ and $x = e$ we get:

$$Work\ Done\ against\ tension = \int_0^e \frac{\lambda x}{l}\, dx = \left[\frac{\lambda x^2}{2l}\right]_0^e = \frac{\lambda e^2}{2l} = Energy\ stored\ by\ string$$

$$EPE = \frac{\lambda e^2}{2l}$$

$$\lambda = modulus\ of\ elasticity\ (N) \quad l = natural\ length\ (m) \quad e = extension\ (m)$$

How much energy? Solutions

In order of size:

Penny	$10\ J$
AK-47 bullet	$2\ kJ$ (or $2 \times 10^3\ J$)
AA battery	$10\ kJ$ (or $1 \times 10^4\ J$)
Drill battery	$100\ kJ$ (or $1.3 \times 10^5\ J$)
Bag of crisps	$700\ kJ$ (or $7 \times 10^5\ J$)
Pint of Guinness	$900\ kJ$ (or $9 \times 10^5\ J$)
4 Weetabix	$2\ MJ$ (or $2 \times 10^6\ J$)
1 litre of petrol	$30\ MJ$ (or $3 \times 10^7\ J$)
UK household daily	$40\ MJ$ (or $4 \times 10^7\ J$)
Lightning strike	$5\ GJ$ (or $5 \times 10^9\ J$)
Voyager 2 space probe	$100\ GJ$ (or $1 \times 10^{11}\ J$)
Grand Coulee Dam daily	$200{,}000\ GJ$ (or $2 \times 10^{14}\ J$)

Values rounded to 1 significant figure.

$1\ GJ = 1000\ MJ,\ 1\ MJ = 1000\ kJ,\ 1\ kJ = 1000J$