

General Certificate of Education
June 2007
Advanced Level Examination



MATHEMATICS
Unit Further Pure 2

MFP2

Tuesday 26 June 2007 1.30 pm to 3.00 pm

For this paper you must have:

- an 8-page answer book
 - the **blue** AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP2.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

- 1 (a) Given that $f(r) = (r - 1)r^2$, show that

$$f(r + 1) - f(r) = r(3r + 1) \quad (3 \text{ marks})$$

- (b) Use the method of differences to find the value of

$$\sum_{r=50}^{99} r(3r + 1) \quad (4 \text{ marks})$$

- 2 The cubic equation

$$z^3 + pz^2 + 6z + q = 0$$

has roots α , β and γ .

- (a) Write down the value of $\alpha\beta + \beta\gamma + \gamma\alpha$. (1 mark)

- (b) Given that p and q are real and that $\alpha^2 + \beta^2 + \gamma^2 = -12$:

- (i) explain why the cubic equation has two non-real roots and one real root; (2 marks)

- (ii) find the value of p . (4 marks)

- (c) One root of the cubic equation is $-1 + 3i$.

Find:

- (i) the other two roots; (3 marks)

- (ii) the value of q . (2 marks)

- 3 Use De Moivre's Theorem to find the smallest positive angle θ for which

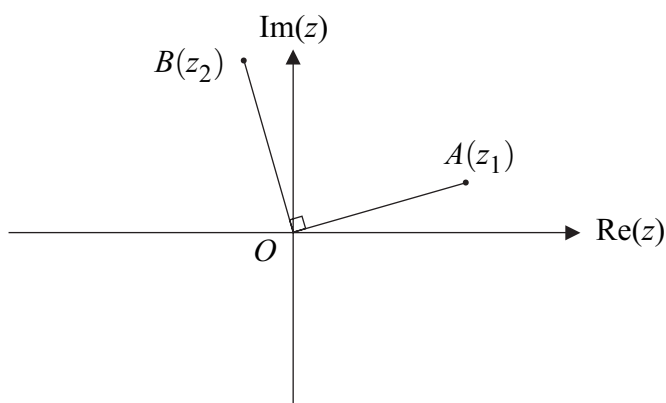
$$(\cos \theta + i \sin \theta)^{15} = -i \quad (5 \text{ marks})$$

- 4 (a) Differentiate $x \tan^{-1} x$ with respect to x . (2 marks)

(b) Show that

$$\int_0^1 \tan^{-1} x \, dx = \frac{\pi}{4} - \ln \sqrt{2} \quad (5 \text{ marks})$$

- 5 The sketch shows an Argand diagram. The points A and B represent the complex numbers z_1 and z_2 respectively. The angle $AOB = 90^\circ$ and $OA = OB$.



- (a) Explain why $z_2 = iz_1$. (2 marks)

(b) On a **single** copy of the diagram, draw:

- (i) the locus L_1 of points satisfying $|z - z_2| = |z - z_1|$; (2 marks)

- (ii) the locus L_2 of points satisfying $\arg(z - z_2) = \arg z_1$. (3 marks)

- (c) Find, in terms of z_1 , the complex number representing the point of intersection of L_1 and L_2 . (2 marks)

- 6 (a) Show that

$$\left(1 - \frac{1}{(k+1)^2}\right) \times \frac{k+1}{2k} = \frac{k+2}{2(k+1)} \quad (3 \text{ marks})$$

- (b) Prove by induction that for all integers $n \geq 2$

$$\left(1 - \frac{1}{2^2}\right) \left(1 - \frac{1}{3^2}\right) \left(1 - \frac{1}{4^2}\right) \dots \left(1 - \frac{1}{n^2}\right) = \frac{n+1}{2n} \quad (4 \text{ marks})$$

Turn over for the next question

Turn over ►

7 A curve has equation $y = 4\sqrt{x}$.

- (a) Show that the length of arc s of the curve between the points where $x = 0$ and $x = 1$ is given by

$$s = \int_0^1 \sqrt{\frac{x+4}{x}} \, dx \quad (4 \text{ marks})$$

- (b) (i) Use the substitution $x = 4 \sinh^2 \theta$ to show that

$$\int \sqrt{\frac{x+4}{x}} \, dx = \int 8 \cosh^2 \theta \, d\theta \quad (5 \text{ marks})$$

- (ii) Hence show that

$$s = 4 \sinh^{-1} 0.5 + \sqrt{5} \quad (6 \text{ marks})$$

8 (a) (i) Given that $z^6 - 4z^3 + 8 = 0$, show that $z^3 = 2 \pm 2i$. (2 marks)

- (ii) Hence solve the equation

$$z^6 - 4z^3 + 8 = 0$$

giving your answers in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$. (6 marks)

- (b) Show that, for any real values of k and θ ,

$$(z - ke^{i\theta})(z - ke^{-i\theta}) = z^2 - 2kz \cos \theta + k^2 \quad (2 \text{ marks})$$

- (c) Express $z^6 - 4z^3 + 8$ as the product of three quadratic factors with real coefficients. (3 marks)

END OF QUESTIONS