

General Certificate of Education  
June 2008  
Advanced Level Examination

**MATHEMATICS**  
**Unit Further Pure 2**

**MFP2**



Thursday 15 May 2008 9.00 am to 10.30 am

**For this paper you must have:**

- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

**Instructions**

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP2.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

**Information**

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

**Advice**

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

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Answer **all** questions.

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- 1 (a) Express

$$5 \sinh x + \cosh x$$

in the form  $Ae^x + Be^{-x}$ , where  $A$  and  $B$  are integers. (2 marks)

- (b) Solve the equation

$$5 \sinh x + \cosh x + 5 = 0$$

giving your answer in the form  $\ln a$ , where  $a$  is a rational number. (4 marks)

- 2 (a) Given that

$$\frac{1}{r(r+1)(r+2)} = \frac{A}{r(r+1)} + \frac{B}{(r+1)(r+2)}$$

show that  $A = \frac{1}{2}$  and find the value of  $B$ . (3 marks)

- (b) Use the method of differences to find

$$\sum_{r=10}^{98} \frac{1}{r(r+1)(r+2)}$$

giving your answer as a rational number. (4 marks)

### 3 The cubic equation

$$z^3 + qz + (18 - 12i) = 0$$

where  $q$  is a complex number, has roots  $\alpha$ ,  $\beta$  and  $\gamma$ .

(a) Write down the value of:

(i)  $\alpha\beta\gamma$ ; (1 mark)

(ii)  $\alpha + \beta + \gamma$ . (1 mark)

(b) Given that  $\beta + \gamma = 2$ , find the value of:

(i)  $\alpha$ ; (1 mark)

(ii)  $\beta\gamma$ ; (2 marks)

(iii)  $q$ . (3 marks)

(c) Given that  $\beta$  is of the form  $ki$ , where  $k$  is real, find  $\beta$  and  $\gamma$ . (4 marks)

### 4 (a) A circle $C$ in the Argand diagram has equation

$$|z + 5 - i| = \sqrt{2}$$

Write down its radius and the complex number representing its centre. (2 marks)

(b) A half-line  $L$  in the Argand diagram has equation

$$\arg(z + 2i) = \frac{3\pi}{4}$$

Show that  $z_1 = -4 + 2i$  lies on  $L$ . (2 marks)

(c) (i) Show that  $z_1 = -4 + 2i$  also lies on  $C$ . (1 mark)

(ii) Hence show that  $L$  touches  $C$ . (3 marks)

(iii) Sketch  $L$  and  $C$  on one Argand diagram. (2 marks)

(d) The complex number  $z_2$  lies on  $C$  and is such that  $\arg(z_2 + 2i)$  has as great a value as possible.

Indicate the position of  $z_2$  on your sketch. (2 marks)

Turn over ►

- 5 (a) Use the definition  $\cosh x = \frac{1}{2}(e^x + e^{-x})$  to show that  $\cosh 2x = 2 \cosh^2 x - 1$ . (2 marks)

- (b) (i) The arc of the curve  $y = \cosh x$  between  $x = 0$  and  $x = \ln a$  is rotated through  $2\pi$  radians about the  $x$ -axis. Show that  $S$ , the surface area generated, is given by

$$S = 2\pi \int_0^{\ln a} \cosh^2 x \, dx \quad (3 \text{ marks})$$

- (ii) Hence show that

$$S = \pi \left( \ln a + \frac{a^4 - 1}{4a^2} \right) \quad (5 \text{ marks})$$

- 6 By using the substitution  $u = x - 2$ , or otherwise, find the exact value of

$$\int_{-1}^5 \frac{dx}{\sqrt{32 + 4x - x^2}} \quad (5 \text{ marks})$$

- 7 (a) Explain why  $n(n + 1)$  is a multiple of 2 when  $n$  is an integer. (1 mark)

- (b) (i) Given that

$$f(n) = n(n^2 + 5)$$

show that  $f(k + 1) - f(k)$ , where  $k$  is a positive integer, is a multiple of 6. (4 marks)

- (ii) Prove by induction that  $f(n)$  is a multiple of 6 for all integers  $n \geq 1$ . (4 marks)

8 (a) (i) Expand

$$\left(z + \frac{1}{z}\right)\left(z - \frac{1}{z}\right) \quad (1 \text{ mark})$$

(ii) Hence, or otherwise, expand

$$\left(z + \frac{1}{z}\right)^4 \left(z - \frac{1}{z}\right)^2 \quad (3 \text{ marks})$$

(b) (i) Use De Moivre's theorem to show that if  $z = \cos \theta + i \sin \theta$  then

$$z^n + \frac{1}{z^n} = 2 \cos n\theta \quad (3 \text{ marks})$$

(ii) Write down a corresponding result for  $z^n - \frac{1}{z^n}$ . (1 mark)

(c) Hence express  $\cos^4 \theta \sin^2 \theta$  in the form

$$A \cos 6\theta + B \cos 4\theta + C \cos 2\theta + D$$

where  $A, B, C$  and  $D$  are rational numbers. (4 marks)

(d) Find  $\int \cos^4 \theta \sin^2 \theta \, d\theta$ . (2 marks)

**END OF QUESTIONS**

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