For this paper you must have:
• an 8-page answer book
• the blue AQA booklet of formulae and statistical tables.
You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
• Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
• Write the information required on the front of your answer book. The Examining Body for this paper is AQA. The Paper Reference is MPC4.
• Answer all questions.
• Show all necessary working; otherwise marks for method may be lost.

Information
• The maximum mark for this paper is 75.
• The marks for questions are shown in brackets.

Advice
• Unless stated otherwise, you may quote formulae, without proof, from the booklet.
Answer all questions.

1. (a) Given that \( \frac{3}{9 - x^2} \) can be expressed in the form \( k \left( \frac{1}{3 + x} + \frac{1}{3 - x} \right) \), find the value of the rational number \( k \). \( (2 \text{ marks}) \)

(b) Show that \( \int_1^2 \frac{3}{9 - x^2} \, dx = \frac{1}{2} \ln \left( \frac{a}{b} \right) \), where \( a \) and \( b \) are integers. \( (3 \text{ marks}) \)

2. (a) The polynomial \( f(x) \) is defined by \( f(x) = 2x^3 + 3x^2 - 18x + 8 \).

(i) Use the Factor Theorem to show that \( (2x - 1) \) is a factor of \( f(x) \). \( (2 \text{ marks}) \)

(ii) Write \( f(x) \) in the form \( (2x - 1)(x^2 + px + q) \), where \( p \) and \( q \) are integers. \( (2 \text{ marks}) \)

(iii) Simplify the algebraic fraction \( \frac{4x^2 + 16x}{2x^3 + 3x^2 - 18x + 8} \). \( (2 \text{ marks}) \)

(b) Express the algebraic fraction \( \frac{2x^2}{(x + 5)(x - 3)} \) in the form \( A + \frac{B + Cx}{(x + 5)(x - 3)} \), where \( A, B \) and \( C \) are integers. \( (4 \text{ marks}) \)

3. (a) Obtain the binomial expansion of \( (1 + x)^{\frac{3}{2}} \) up to and including the term in \( x^2 \). \( (2 \text{ marks}) \)

(b) Hence obtain the binomial expansion of \( \sqrt{1 + \frac{3}{2}x} \) up to and including the term in \( x^2 \). \( (2 \text{ marks}) \)

(c) Hence show that \( \sqrt{\frac{2 + 3x}{8}} \approx a + bx + cx^2 \) for small values of \( x \), where \( a, b \) and \( c \) are constants to be found. \( (2 \text{ marks}) \)
4 David is researching changes in the selling price of houses. One particular house was sold on 1 January 1885 for £20. Sixty years later, on 1 January 1945, it was sold for £2000. David proposes a model

\[ P = Ak^t \]

for the selling price, \( P \), of this house, where \( t \) is the time in years after 1 January 1885 and \( A \) and \( k \) are constants.

(a) (i) Write down the value of \( A \). 

(ii) Show that, to six decimal places, \( k = 1.079775 \). 

(iii) Use the model, with this value of \( k \), to estimate the selling price of this house on 1 January 2008. Give your answer to the nearest £1000. 

(b) For another house, which was sold for £15 on 1 January 1885, David proposes the model

\[ Q = 15 \times 1.082709^t \]

for the selling price, \( Q \), of this house \( t \) years after 1 January 1885. Calculate the year in which, according to these models, these two houses would have had the same selling price.

5 A curve is defined by the parametric equations \( x = 2t + \frac{1}{t^2}, \quad y = 2t - \frac{1}{t^2} \).

(a) At the point \( P \) on the curve, \( t = \frac{1}{2} \).

(i) Find the coordinates of \( P \). 

(ii) Find an equation of the tangent to the curve at \( P \). 

(b) Show that the cartesian equation of the curve can be written as

\[ (x - y)(x + y)^2 = k \]

where \( k \) is an integer.
6. A curve has equation \(3xy - 2y^2 = 4\).

Find the gradient of the curve at the point (2, 1). \(5\) marks

7. (a) (i) Express \(6\sin \theta + 8\cos \theta\) in the form \(R\sin(\theta + \alpha)\), where \(R > 0\) and \(0^\circ < \alpha < 90^\circ\). Give your value for \(\alpha\) to the nearest 0.1°. \(2\) marks

(ii) Hence solve the equation \(6\sin 2x + 8\cos 2x = 7\), giving all solutions to the nearest 0.1° in the interval \(0^\circ < x < 360^\circ\). \(4\) marks

(b) (i) Prove the identity \(\frac{\sin 2x}{1 - \cos 2x} = \frac{1}{\tan x}\). \(4\) marks

(ii) Hence solve the equation \(\frac{\sin 2x}{1 - \cos 2x} = \tan x\), giving all solutions in the interval \(0^\circ < x < 360^\circ\). \(4\) marks

8. Solve the differential equation \(\frac{dy}{dx} = \frac{3\cos 3x}{y}\)

given that \(y = 2\) when \(x = \frac{\pi}{2}\). Give your answer in the form \(y^2 = f(x)\). \(5\) marks

9. The points \(A\) and \(B\) lie on the line \(l_1\) and have coordinates (2, 5, 1) and (4, 1, 0) respectively.

(a) (i) Find the vector \(\overrightarrow{AB}\). \(2\) marks

(ii) Find a vector equation of the line \(l_1\), with parameter \(\lambda\). \(1\) mark

(b) The line \(l_2\) has equation \(r = \begin{bmatrix} 1 \\ -3 \\ -1 \end{bmatrix} + \mu \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}\).

(i) Show that the point \(P(-2, -3, 5)\) lies on \(l_2\). \(2\) marks

(ii) The point \(Q\) lies on \(l_1\) and is such that \(PQ\) is perpendicular to \(l_2\). Find the coordinates of \(Q\). \(6\) marks

END OF QUESTIONS