Summary Method C4 (Jun '11)

1.
a)
Substitute the value $x = -2$ into the function.
b)
Recall that: $(ax + b)\ is\ a\ factor \iff x = -\frac{b}{a}\ is\ a\ root.$
c)
 Factorise the numerator, write $f(x)$ in its factorised form and cancel any common factors.

2.
a)
i.
Substitute known values from the initial conditions ($P = 80\ at\ t = 0$) and solve for A.
ii.
Substitute values from the second set of conditions ($P = 2000\ at\ t = 25$) and solve for k.
b)
Write out the expression with the calculated values of A and k, and solve the inequality achieved by putting $P > 100000$.

3.
a)
i.
Use the formula from the formula book.
ii.
Rearrange the bracket to change the first term into a 1. Remember that 'taking out' the 127 will mean a factor of $127^{\frac{1}{3}}$ outside the bracket. Substitute your resulting second term in place of x in your answer to part i and simplify.
b)
Set the inside of the bracket equal to 119 to find the appropriate value of x to use, and substitute into your answer for part a i.

4.
a)
i.
Find $\frac{dx}{dt}$ and $\frac{dy}{dt}$, and divide one by the other to find $\frac{dy}{dx}$.
ii.
Substitute the given value of θ into the expressions for x, y and $\frac{dy}{dx}$, and recall the gradient of a normal is given by $m = \frac{-1}{\frac{dy}{dx}}$.
b)
Use the double angle formula for $\cos 2x$ in terms of $\sin^2 x$ to rewrite, then integrate (remembering to divide by the differential of $2x$ when integrating $\cos 2x$).

5.
a)
Subtract one point from the other to find $\overrightarrow{AB}$ or $\overrightarrow{BA}$ - this will be the direction vector. Use either of the points for the position vector and construct the line equation: $r = [position] + \lambda[direction]$.

b)

i.

Find the general point of the line from part a, and set this equal to the general point of the line l. Solve the resulting three simultaneous equations. If all the solutions are consistent (ie, there are no contradictions), then the lines intersect, and they intersect at the points given by the values of λ and μ you have found.

ii.

Draw a diagram to visualise the situation. If PBC has a right angle at B then $\overrightarrow{PB}$ is perpendicular to $\overrightarrow{BC}$. This means their dot product is zero. Since C lies on l, it is equal to the general point for some value of μ. The other two points you know, so find the dot product in terms of μ, set equal to 0 and solve for μ. Substitute this into the general point and you have C.

6.

a)

Substitute the values given for x and y into the equation, rearrange and simplify, solving for C. Note: the question asks for an exact value, so leave in terms of e. Recall that $e^0 = 1$.

b)

Differentiate implicitly: y will differentiate to $\frac{dy}{dx}$, and more complex expressions will require chain or

product rule. Finally, rearrange to make $\frac{dy}{dx}$ the subject.

c)

Substitute the values given for x and y, this time into your expression for $\frac{dy}{dx}$. Recall that at all stationary

points, $\frac{dy}{dx} = 0$.

7.

a)

Since $k > 0$, you need to take into account the fact that this is exponential *decay* rather than growth in

your choice of sign. The rate at which the surface area is increasing is $\frac{dA}{dt}$ (and since it is decreasing, this

will be negative).

b)

i.

First, rearrange and solve the differential equation using separation of variables. Then use the surface area formula to find values for your initial conditions, substitute these into your solution to find the arbitrary constant and rewrite in full. Rearrange to the form required.

ii.

To melt completely the snowball will have $A = 0$. Substitute this into the formula from part i and solve for t.

8.

a)

Add the fractions from the right hand side, ensuring they first have a common denominator (make it the same as the one on the left). Then either compare coefficients (constants, x terms and x^2 terms) from the numerators, or choose values to substitute in for x to eliminate all but one of A, B and C in order to find their values.

b)

Separate the variables, then compare the required integration to part a to simplify. Part of this can be

solved using the rule $\int \frac{f'(x)}{f(x)} = \ln f(x)$. The other part can be solved either by inspection (work

backwards from chain rule) or, more straightforwardly if slightly longer, by a simple substitution $(u = 1 - x)$. Substitute the values given for initial conditions and rearrange into the required form.