



**General Certificate of Education**

**Mathematics 6360**

**MFP2**

**Further Pure 2**

**Mark Scheme**

*2008 examination - January series*

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

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## Key to mark scheme and abbreviations used in marking

M	mark is for method		
m or dM	mark is dependent on one or more M marks and is for method		
A	mark is dependent on M or m marks and is for accuracy		
B	mark is independent of M or m marks and is for method and accuracy		
E	mark is for explanation		
✓ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
− <i>x</i> EE	deduct <i>x</i> marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

## No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

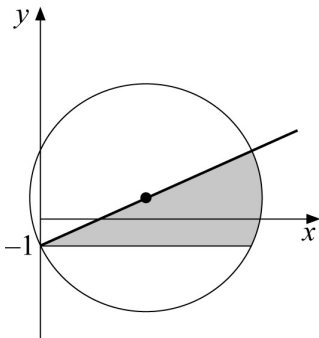
Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**

**MFP2**

<b>Q</b>	<b>Solution</b>	<b>Marks</b>	<b>Total</b>	<b>Comments</b>
<b>1(a)</b>	Any method for finding $r$ or $\theta$ $r = 4\sqrt{2}$ , $\theta = \frac{\pi}{4}$	M1 A1A1	3	
<b>(b)</b>	$z^5 = 4\sqrt{2} e^{\frac{\pi i}{4}}$ $z = \sqrt{2} e^{\frac{\pi i}{20} + \frac{2k\pi i}{5}}$ $z = \sqrt{2} e^{\frac{\pi i}{20}}, \sqrt{2} e^{\frac{9\pi i}{20}}, \sqrt{2} e^{\frac{17\pi i}{20}}, \sqrt{2} e^{\frac{-7\pi i}{20}}, \sqrt{2} e^{\frac{-15\pi i}{20}}$	M1 A1F A1F  A2,1,0 F	5	M1 needs some reference to $a + 2k\pi i$ A1 for $r$ A1 for $\theta$ ] incorrect $r, \theta$ part (a)  Accept $r$ in any form eg $32^{\frac{1}{10}}$ Correct but some answers outside range allow A1 ft incorrect $r, \theta$ in part (a)
	<b>Total</b>		<b>8</b>	
<b>2(a)</b>	Attempt to expand $(2r+1)^3 - (2r-1)^3$ $(2r+1)^3$ or $(2r-1)^3$ expanded $24r^2 + 2$	M1 A1 A1	3	AG
<b>(b)</b>	$r=1 \quad 3^3 - 1^3 = 24 \times 1^2 + 2$ $r=2 \quad 5^3 - 3^3 = 24 \times 2^2 + 2$  $r=n \quad (2n+1)^3 - (2n-1)^3 = 24 \times n^2 + 2$  $(2n+1)^3 - 1 = 24 \sum_{r=1}^n r^2 + 2n$  $8n^3 + 12n^2 + 6n + 1 - 1 - 2n = 24 \sum_{r=1}^n r^2$  $8n^3 + 12n^2 + 4n = 24 \sum_{r=1}^n r^2$  $\sum_{r=1}^n r^2 = \frac{1}{6} n(n+1)(2n+1)$	M1A1    A1  M1  A1  A1	6	3 rows seen Do not allow M1 for $(2n+1)^3 - 1$ not equal to anything   M1 for multiplication of bracket or taking $(2n+1)$ out as a factor  CAO  AG
	<b>Total</b>		<b>9</b>	

