For this paper you must have:
* an 8-page answer book
* the blue AQA booklet of formulae and statistical tables.
You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
• Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
• Write the information required on the front of your answer book. The Examining Body for this paper is AQA. The Paper Reference is MPC3.
• Answer all questions.
• Show all necessary working; otherwise marks for method may be lost.

Information
• The maximum mark for this paper is 75.
• The marks for questions are shown in brackets.

Advice
• Unless stated otherwise, you may quote formulae, without proof, from the booklet.
Answer all questions.

1. (a) Find $\frac{dy}{dx}$ when:

   (i) \( y = (2x^2 - 5x + 1)^2 \);  
   (ii) \( y = x \cos x \).  

(b) Given that 
   
   \[ y = \frac{x^3}{x - 2} \]

   show that 
   
   \[ \frac{dy}{dx} = \frac{kx^2(x - 3)}{(x - 2)^2} \]

   where \( k \) is a positive integer.

2. (a) Solve the equation \( \cot x = 2 \), giving all values of \( x \) in the interval \( 0 \leq x \leq 2\pi \) in radians to two decimal places.

(b) Show that the equation \( \csc^2 x = \frac{3 \cot x + 4}{2} \) can be written as 
   
   \[ 2 \cot^2 x - 3 \cot x - 2 = 0 \]

(c) Solve the equation \( \csc^2 x = \frac{3 \cot x + 4}{2} \), giving all values of \( x \) in the interval \( 0 \leq x \leq 2\pi \) in radians to two decimal places.
3 The equation
\[ x + (1 + 3x)^{\frac{1}{4}} = 0 \]
has a single root, \( a \).

(a) Show that \( a \) lies between \(-0.33\) and \(-0.32\). (2 marks)

(b) Show that the equation \( x + (1 + 3x)^{\frac{1}{4}} = 0 \) can be rearranged into the form
\[ x = \frac{1}{3}(x^4 - 1) \] (2 marks)

(c) Use the iteration \( x_{n+1} = \frac{(x_n^4 - 1)}{3} \) with \( x_1 = -0.3 \) to find \( x_4 \), giving your answer to three significant figures. (3 marks)

4 The functions \( f \) and \( g \) are defined with their respective domains by
\[ f(x) = x^3, \quad \text{for all real values of } x \]
\[ g(x) = \frac{1}{x - 3}, \quad \text{for real values of } x, x \neq 3 \]

(a) State the range of \( f \). (1 mark)

(b) (i) Find \( fg(x) \). (1 mark)

(ii) Solve the equation \( fg(x) = 64 \). (3 marks)

(c) (i) The inverse of \( g \) is \( g^{-1} \). Find \( g^{-1}(x) \). (3 marks)

(ii) State the range of \( g^{-1} \). (1 mark)

5 (a) (i) Given that \( y = 2x^2 - 8x + 3 \), find \( \frac{dy}{dx} \). (1 mark)

(ii) Hence, or otherwise, find
\[ \int_{4}^{6} \frac{x - 2}{2x^2 - 8x + 3} \, dx \]
giving your answer in the form \( k \ln 3 \), where \( k \) is a rational number. (4 marks)

(b) Use the substitution \( u = 3x - 1 \) to find \( \int x\sqrt{3x - 1} \, dx \), giving your answer in terms of \( x \). (4 marks)

**Turn over for the next question**
6 (a) Sketch the curve with equation \( y = \csc x \) for \( 0 < x < \pi \). (2 marks)

(b) Use the mid-ordinate rule with four strips to find an estimate for \( \int_{0.1}^{0.5} \csc x \, dx \), giving your answer to three significant figures. (4 marks)

7 (a) Describe a sequence of two geometrical transformations that maps the graph of \( y = x^2 \) onto the graph of \( y = 4x^2 - 5 \). (4 marks)

(b) Sketch the graph of \( y = |4x^2 - 5| \), indicating the coordinates of the point where the curve crosses the \( y \)-axis. (3 marks)

(c) (i) Solve the equation \( |4x^2 - 5| = 4 \). (3 marks)

(ii) Hence, or otherwise, solve the inequality \( |4x^2 - 5| \geq 4 \). (2 marks)

8 (a) Given that \( e^{-2x} = 3 \), find the exact value of \( x \). (2 marks)

(b) Use integration by parts to find \( \int xe^{-2x} \, dx \). (4 marks)

(c) A curve has equation \( y = e^{-2x} + 6x \).

(i) Find the exact values of the coordinates of the stationary point of the curve. (4 marks)

(ii) Determine the nature of the stationary point. (2 marks)

(iii) The region \( R \) is bounded by the curve \( y = e^{-2x} + 6x \), the \( x \)-axis and the lines \( x = 0 \) and \( x = 1 \).

Find the volume of the solid formed when \( R \) is rotated through \( 2\pi \) radians about the \( x \)-axis, giving your answer to three significant figures. (5 marks)

END OF QUESTIONS