General Certificate of Education
January 2008
Advanced Subsidiary Examination

MATHEMATICS **MPC1**
Unit Pure Core 1

Wednesday 9 January 2008 1.30 pm to 3.00 pm

For this paper you must have: • an 8-page answer book • the blue AQA booklet of formulae and statistical tables. You must **not** use a calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The use of calculators (scientific and graphics) is **not** permitted.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The triangle ABC has vertices $A(-2, 3)$, $B(4, 1)$ and $C(2, -5)$.

(a) Find the coordinates of the mid-point of BC. *(2 marks)*

(b) (i) Find the gradient of AB, in its simplest form. *(2 marks)*

(ii) Hence find an equation of the line AB, giving your answer in the form $x + qy = r$, where q and r are integers. *(2 marks)*

(iii) Find an equation of the line passing through C which is parallel to AB. *(2 marks)*

(c) Prove that angle ABC is a right angle. *(3 marks)*

2 The curve with equation $y = x^4 - 32x + 5$ has a single stationary point, M.

(a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. *(3 marks)*

(b) Hence find the x-coordinate of M. *(3 marks)*

(c) (i) Find $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$. *(1 mark)*

(ii) Hence, or otherwise, determine whether M is a maximum or a minimum point. *(2 marks)*

(d) Determine whether the curve is increasing or decreasing at the point on the curve where $x = 0$. *(2 marks)*

3 (a) Express $5\sqrt{8} + \dfrac{6}{\sqrt{2}}$ in the form $n\sqrt{2}$, where n is an integer. *(3 marks)*

(b) Express $\dfrac{\sqrt{2} + 2}{3\sqrt{2} - 4}$ in the form $c\sqrt{2} + d$, where c and d are integers. *(4 marks)*

4 A circle with centre C has equation $x^2 + y^2 - 10y + 20 = 0$.

(a) By completing the square, express this equation in the form

$$x^2 + (y - b)^2 = k$$ *(2 marks)*

(b) Write down:

(i) the coordinates of C; *(1 mark)*

(ii) the radius of the circle, leaving your answer in surd form. *(1 mark)*

(c) A line has equation $y = 2x$.

(i) Show that the x-coordinate of any point of intersection of the line and the circle satisfies the equation $x^2 - 4x + 4 = 0$. *(2 marks)*

(ii) Hence show that the line is a tangent to the circle and find the coordinates of the point of contact, P. *(3 marks)*

(d) Prove that the point $Q(-1, 4)$ lies inside the circle. *(2 marks)*

5 (a) Factorise $9 - 8x - x^2$. *(2 marks)*

(b) Show that $25 - (x + 4)^2$ can be written as $9 - 8x - x^2$. *(1 mark)*

(c) A curve has equation $y = 9 - 8x - x^2$.

(i) Write down the equation of its line of symmetry. *(1 mark)*

(ii) Find the coordinates of its vertex. *(2 marks)*

(iii) Sketch the curve, indicating the values of the intercepts on the x-axis and the y-axis. *(3 marks)*

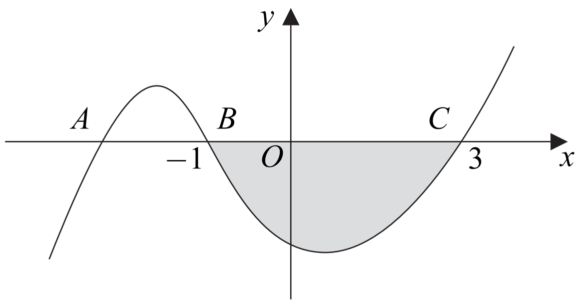
Turn over for the next question

6 (a) The polynomial $\mathrm{p}(x)$ is given by $\mathrm{p}(x) = x^3 - 7x - 6$.

(i) Use the Factor Theorem to show that $x + 1$ is a factor of $\mathrm{p}(x)$. *(2 marks)*

(ii) Express $\mathrm{p}(x) = x^3 - 7x - 6$ as the product of three linear factors. *(3 marks)*

(b) The curve with equation $y = x^3 - 7x - 6$ is sketched below.



The curve cuts the x-axis at the point A and the points $B(-1, 0)$ and $C(3, 0)$.

(i) State the coordinates of the point A. *(1 mark)*

(ii) Find $\displaystyle\int_{-1}^{3} (x^3 - 7x - 6)\,\mathrm{d}x$. *(5 marks)*

(iii) Hence find the area of the shaded region bounded by the curve $y = x^3 - 7x - 6$ and the x-axis between B and C. *(1 mark)*

(iv) Find the gradient of the curve $y = x^3 - 7x - 6$ at the point B. *(3 marks)*

(v) Hence find an equation of the normal to the curve at the point B. *(3 marks)*

7 The curve C has equation $y = x^2 + 7$. The line L has equation $y = k(3x + 1)$, where k is a constant.

(a) Show that the x-coordinates of any points of intersection of the line L with the curve C satisfy the equation

$$x^2 - 3kx + 7 - k = 0$$ *(1 mark)*

(b) The curve C and the line L intersect in two distinct points. Show that

$$9k^2 + 4k - 28 > 0$$ *(3 marks)*

(c) Solve the inequality $9k^2 + 4k - 28 > 0$. *(4 marks)*

END OF QUESTIONS

Copyright © 2008 AQA and its licensors. All rights reserved.