

General Certificate of Education  
January 2007  
Advanced Level Examination



**MATHEMATICS**  
**Unit Further Pure 2**

**MFP2**

Thursday 1 February 2007 9.00 am to 10.30 am

**For this paper you must have:**

- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

**Instructions**

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP2.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

**Information**

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

**Advice**

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

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Answer **all** questions.

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- 1 (a) Given that

$$4 \cosh^2 x = 7 \sinh x + 1$$

find the two possible values of  $\sinh x$ . (4 marks)

- (b) Hence obtain the two possible values of  $x$ , giving your answers in the form  $\ln p$ . (3 marks)

- 2 (a) Sketch on one diagram:

(i) the locus of points satisfying  $|z - 4 + 2i| = 2$ ; (3 marks)

(ii) the locus of points satisfying  $|z| = |z - 3 - 2i|$ . (3 marks)

- (b) Shade on your sketch the region in which

both  $|z - 4 + 2i| \leq 2$

and  $|z| \leq |z - 3 - 2i|$  (2 marks)

- 3 The cubic equation

$$z^3 + 2(1 - i)z^2 + 32(1 + i) = 0$$

has roots  $\alpha$ ,  $\beta$  and  $\gamma$ .

- (a) It is given that  $\alpha$  is of the form  $ki$ , where  $k$  is real. By substituting  $z = ki$  into the equation, show that  $k = 4$ . (5 marks)
- (b) Given that  $\beta = -4$ , find the value of  $\gamma$ . (2 marks)

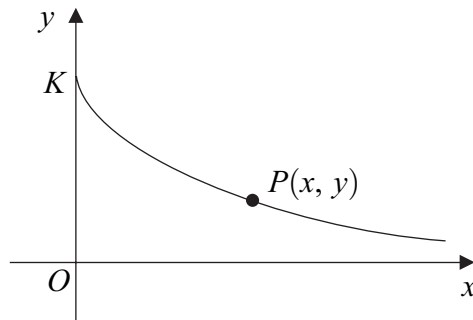
4 (a) Given that  $y = \operatorname{sech} t$ , show that:

(i)  $\frac{dy}{dt} = -\operatorname{sech} t \tanh t$ ; (3 marks)

(ii)  $\left(\frac{dy}{dt}\right)^2 = \operatorname{sech}^2 t - \operatorname{sech}^4 t$ . (2 marks)

(b) The diagram shows a sketch of part of the curve given parametrically by

$$x = t - \tanh t \quad y = \operatorname{sech} t$$



The curve meets the  $y$ -axis at the point  $K$ , and  $P(x, y)$  is a general point on the curve. The arc length  $KP$  is denoted by  $s$ . Show that:

(i)  $\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = \tanh^2 t$ ; (4 marks)

(ii)  $s = \ln \cosh t$ ; (3 marks)

(iii)  $y = e^{-s}$ . (2 marks)

(c) The arc  $KP$  is rotated through  $2\pi$  radians about the  $x$ -axis. Show that the surface area generated is

$$2\pi(1 - e^{-s}) \quad \text{(4 marks)}$$

**Turn over for the next question**

**Turn over ►**

- 5 (a) Prove by induction that, if  $n$  is a positive integer,

$$(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta \quad (5 \text{ marks})$$

- (b) Find the value of  $\left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}\right)^6$ . (2 marks)

- (c) Show that

$$(\cos \theta + i \sin \theta)(1 + \cos \theta - i \sin \theta) = 1 + \cos \theta + i \sin \theta \quad (3 \text{ marks})$$

- (d) Hence show that

$$\left(1 + \cos \frac{\pi}{6} + i \sin \frac{\pi}{6}\right)^6 + \left(1 + \cos \frac{\pi}{6} - i \sin \frac{\pi}{6}\right)^6 = 0 \quad (4 \text{ marks})$$

- 6 (a) Find the three roots of  $z^3 = 1$ , giving the non-real roots in the form  $e^{i\theta}$ , where  $-\pi < \theta \leq \pi$ . (2 marks)

- (b) Given that  $\omega$  is one of the non-real roots of  $z^3 = 1$ , show that

$$1 + \omega + \omega^2 = 0 \quad (2 \text{ marks})$$

- (c) By using the result in part (b), or otherwise, show that:

$$(i) \quad \frac{\omega}{\omega + 1} = -\frac{1}{\omega}; \quad (2 \text{ marks})$$

$$(ii) \quad \frac{\omega^2}{\omega^2 + 1} = -\omega; \quad (1 \text{ mark})$$

$$(iii) \quad \left(\frac{\omega}{\omega + 1}\right)^k + \left(\frac{\omega^2}{\omega^2 + 1}\right)^k = (-1)^k 2 \cos \frac{2}{3}k\pi, \text{ where } k \text{ is an integer.} \quad (5 \text{ marks})$$

- 7 (a) Use the identity  $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$  with  $A = (r + 1)x$  and  $B = rx$  to show that

$$\tan rx \tan(r + 1)x = \frac{\tan(r + 1)x}{\tan x} - \frac{\tan rx}{\tan x} - 1 \quad (4 \text{ marks})$$

- (b) Use the method of differences to show that

$$\tan \frac{\pi}{50} \tan \frac{2\pi}{50} + \tan \frac{2\pi}{50} \tan \frac{3\pi}{50} + \dots + \tan \frac{19\pi}{50} \tan \frac{20\pi}{50} = \frac{\tan \frac{2\pi}{5}}{\tan \frac{\pi}{50}} - 20 \quad (5 \text{ marks})$$

**END OF QUESTIONS**

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