General Certificate of Education
January 2006
Advanced Level Examination

ASSESSMENT and QUALIFICATIONS ALLIANCE

MATHEMATICS **MPC3**
Unit Pure Core 3

Wednesday 25 January 2006 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables
- an insert for use in Question 6 (enclosed)

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC3.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- Fill in the boxes at the top of the insert.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 (a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ when $y = \tan 3x$. *(2 marks)*

(b) Given that $y = \dfrac{3x+1}{2x+1}$, show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{(2x+1)^2}$. *(3 marks)*

2 Use Simpson's rule with 5 ordinates (4 strips) to find an approximation to

$$\int_1^3 \frac{1}{\sqrt{1+x^3}}\,\mathrm{d}x$$

giving your answer to three significant figures. *(4 marks)*

3 (a) (i) Given that $\mathrm{f}(x) = x^4 + 2x$, find $\mathrm{f}'(x)$. *(1 mark)*

(ii) Hence, or otherwise, find $\displaystyle\int \frac{2x^3+1}{x^4+2x}\,\mathrm{d}x$. *(2 marks)*

(b) (i) Use the substitution $u = 2x + 1$ to show that

$$\int x\sqrt{2x+1}\,\mathrm{d}x = \frac{1}{4}\int \left(u^{\frac{3}{2}} - u^{\frac{1}{2}}\right)\mathrm{d}u \qquad \textit{(3 marks)}$$

(ii) Hence show that $\displaystyle\int_0^4 x\sqrt{2x+1}\,\mathrm{d}x = 19.9$ correct to three significant figures. *(4 marks)*

4 It is given that $2\mathrm{cosec}^2 x = 5 - 5\cot x$.

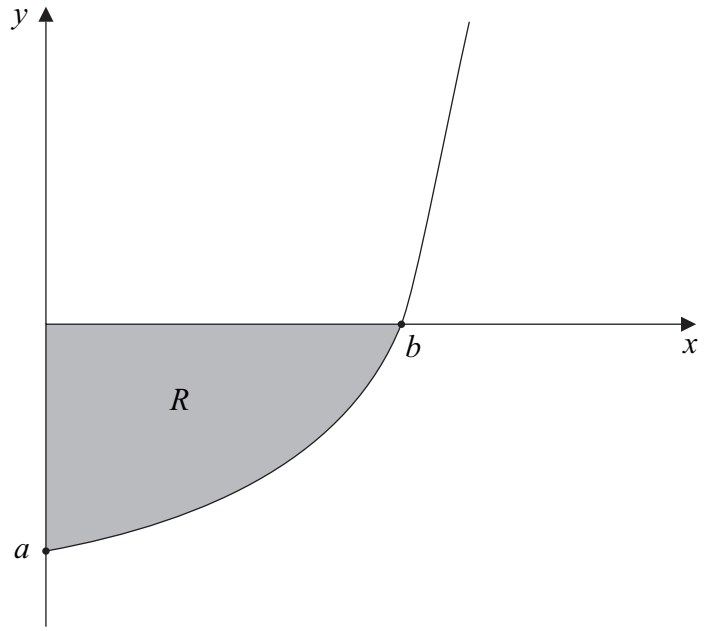
(a) Show that the equation $2\mathrm{cosec}^2 x = 5 - 5\cot x$ can be written in the form

$$2\cot^2 x + 5\cot x - 3 = 0 \qquad \textit{(2 marks)}$$

(b) Hence show that $\tan x = 2$ or $\tan x = -\frac{1}{3}$. *(2 marks)*

(c) Hence, or otherwise, solve the equation $2\mathrm{cosec}^2 x = 5 - 5\cot x$, giving all values of x in radians to one decimal place in the interval $-\pi < x \leqslant \pi$. *(3 marks)*

5 The diagram shows part of the graph of $y = e^{2x} - 9$. The graph cuts the coordinate axes at $(0, a)$ and $(b, 0)$.



(a) State the value of a, and show that $b = \ln 3$. *(3 marks)*

(b) Show that $y^2 = e^{4x} - 18e^{2x} + 81$. *(1 mark)*

(c) The shaded region R is rotated through 360° about the x-axis. Find the volume of the solid formed, giving your answer in the form $\pi(p \ln 3 + q)$, where p and q are integers. *(6 marks)*

(d) Sketch the curve with equation $y = |e^{2x} - 9|$ for $x \geqslant 0$. *(2 marks)*

Turn over for the next question

6 [**Figure 1**, printed on the insert, is provided for use in this question.]

