Summary Method C2 (Jun '13)

1.

a)

Use the formula from the formula book: $U_n = ar^{n-1}$.

b)

Use the formula from the formula book: $S_\infty = \frac{a}{1-r}$.

c)

Use the formula from the formula book: $S_n = \frac{a(1-r^n)}{1-r}$.

2.

a)

Use the formula for arc length: $l = r\theta$. This can be generated from the circumference formula: $\frac{\theta}{2\pi}(2\pi r)$.

b)

Use the formula for sector area: $A = \frac{1}{2}r^2\theta$. This can be generated from the area formula: $\frac{\theta}{2\pi}(\pi r^2)$.

c)

Use the sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ and remember that you are looking for an obtuse angle, which in radians means $\frac{\pi}{2} < \theta < \pi$. Note that the $\sin^{-1} x$ function only gives the primary solution.

3.

a) i.

Use the binomial expansion formula (given in the formula book) as suggested by the question. You would still get the marks for this particular question if you multiplied out the brackets, but this takes longer and is sometimes much more impractical (eg a power of 5).

ii.

The word 'hence' means you should use the expression found in part a)i. Substitute $y = x^{-2}$ into your expansion to generate an expression for $(2 + x^{-2})^3$, and $y = -x^{-2}$ to generate an expression for $(2 - x^{-2})^3$. Finally, add the two expressions.

b) i.

Use the rule $\int x^n \, dx = \frac{x^{n+1}}{n+1}$ to integrate your expression.

ii.

Substitute the values 2 and 1 into your solution to b)i. and subtract to find the value of the definite integral.

4.

a)

Recall that an exponential graph has a constantly increasing gradient, crosses the y-axis at $(0,1)$ and has an asymptote at $y = 0$ (the x-axis).

b)

Take logs of both sides, and use the rule $\log x^n = n \log x$.

c)

Use the fact that a reflection in the y-axis is a stretch in the x direction of scale factor -1.

5.

a)

Use the trapezium rule from the formula book, making sure to make a table of values for x_0 to x_4 from 0 to 2, and a list of values for y_0 to y_4.

b)
Determine what x would need to be replaced with to convert $\sqrt{x^3+1}$ into $\sqrt{8x^3+1}$. Interpret this as a stretch.
c)
Apply the translation by using the rules for horizontal and vertical translations. Finally, substitute $x=4$ into the new expression.

6.
a)
Write all the x terms in index form, then divide each part of the numerator by the denominator.
b) i.
Differentiate using the rule $y=x^n \quad \Rightarrow \quad \frac{dy}{dx}=nx^{n-1}$.
ii.
Substitute $x=4$ into your expression for $\frac{dy}{dx}$, since this gives the gradient of the curve. Use the fact that the gradients of the tangent (same as curve) and the normal are related by the formula $m_1m_2=-1$.
iii.
Use the fact that stationary points occur when the gradient equals 0. Solve the resulting equation.

7.
a)
Use the iterative formula to form an equation in p and q. Use $U_{n+1}=U_n=24$ to form a second equation in p and q. Solve simultaneously.
b)
Use the value of p found in part a), and the value of q which can be found alongside it, to form a complete iterative formula for U_{n+1}. Use this to generate U_3.

8.
a)
Use the definition of logarithms to rearrange the expression. Note that you can check that your rearrangement works by substituting some numbers in and testing with a calculator.
b)
Simplify the left hand side into a single logarithm using the log rules, then reverse the logarithm (use the fact that 2^x is the inverse function of $\log_2 x$) and simplify.

9.
a) i.
If you can't recall the exact shape of the $y=\tan x$ graph, use your calculator to generate a few values. Note that asymptotes and crossing points should be accurately shown.
ii.
Use $\tan^{-1}(-1)$ to generate a primary solution in degrees, and use the graph (or the 180° period of the $\tan x$ function) to generate any other solutions in the given interval. You can verify your solutions by substituting them into the original equation.
b) i.
Use the formulae $\tan x=\frac{\sin x}{\cos x}$ and $\sin^2 x+\cos^2 x=1$ to rewrite the left hand side initially in terms of $\sin\theta$ and $\cos\theta$, rearrange the equation to get rid of the denominator and convert any $\sin^2\theta$ terms to terms involving $\cos^2\theta$.
ii.
'Hence' means you should use the result from part b)i. Solve the quadratic equation to find potential solutions for θ (ruling out any whose *cosine* is not between 1 and -1), but find all solutions within the modified range for $3x$, and finally divide all solutions by 3 to generate solutions for x.