**MFP2 (cont)**

<b>Q</b>	<b>Solution</b>	<b>Marks</b>	<b>Total</b>	<b>Comments</b>
<b>3(a)(i)</b>	$z = -i \quad  -2\sqrt{3} - 2i  = \sqrt{12 + 4} = 4$	M1 A1	2	$ -2\sqrt{3} - 2i $ 4
<b>(ii)</b>	Centre of circle is $2\sqrt{3} + i$  Substitute into line $\arg(2\sqrt{3} + 2i) = \frac{\pi}{6}$ shown	B1  M1 A1	3	Do not accept $(2\sqrt{3}, 1)$ unless attempt to solve using trig
<b>(b)</b>				
	Circle: centre correct through $(0, -1)$ Half line: through $(0, -1)$ through centre of circle	B1 B1 B1 B1	4	
<b>(c)</b>	Shading inside circle and below line Bounded by $y = -1$	B1F B1	2	
<b>Total</b>			<b>11</b>	
<b>4(a)(i)</b>	$\sum \alpha = -i$	B1	1	
<b>(ii)</b>	$\sum \alpha\beta = 3$	B1	1	
<b>(iii)</b>	$\alpha\beta\gamma = 1 + i$	B1	1	
<b>(b)(i)</b>	$\sum \alpha^2 = (\sum \alpha)^2 - 2\sum \alpha\beta$ used $= (-i)^2 - 2 \times 3$ $= -7$	M1 A1F A1F	3	Allow if sign error or 2 missing  ft errors in (a)
<b>(ii)</b>	$\sum \alpha^2 \beta^2 = (\sum \alpha\beta)^2 - 2\sum \alpha\beta \cdot \beta\gamma$ $= (\sum \alpha\beta)^2 - 2\alpha\beta\gamma \sum \alpha$ $= 9 - 2(1+i)(-i)$ $= 7 + 2i$	M1 A1 A1F A1F	4	Allow if sign error in 2 missing  ft errors in (a) ft errors in (a)
<b>(iii)</b>	$\alpha^2 \beta^2 \gamma^2 = (1+i)^2 = 2i$	M1 A1F	2	ft sign error in $\alpha\beta\gamma$
<b>(c)</b>	$z^3 + 7z^2 + (7 + 2i)z - 2i = 0$	B1F B1F	2	Correct numbers in correct places Correct signs
<b>Total</b>			<b>14</b>	

**MFP2 (cont)**

<b>Q</b>	<b>Solution</b>	<b>Marks</b>	<b>Total</b>	<b>Comments</b>
<b>5</b>	<p>Assume result true for <math>n = k</math></p> <p>Then <math>\sum_{r=1}^{k+1} (r^2 + 1)r!</math></p> <p><math>= ((k+1)^2 + 1)(k+1)! + k(k+1)!</math></p> <p>Taking out <math>(k+1)!</math> as factor</p> <p><math>= (k+1)!(k^2 + 2k + 1 + k)</math></p> <p><math>= (k+1)(k+2)!</math></p> <p><math>k=1</math> shown <math>(1^2 + 1)1! = 2</math>  <math>1 \times 2! = 2</math> ]</p> <p><math>P_k \Rightarrow P_{k+1}</math> and <math>P_1</math> true</p>	<p>M1A1</p> <p>m1</p> <p>A1</p> <p>A1</p> <p>B1</p> <p>E1</p>	<p>7</p>	<p>If all 6 marks earned</p>
	<b>Total</b>		<b>7</b>	
<b>6(a)(i)</b>	<p><math>\cos 3\theta + i \sin 3\theta = (\cos \theta + i \sin \theta)^3</math></p> <p><math>= \cos^3 \theta + 3i \cos^2 \theta \sin \theta + 3i^2 \cos \theta \sin^2 \theta + i^3 \sin^3 \theta</math></p> <p>Real parts: <math>\cos 3\theta = \cos^3 \theta - 3 \cos \theta \sin^2 \theta</math></p>	<p>M1</p> <p>A1</p> <p>A1</p>	<p>3</p>	<p>AG</p>
<b>(ii)</b>	<p>Imaginary parts:</p> <p><math>\sin 3\theta = 3 \cos^2 \theta \sin \theta - \sin^3 \theta</math></p>	<p>A1F</p>	<p>1</p>	
<b>(iii)</b>	<p><math>\tan 3\theta = \frac{\sin 3\theta}{\cos 3\theta}</math></p> <p><math>= \frac{3 \cos^2 \theta \sin \theta - \sin^3 \theta}{\cos^3 \theta - 3 \sin^2 \theta \cos \theta}</math></p> <p><math>= \frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta}</math></p> <p><math>= \frac{\tan^3 \theta - 3 \tan \theta}{3 \tan^2 \theta - 1}</math></p>	<p>M1</p> <p>A1F</p> <p>A1</p>	<p>3</p>	<p>Used</p> <p>Error in <math>\sin 3\theta</math></p> <p>AG</p>
<b>(b)(i)</b>	<p><math>\tan \frac{3\pi}{12} = 1</math></p> <p><math>\tan \frac{\pi}{12}</math> is a root of <math>1 = \frac{x^3 - 3x}{3x^2 - 1}</math></p> <p><math>x^3 - 3x^2 - 3x + 1 = 0</math></p>	<p>B1</p> <p>M1</p> <p>A1</p>	<p>3</p>	<p>Used (possibly implied)</p> <p>Must be hence</p>
<b>(ii)</b>	<p>Other roots are <math>\tan \frac{5\pi}{12}, \tan \frac{9\pi}{12}</math></p>	<p>B1B1</p>	<p>2</p>	
<b>(c)</b>	<p><math>\tan \frac{\pi}{12} + \tan \frac{5\pi}{12} + \tan \frac{9\pi}{12} = 3</math></p> <p><math>\tan \frac{\pi}{12} + \tan \frac{5\pi}{12} = 4</math></p>	<p>M1</p> <p>A1</p>	<p>2</p>	<p>Must be hence</p>
	<b>Total</b>		<b>14</b>	