The curve $y = x^3 + 4x - 3$ intersects the x-axis at the point A where $x = \alpha$.

(a) Show that α lies between 0.5 and 1.0. *(2 marks)*

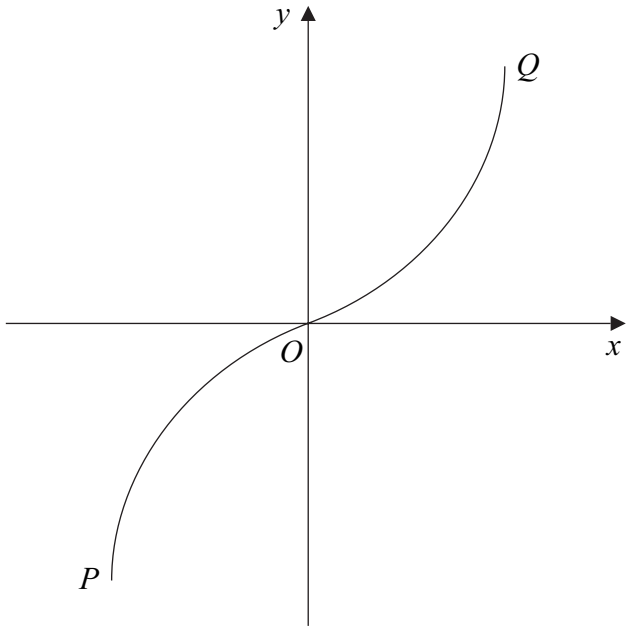
(b) Show that the equation $x^3 + 4x - 3 = 0$ can be rearranged into the form $x = \dfrac{3 - x^3}{4}$. *(1 mark)*

(c) (i) Use the iteration $x_{n+1} = \dfrac{3 - x_n^{\,3}}{4}$ with $x_1 = 0.5$ to find x_3, giving your answer to two decimal places. *(3 marks)*

(ii) The sketch on **Figure 1** shows parts of the graphs of $y = \dfrac{3 - x^3}{4}$ and $y = x$, and the position of x_1.

On **Figure 1**, draw a cobweb or staircase diagram to show how convergence takes place, indicating the positions of x_2 and x_3 on the x-axis. *(3 marks)*

7 (a) The sketch shows the graph of $y = \sin^{-1} x$.



Write down the coordinates of the points P and Q, the end-points of the graph. *(2 marks)*

(b) Sketch the graph of $y = -\sin^{-1}(x - 1)$. *(3 marks)*

8 The functions f and g are defined with their respective domains by

$$\mathrm{f}(x) = x^2 \qquad \text{for all real values of } x$$

$$\mathrm{g}(x) = \frac{1}{x+2} \qquad \text{for real values of } x, \quad x \neq -2$$

(a) State the range of f. *(1 mark)*

(b) (i) Find $\mathrm{fg}(x)$. *(1 mark)*

(ii) Solve the equation $\mathrm{fg}(x) = 4$. *(4 marks)*

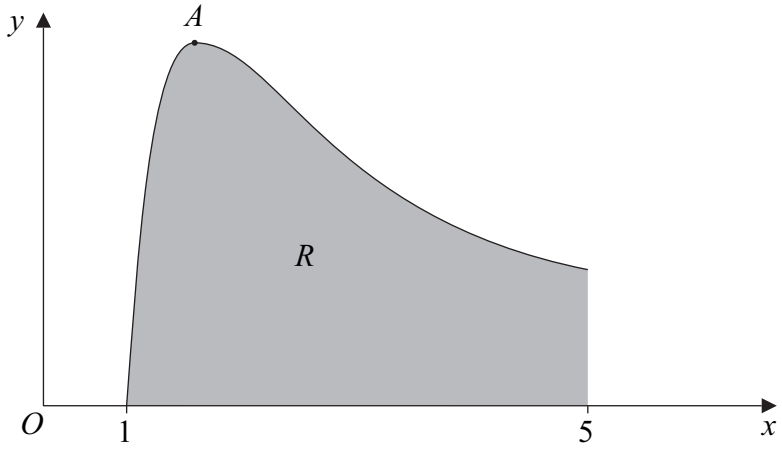
(c) (i) Explain why the function f does **not** have an inverse. *(1 mark)*

