

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										



General Certificate of Education  
Advanced Level Examination  
June 2010

# Mathematics

## MFP2

### Unit Further Pure 2

Wednesday 9 June 2010 1.30 pm to 3.00 pm

#### For this paper you must have:

- the blue AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

#### Time allowed

- 1 hour 30 minutes

#### Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

#### Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

#### Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
TOTAL	



J U N 1 0 M F P 2 0 1

Answer **all** questions in the spaces provided.

**1 (a)** Show that

$$9 \sinh x - \cosh x = 4e^x - 5e^{-x} \quad (2 \text{ marks})$$

**(b)** Given that

$$9 \sinh x - \cosh x = 8$$

find the exact value of  $\tanh x$ . (7 marks)

QUESTION  
PART  
REFERENCE



QUESTION  
PART  
REFERENCE

Turn over ►



**2 (a)** Express  $\frac{1}{r(r+2)}$  in partial fractions. (3 marks)

**(b)** Use the method of differences to find

$$\sum_{r=1}^{48} \frac{1}{r(r+2)}$$

giving your answer as a rational number.

(5 marks)

QUESTION  
PART  
REFERENCE



QUESTION  
PART  
REFERENCE



**3** Two loci,  $L_1$  and  $L_2$ , in an Argand diagram are given by

$$L_1 : |z + 1 + 3i| = |z - 5 - 7i|$$

$$L_2 : \arg z = \frac{\pi}{4}$$

**(a)** Verify that the point represented by the complex number  $2 + 2i$  is a point of intersection of  $L_1$  and  $L_2$ . (2 marks)

**(b)** Sketch  $L_1$  and  $L_2$  on one Argand diagram. (5 marks)

**(c)** Shade on your Argand diagram the region satisfying

both  $|z + 1 + 3i| \leq |z - 5 - 7i|$

and  $\frac{\pi}{4} \leq \arg z \leq \frac{\pi}{2}$  (2 marks)

QUESTION  
PART  
REFERENCE



QUESTION  
PART  
REFERENCE

Turn over ►



**4** The roots of the cubic equation

$$z^3 - 2z^2 + pz + 10 = 0$$

are  $\alpha$ ,  $\beta$  and  $\gamma$ .

It is given that  $\alpha^3 + \beta^3 + \gamma^3 = -4$ .

**(a)** Write down the value of  $\alpha + \beta + \gamma$ . (1 mark)

**(b) (i)** Explain why  $\alpha^3 - 2\alpha^2 + p\alpha + 10 = 0$ . (1 mark)

**(ii)** Hence show that

$$\alpha^2 + \beta^2 + \gamma^2 = p + 13$$
 (4 marks)

**(iii)** Deduce that  $p = -3$ . (2 marks)

**(c) (i)** Find the real root  $\alpha$  of the cubic equation  $z^3 - 2z^2 - 3z + 10 = 0$ . (2 marks)

**(ii)** Find the values of  $\beta$  and  $\gamma$ . (3 marks)

QUESTION  
PART  
REFERENCE





QUESTION  
PART  
REFERENCE

Turn over ►



**5 (a)** Using the identities

$$\cosh^2 t - \sinh^2 t = 1, \quad \tanh t = \frac{\sinh t}{\cosh t} \quad \text{and} \quad \operatorname{sech} t = \frac{1}{\cosh t}$$

show that:

$$(i) \quad \tanh^2 t + \operatorname{sech}^2 t = 1; \quad (2 \text{ marks})$$

$$(ii) \quad \frac{d}{dt}(\tanh t) = \operatorname{sech}^2 t; \quad (3 \text{ marks})$$

$$(iii) \quad \frac{d}{dt}(\operatorname{sech} t) = -\operatorname{sech} t \tanh t. \quad (3 \text{ marks})$$

**(b)** A curve  $C$  is given parametrically by

$$x = \operatorname{sech} t, \quad y = 4 - \tanh t$$

(i) Show that the arc length,  $s$ , of  $C$  between the points where  $t = 0$  and  $t = \frac{1}{2}\ln 3$  is given by

$$s = \int_0^{\frac{1}{2}\ln 3} \operatorname{sech} t \, dt \quad (4 \text{ marks})$$

(ii) Using the substitution  $u = e^t$ , find the exact value of  $s$ . (6 marks)

QUESTION  
PART  
REFERENCE



QUESTION  
PART  
REFERENCE



QUESTION  
PART  
REFERENCE

Turn over ►



**6 (a)** Show that  $\frac{1}{(k+2)!} - \frac{k+1}{(k+3)!} = \frac{2}{(k+3)!}$ . (2 marks)

**(b)** Prove by induction that, for all positive integers  $n$ ,

$$\sum_{r=1}^n \frac{r \times 2^r}{(r+2)!} = 1 - \frac{2^{n+1}}{(n+2)!} \quad (6 \text{ marks})$$

QUESTION  
PART  
REFERENCE



QUESTION  
PART  
REFERENCE



**7 (a) (i)** Express each of the numbers  $1 + \sqrt{3}i$  and  $1 - i$  in the form  $re^{i\theta}$ , where  $r > 0$ .  
(3 marks)

**(ii)** Hence express

$$(1 + \sqrt{3}i)^8(1 - i)^5$$

in the form  $re^{i\theta}$ , where  $r > 0$ . (3 marks)

**(b)** Solve the equation

$$z^3 = (1 + \sqrt{3}i)^8(1 - i)^5$$

giving your answers in the form  $a\sqrt{2}e^{i\theta}$ , where  $a$  is a positive integer and  $-\pi < \theta \leq \pi$ . (4 marks)

QUESTION  
PART  
REFERENCE







QUESTION  
PART  
REFERENCE



QUESTION  
PART  
REFERENCE

END OF QUESTIONS



**There are no questions printed on this page**

**DO NOT WRITE ON THIS PAGE  
ANSWER IN THE SPACES PROVIDED**