**MFP2 (cont)**

<b>Q</b>	<b>Solution</b>	<b>Marks</b>	<b>Total</b>	<b>Comments</b>
<b>7(a)</b>	$\frac{dy}{dx} = \frac{1}{\tanh \frac{x}{2}} \dots$ $\operatorname{sech}^2 \frac{x}{2} \dots$ $\frac{1}{2}$ $= \frac{1}{2 \frac{\sinh \frac{x}{2}}{\cosh \frac{x}{2}} \cosh^2 \frac{x}{2}}$ $= \frac{1}{2 \sinh \frac{x}{2} \cosh \frac{x}{2}}$ $= \frac{1}{\sinh x}$ $= \operatorname{cosech} x$ <p><b>Alternative</b></p> $\ln \sinh \frac{x}{2} - \ln \cosh \frac{x}{2}$ $\frac{1}{2} \frac{\cosh \frac{x}{2}}{\sinh \frac{x}{2}} - \frac{1}{2} \frac{\sinh \frac{x}{2}}{\cosh \frac{x}{2}}$ $\frac{\cosh^2 \frac{x}{2} - \sinh^2 \frac{x}{2}}{2 \sinh \frac{x}{2} \cosh \frac{x}{2}}$ <p>Use of <math>\sinh 2A = 2 \sinh A \cosh A</math> result</p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>(B1)</p> <p>(B1B1)</p> <p>(M1)</p> <p>(M1)</p> <p>(A1)</p>	6	<p>OE ie expressing in <math>\sinh \frac{x}{2}</math> and <math>\cosh \frac{x}{2}</math></p> <p>ie use of <math>\sinh 2A = 2 \sinh A \cosh A</math></p> <p>AG</p>
<b>(b)(i)</b>	$s = \int_1^2 \sqrt{1 + \operatorname{cosech}^2 x} \, dx$ $= \int_1^2 \coth x \, dx$	<p>M1</p> <p>A1</p>	2	AG
<b>(ii)</b>	$s = [\ln \sinh x]_1^2$ $= \ln \sinh 2 - \ln \sinh 1$ $= \ln \frac{2 \sinh 1 \cosh 1}{\sinh 1}$ $= \ln(2 \cosh 1)$	<p>M1</p> <p>A1</p> <p>A1F</p> <p>A1</p>	4	<p>needs to be correct</p> <p>must be seen</p> <p>AG</p>
	<b>Total</b>		<b>12</b>	
	<b>TOTAL</b>		<b>75</b>	