(ii) The inverse of g is g^{-1}. Find $\mathrm{g}^{-1}(x)$. *(3 marks)*

9 (a) Given that $y = x^{-2} \ln x$, show that $\dfrac{dy}{dx} = \dfrac{1 - 2\ln x}{x^3}$. *(4 marks)*

(b) Using integration by parts, find $\int x^{-2} \ln x \, dx$. *(4 marks)*

(c) The sketch shows the graph of $y = x^{-2} \ln x$.



(i) Using the answer to part (a), find, in terms of e, the x-coordinate of the stationary point A. *(2 marks)*

(ii) The region R is bounded by the curve, the x-axis and the line $x = 5$. Using your answer to part (b), show that the area of R is

$$\frac{1}{5}(4 - \ln 5)$$ *(3 marks)*

END OF QUESTIONS

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Surname		Other Names	
Centre Number			Candidate Number				
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Insert

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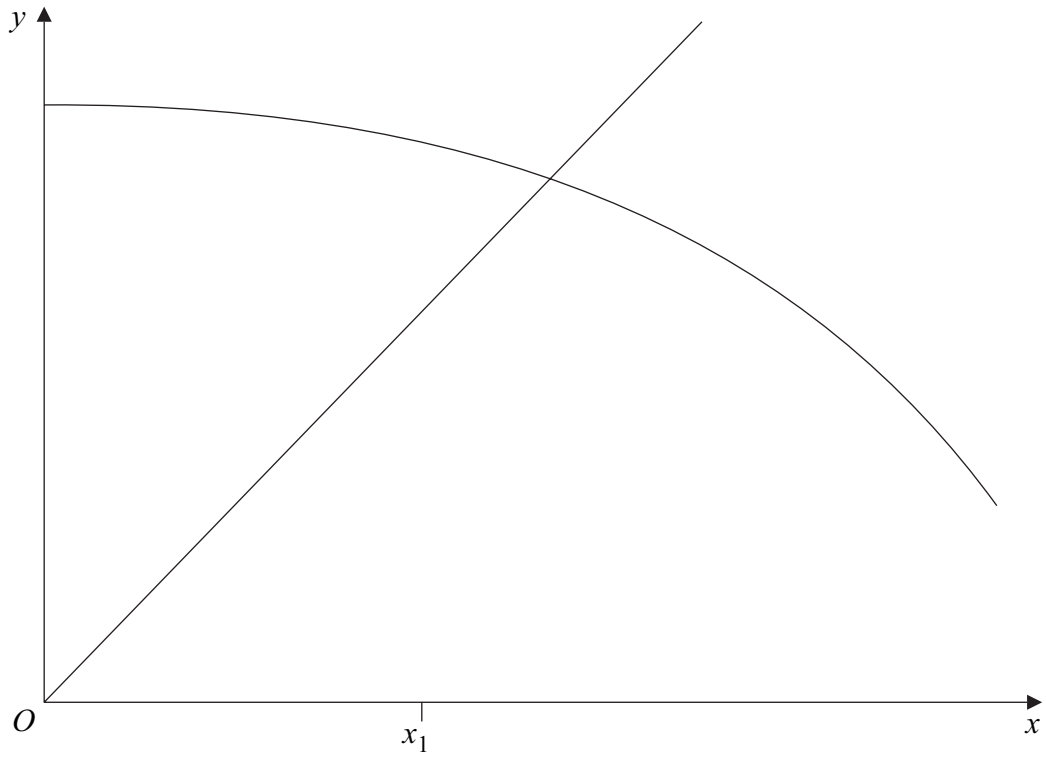
Insert for use in **Question 6**.

Fill in the boxes at the top of this page.

Attach this insert securely to your answer book.

Turn over for Figure 1

Figure 1 (for Question 6)



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